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A SPLITTING CRITERION FOR ABELIAN GROUPS

Ladislav BICAN, Praha

Abstract: The purpose of this paper is to present some necesary and sufficient conditions for splitting of an arbitrary mixed abelian group. An example of a non-splitting abelian group G with the torsion part T such that every pure subgroup of finite rank of G containing T splits is given.

Key words: Splitting group, generalized p-height, increasing p-height ordering, basis, generalized p-sequence.

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1. Introduction. The splitting problem is one of the most important and serious problems in abelian group theory. Numerous authors have studied several aspects of this problem. In 1974, an interesting result of Stratton [10] has appeared. Stratton's theorem concerning the groups of finite rank generalizes the previous criteria for aplitting of mixed groups of rank one discoverd independently by A.E. Stratton [9] and the author [2] in 1970. The general criterion for splitting presented here can be used for the characterization of factor-splitting torsionfree groups. The results of this kind will appear elsewhere (see [5]).

By the word "group" we shall always mean an additively written abelian group. If M is a subset of a group G then $\langle M \rangle$ denotes the subgroup of G generated by M. As in

[2], we shall deal with the notions "characteristic" and "type" in mixed groups. In this paper we shall denote by $h_p^G(g)$, $\tau^G(g)$, $\hat{\tau}^G(g)$ the p-height, the characteristic and the type of the element g in the group G, respectively. The rank of a mixed group G with the maximal torsion subgroup T is the rank of the factor-group G/T.

In what follows we shall deal with a mixed group G with maximal torsion subgroup T and $\overline{G}$ will denote the factor-group G/T. The bar over the elements will denote the elements from $\overline{G}$. For the sake of simplicity we shall write briefly $\tau(g)$, $\tau(\overline{g})$, $\hat{\tau}(g)$ etc. in place of $\tau^G(g)$, $\tau^{\overline{G}}(\overline{g})$, $\hat{\tau}^G(g)$ etc. We say that a set $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of elements of G is a basis of G if the subset $\overline{M} = \{\overline{a}_\lambda \mid \lambda \in \Lambda\}$ is a basis of $\overline{G}$, i.e. a maximal linearly independent subset of elements of $\overline{G}$. A sequence $g_0, g_1, \dots$ of elements of a mixed group G is said to be a p-sequence of g_0 if $pg_{i+1} = g_i$, i = = 1,... . If M is a subset of a torsionfree group G then $\langle M \rangle_*$ is the pure closure of M in G, i.e. the largest subgroup of G such that $\langle M \rangle_* / \langle M \rangle$ is torsion.

All the results stated below can be formulated for modules over an associative and commutative principal ideal domain. However, this generalization seems to be rather formal and consequently we restrict ourselves to the abelian group case only.

2. Main results.

Definition 1: Let $M = \{a_\alpha \mid \alpha < \mu\}$, μ being an ordinal number, be a well-ordered basis of a group G. We define the generalized p-height $H_p(a_\alpha)$ of the element a_α as the p-height of $a_\alpha + \sum_{\beta<\alpha}\langle a_\beta\rangle$ in the group $G/\sum_{\beta<\alpha}\langle a_\beta\rangle$. The well-ordering on M is said to be an increasing p-height ordering if $H_p(a_\alpha) \leqq H_p(a_\beta)$ whenever $\alpha \leqq \beta < \mu$.

Definition 2: Let U be a torsionfree subgroup of a mixed group G and let $g \in G\setminus U$ be an element of infinite order. If $h_p^{G/U}(g + U) = \infty$ then every sequence $g = x_0, x_1, \dots$ of elements of G such that $p(x_{i+1} + U) = x_i + U$, $i = 0,1,\dots$, is called a generalized p-sequence of g with respect to U.

