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ON BICOMPACTA WHICH ARE UNIONS OF TWO SUBSPACES OF A CERTAIN TYPE

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Abstract: Let X be a bicompact space, X = Y ∪ Z, and suppose that we have some information about Y and Z. What can be said then about X ? About Fr (Y) ? The