Definition 3: We say that a basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of a mixed group G has the property (S) if $\tau^G(a) = \tau^{\bar{G}}(\bar{a})$ for every element $a \in \langle a_\lambda \mid \lambda \in \Lambda\rangle$. If p is a prime then M is said to have the property $(\widetilde{Sp})$ if for every subset $N \subsetneqq M$ there is an essential p-pure torsionfree extension U of $\langle N\rangle$ in G such that every element $a \in M\setminus N$ with $h_p^{G/\langle N\rangle}(a + \langle N\rangle) = \infty$ has a generalized p-sequence with respect to U. Further, M is said to have the property $(\widetilde{\widetilde{Sp}})$ if there is a subset $N \subseteqq M$ having an essential p-pure torsionfree extension U in G such that every element $a \in M\setminus N$ has a generalized p-sequence with respect to U. An increasingly p-height ordered basis $M = \{a_\alpha \mid \alpha < \mu\}$, where $H_p^G(a_\alpha) = n_\alpha < \infty$ if and only if $\alpha < \nu$, is said to have the property (Sp) if for every $\alpha < \nu$ there is $x_\alpha \in G$ such that $p^{n_\alpha}(x_\alpha + \sum_{\beta<\alpha}\langle a_\beta\rangle) = a_\alpha + \sum_{\beta<\alpha}(a_\beta)$ and every element a_γ, $\nu \leqq \gamma < \mu$, has a generalized p-sequence with respect to $U = \langle x_\alpha \mid \alpha < \nu\rangle$. In this case we also say that the well-ordering on M has the property (Sp).

Theorem: Let G be a mixed group with the torsion part T. Then the following conditions are equivalent:

(1) G splits,

(2) for every basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of G there exist non-zero integers m_λ, $\lambda \in \Lambda$, such that the basis $\widetilde{M} = \{m_\lambda a_\lambda \mid \lambda \in \Lambda\}$ has the property (S) and for each prime p every increasing p-height ordering on $\widetilde{M}$ has the property (Sp),

(3) for every basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ there exist non-zero integers m_λ, $\lambda \in \Lambda$ such that the basis $\widetilde{M} = \{m_\lambda a_\lambda \mid \lambda \in \Lambda\}$ has the property (S) and, for each prime p, there exists an increasing p-height ordering on M having the property (Sp),

(4) there is a basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of G having the property (S), and, for each prime p, every increasing p-height ordering on M has the property (Sp),

(5) there is a basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of G having the property (S) and, for each prime p, there exists an increasing p-height ordering on M having the property (Sp),

(6) for every basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ there exist non-zero integers m_λ, $\lambda \in \Lambda$ such that the basis $\widetilde{M} = \{m_\lambda a_\lambda \mid \lambda \in \Lambda\}$ has the properties (S) and $\widetilde{(Sp)}$ for each prime p,

(7) there is a basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of G having the properties (S) and $\widetilde{(Sp)}$ for each prime p,

(8) for every basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of G there exist non-zero integers m_λ, $\lambda \in \Lambda$ such that the basis $\widetilde{M} = \{m_\lambda a_\lambda \mid \lambda \in \Lambda\}$ has the properties (S) and $\widetilde{\widetilde{(Sp)}}$ for each prime p,

(9) there is a basis $M = \{a_\lambda \mid \lambda \in \Lambda\}$ of G having the properties (S) and $\widetilde{\widetilde{(Sp)}}$ for each prime p.

<u>Corollary</u>: Let G be a mixed group of finite rank. Then the following conditions are equivalent:

(10) G splits,

(11) for every basis $M = \{a_1, a_2, \dots, a_n\}$ of G there ex-

ists a non-zero integer m such that the basis $\widetilde{M} = \{ma_1, ma_2, \ldots, ma_n\}$ has the property (S) and for each prime p every increasing p-height ordering on $\widetilde{M}$ has the property (Sp),

(12) for every basis $M = \{a_1, a_2, \ldots, a_n\}$ of G there exists a non-zero integer m such that the basis $\widetilde{M} = (ma_1, ma_2, \ldots, ma_n)$ has the property (S) and for each prime p there exists an increasing p-height ordering on $\widetilde{M}$ having the property (Sp),

(13) for every basis $M = \{a_1, a_2, \ldots, a_n\}$ of G there exists a non-zero integer m such that the basis $\widetilde{M} = \{ma_1, ma_2, \ldots, ma_n\}$ has the properties (S) and $(\widetilde{Sp})$ for each prime p,

(14) for every basis $M = \{a_1, a_2, \ldots, a_n\}$ of G there exists a non-zero integer m such that the basis $\widetilde{M} = \{ma_1, ma_2, \ldots, ma_n\}$ has the properties (S) and $(\widetilde{\widetilde{Sp}})$ for each prime p.

Proof: It follows immediately from Theorem.

<u>Remark</u>: The preceding Corollary generalizes the result of Stratton [10].

3. <u>Some auxiliary results</u>.

<u>Lemma 1</u>: Let U be a pure torsionfree subgroup of a mixed group G. If G/U splits then G splits, too.

Proof: First, let us show that $\langle T \cup U \rangle / U \cong T$ is the torsion part of G/U. If $g + U$ is a torsion element of G/U then $mg \in U$ for some non-zero integer m. Since U is pure in G, there is $u \in U$ with $mu = mg$. Thus $g = u + t$, $t \in T$, as desired.

Now suppose that G/U splits, $G/U = \langle T \cup U \rangle / U \oplus V/U$. Then $\langle T \cup V \rangle = \langle T \cup U \cup V \rangle = G$ and $T \cap V \subseteq \langle T \cup U \rangle \cap V \subseteq U$ yields $T \cap V \subseteq T \cap U = 0$ and $G = T \oplus V$ splits.

Lemma 2: Let $M = \{a_\lambda \mid \lambda \in \Lambda\}$ be a basis of a mixed group G such that $\tau(a_\lambda) = \tau(\bar{a}_\lambda)$ for every $\lambda \in \Lambda$. If $\bar{G}$ is divisible and every element a_λ, $\lambda \in \Lambda$, has a p-sequence in G then G splits, $G = T \oplus V$, and V can be chosen such that $M \subseteq V$.

Proof: It follows immediately from the proof of Theorem 1 in [3].

Lemma 3: A mixed group G splits if and only if

a) $Z_p \otimes G$ x) splits for each prime p, $Z_p \otimes G = T^{(p)} \oplus H^{(p)}$ and

b) there is a basis M of G such that $Z_p \otimes \langle M \rangle \subseteq H^{(p)}$ for each prime p.

Proof: See [10], Proposition 5.2.

Lemma 4: Let $M = \{a_\alpha \mid \alpha < \mu\}$ be an increasingly p-height ordered basis of a mixed group G such that $H_p^G(a_\alpha) = n_\alpha < \infty$ for each $\alpha < \mu$. If $p^{n_\alpha}(x_\alpha) = a_\alpha + \sum_{\beta < \alpha} r_\beta^{(\alpha)} a_\beta$ (finite sum) then the subgroup $U = \langle x_\alpha \mid \alpha < \mu \rangle$ is p-pure in G.

Proof: It clearly suffices to show that the equation $px = \sum r_\beta x_\beta$ is solvable in G if and only if $p \mid r_\beta$ for all $\beta < \mu$. Let $pg = \sum_{i=1}^{k} r_i x_{\beta_i}$, $\beta_1 < \beta_2 < \dots < \beta_k$, $n_i = H_p^G(a_{\beta_i})$, $i = 1,2,\dots,k$, and suppose that β_k is the smallest ordinal number such that this equality does not imply $p \mid r_i$, $i = 1,2,\dots,k$. Then obviously $(r_k,p) = 1$ and we have

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x) R_p is the ring of rationals with denominators prime to p and Z_p is its additive group.

$$p^{n_k+1} g = \sum_{i=1}^{k} p^{n_k-n_i} r_i (a_{\beta_i} + \sum_{\gamma<\beta_i} r_\gamma^{(\beta_i)} a_\gamma) = r_k a_{\beta_k} +$$

$+ \sum_{\gamma<\beta_k} r'_\gamma a_\gamma = h$. Now $n_k + 1 \leq h_p^G(h) \leq H_p^G(a_{\beta_k}) = n_k$ - a contradiction finishing the proof.

Lemma 5: Let G be a mixed R_p-module and let $M = \{ a_\alpha \mid \alpha < \mu \}$ be an increasingly p-height ordered basis of G. If M has the property (S) and $H_p^G(a_\alpha)$ is finite for every $\alpha < \mu$ then G splits, $G = T \oplus U$, and U can be chosen to contain M.

Proof: By hypothesis there are elements x_α, $\alpha < \mu$, such that $p^{n_\alpha} x_\alpha = a_\alpha + \sum_{\beta<\alpha} r_\beta^{(\alpha)} a_\beta$ where the last sum is finite and $n_\alpha = H_p(a_\alpha)$. The subgroup $U = \langle x_\alpha \mid \alpha < \mu \rangle$ obviously contains M.

If $g \in G$ is an arbitrary element then $p^r \bar{g} = \sum r_\beta \bar{a}_\beta$ (finite sum) for some non-negative integer r. Since M has the property (S), G contains an element h such that $p^r h = \sum r_\beta a_\beta$ and consequently there is $u \in U$ with $p^r u = \sum r_\beta a_\beta$, U being pure in G by Lemma 4. However, $p^r g = \sum r_\beta a_\beta + t = p^r u + t$, $t \in T$, hence $g - u \in T$ and $G = \langle T \cup U \rangle$.

Let $0 \neq u = \sum_{i=1}^{k} s_i x_{\beta_i} \in T \cap U$, $\beta_1 < \beta_2 < \dots < \beta_k$, $s_1 s_2 \dots s_k \neq 0$. Denoting $n_i = H_p(a_{\beta_i})$, we have $n_1 \leq n_2 \leq \dots \leq n_k$ by hypothesis, and

$$p^{n_k} u = \sum_{i=1}^{k} s_i p^{n_k-n_i} (a_{\beta_i}) + \sum_{\gamma<\beta_i} r_\gamma^{(\beta_i)} a_\gamma \in \langle M \rangle \cap T = 0.$$

Thus $a_k = 0$, which contradicts the choice of u. Hence $T \cap U = 0$ and $G = T \oplus U$ as desired.

4. Proof of Theorem. The implications (2) $\Rightarrow$ (3), (2) $\Rightarrow$ (4), (3) $\Rightarrow$ (5), (4) $\Rightarrow$ (5), (6) $\Rightarrow$ (7) and (8) $\Rightarrow$ (9) are obvious and it is easily seen that it suffices to prove the

implications (1)$\Longrightarrow$(6), (1)$\Longrightarrow$(8), (5)$\Longrightarrow$(1), (6)$\Longrightarrow$(2), (7)$\Longrightarrow$(4) and (9)$\Longrightarrow$(1).

(1)$\Longrightarrow$(6). Let G split, $G = T \oplus V$ and $M = \{a_\lambda \mid \lambda \in \Lambda\}$ be an arbitrary basis of G. If $a_\lambda = t_\lambda + v_\lambda$, $t_\lambda \in T$, $v_\lambda \in V$, $\lambda \in \Lambda$, and m_λ is the order if t_λ then the basis $\widetilde{M} = \{m_\lambda a_\lambda, \lambda \in \Lambda\}$ clearly has the property (S). Let p be a prime, $N \subsetneq \widetilde{M}$ and $a \in \widetilde{M} \setminus N$ be an element with $h_p^{G/\langle N\rangle}(a + \langle N\rangle) = \infty$. If U is the p-pure closure of N in V then $h_p^{G/U}(a + U) = \infty$. Hence there are elements $x_i \in G$, $u_i \in U$ such that $p^i x_i = a + u_i$, $i = 1,2,\dots$. Since V is a direct summand of G and $a + u_i \in V$, we can assume that $x_i \in V$. Further, $p^i(px_{i+1} - x_i) = u_{i+1} - u_i$ and $p^i u_i' = u_{i+1} - u_i$ for some $u_i' \in U$, $i = 1,2,\dots$, U being p-pure in V. Thus $px_{i+1} = x_i + u_i'$ and $a = x_0, x_1, \dots$ is a generalized p-sequence of a with respect to U.

(1)$\Longrightarrow$(8). Let G split, $G = T \oplus V$ and $M = \{a_\lambda \mid \lambda \in \Lambda\}$ be an arbitrary basis of G. If $a_\lambda = t_\lambda + v_\lambda$, $t_\lambda \in T$, $v_\lambda \in V$, $\lambda \in \Lambda$ and m_λ is the order of t_λ then the basis $\widetilde{M} = \{m_\lambda a_\lambda \mid \lambda \in \Lambda\}$ clearly has the property (S). Let p be a prime and $\{m_\alpha a_\alpha \mid \alpha < \mu\}$ be an increasingly p-height ordering on $\widetilde{M}$. If $N = \{m_\alpha a_\alpha \mid H_p^G(a_\alpha) < \infty\}$ and U is the p-pure closure of N in V then for each $a \in \widetilde{M} \setminus N$ we have $h_p^{G/U}(a + U) = \infty$ by the definition of increasing p-height ordering. Similarly as in the above part one can show that a has a generalized p-sequence with respect to U.

(5)$\Longrightarrow$(1). In view of Lemma 3, it suffices to show that the R_p-module $Z_p \otimes G$ splits, $Z_p \otimes G = T^{(p)} \oplus H^{(p)} = G^{(p)}$ and $Z_p \otimes \langle M\rangle \subseteq H^{(p)}$ for each prime p. Suppose that $\{a_\alpha \mid \alpha < \mu\}$

is an increasing p-height ordering on M having the property (Sp) and let $H_p^G(a_\alpha) = n_\alpha < \infty$ if and only if $\alpha < \nu$. If K is a subgroup of G such that $K/T = \langle \bar{a}_\alpha \mid \alpha < \nu \rangle_*^{\bar{G}}$, then $Z_p \otimes K$ splits by Lemma 5, $Z_p \otimes K = T^{(p)} \oplus U^{(p)}$, where $U^{(p)}$ can be chosen to contain $Z_p \otimes \langle a_\alpha \mid \alpha < \nu \rangle$. It is easily seen that every element $1 \otimes a_\gamma$, $\nu \leq \gamma < \mu$, has a generalized p-sequence with respect to $U^{(p)}$, so that $G^{(p)}/U^{(p)}$ splits by Lemma 2. Hence $G^{(p)}$ splits by Lemma 1, $G^{(p)} = T^{(p)} \oplus H^{(p)}$, and $Z_p \otimes \langle M \rangle \subseteq H^{(p)}$.

(6) $\Longrightarrow$ (2). Since we shall treat the basis $\widetilde{M}$, we can assume that $m_\lambda = 1$ for all $\lambda \in \Lambda$. Suppose that $\{a_\alpha \mid \alpha < \mu\}$ is any increasing p-height ordering on M such that $H_p^G(a_\alpha) = n_\alpha < \infty$ if and only if $\alpha < \nu$. By hypothesis, there is a p-pure torsion free subgroup U of G such that $\langle a_\alpha \mid \alpha < \nu \rangle \subseteq U$ and every element a_β, $\nu \leq \beta < \mu$, has a generalized p-sequence with respect to U. There are elements $x_\alpha \in U$, $\alpha < \nu$, in G such that $p^{n_\alpha}(x_\alpha + \sum_{\beta < \alpha} \langle a_\beta \rangle) = a_\alpha + \sum_{\beta < \alpha} \langle a_\beta \rangle$, U being p-pure in G. Setting $V = \langle x_\alpha \mid \alpha < \nu \rangle$, we are going to show that every element a_β, $\nu \leq \beta < \mu$, has a generalized p-sequence with respect to V.

Let $a_\beta = y_0, y_1, \dots$ be a generalized p-sequence of a_β with respect to U. Then $py_{i+1} = y_i + u_i$, where $u_i \in U$ and $m_i u_i = v_i \in V$, $(m_i, p) = 1$, $i = 1, 2, \dots$, V being p-pure and essential in U by the hypothesis and Lemma 4. Hence there are integers ϱ_i, σ_i with $m_i \varrho_i + p\sigma_i = 1$, $i = 1, 2, \dots$. If we put $z_i = y_i - \sum_{j=0}^{i-1} \sigma_j^{i-j} u_j$ then we have $pz_{i+1} = py_{i+1} - p\sum_{j=0}^{i} \sigma_j^{i+1-j} u_j = y_i + u_i - p\sum_{j=0}^{i} \sigma_j^{i+1-j} u_j = z_i + \sum_{j=0}^{i-1} \sigma_j^{i-j} u_j + u_i - \sum_{j=0}^{i} \sigma_j^{i-j} (u_j - m_j \varrho_j u_j) = z_i +$

$+\sum_{j=0}^{i} \sigma_j^{i-j} \rho_j v_j$ and $a_\beta = z_0, z_1, \dots$ is a generalized p-sequence of a_β with respect to V.

The implication (7) $\Longrightarrow$ (4) can be proved similarly.

(9) $\Longrightarrow$ (1). Let p be a prime. Since M has properties (S) and $(\widetilde{Sp})$ the factor-module $Z_p \otimes G/Z_p \otimes U$ splits by Lemma 2 and consequently $Z_p \otimes G$ splits by Lemma 1, $Z_p \otimes U$ being torsionfree and pure in $Z_p \otimes G$. Moreover, as it is easy to check, the torsionfree factor of $Z_p \otimes G$ can be chosen to contain $Z_p \otimes \langle M \rangle$. Hence G splits by Lemma 3.

5. Example. In this final section we shall present an example of a non-splitting group G with the torsion part T such that every rank finite pure subgroup of G containing T splits.

Let $H = \langle a \rangle \oplus \sum_{i=1}^{\infty} \langle a_i \rangle \oplus \sum_{i=1}^{\infty} \langle x_i \rangle + \sum_{i=1}^{\infty} \langle y_i \rangle$ be a free group and $K = \langle a_i + p_i^2 y_i,\ p_i a + p_i^2 x_i + a \mid i = 1,2,\dots \rangle$, $L = \langle a + p_i(x_i - y_i),\ a_i + p_i^2 y_i \mid i = 1,2,\dots \rangle$ be its subgroups. We have $p_i(a + p_i(x_i - y_i)) = p_i a + a_i + p_i^2 x_i - (a_i + p_i^2 y_i) \in K$ so that $K \subseteq L \subseteq K_*$. On the other hand, if $p_j(\lambda a + \sum_{i=1}^{n} (\lambda_i a_i + \mu_i x_i + \nu_i y_i)) = \sum_{i=1}^{n} (\rho_i (a + p_i(x_i - y_i)) + \sigma_i (a_i + p_i^2 y_i))$ then

(15) $\quad p_j \lambda = \sum_{i=1}^{n} \rho_i$,

(16) $\quad p_j \lambda_i = \sigma_i$, $i = 1,2,\dots,n$,

(17) $\quad p_j \mu_i = p_i \rho_i$, $i = 1,2,\dots,n$.

By (17) $p_j \mid \rho_i$, $i = 1,2,\dots,n$, $i \neq j$, and so $p_j \mid \rho_j$ by (15). Since by (16) $p_j \mid \sigma_i$, $i = 1,2,\dots,n$, L is pure in H and $L = K_*$.

Now $a + L = p_j(x_j - y_j) + L$ so that $h^{H/L}_{p_j}(a + L) \geq 1$. Let the equation $p_j(x + K) = ma + K$ be solvable in H/K. Then $p_j(\lambda a + \sum_{i=1}^{n} (\lambda_i a_i + \mu_i x_i + \nu_i y_i) = ma + \sum_{i=1}^{n} (\rho_i(a_i + p_i^2 y_i) + \sigma_i(p_i a + a_i + p_i^2 x_i))$ and so

$$p_j \lambda = m + \sum_{i=1}^{n} p_i \sigma_i,$$

$$p_j \mu_i = p_i^2 \sigma_i, \quad i = 1,2,\dots,n.$$

Thus $p_j \mid \sigma_i$, $i = 1,2,\dots,n$, $i \neq j$, and hence $p_j \mid m$. We have shown that there is no non-zero multiple ma of a such that $\tau^{G/K}(ma + K) = \tau^{G/L}(ma + L)$ and consequently the factor-group $G = H/K$ does not split.

If $X_n = \langle \{a, a_1, \dots, a_n, x_1, \dots, x_n, y_1, \dots, y_n\} \cup K \rangle$ then it is easy to see that the torsion part $(L \cap X_n)/K$ of X_n/K is finite. If S/K is a pure subgroup of G of finite rank then S/K is contained in X_n/K for some n. Thus the torsion part of S/K is finite and S/K splits.

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Matematicko-fyzikální fakulta
Universita Karlova
Sokolovská 83, 18600 Praha 8
Československo

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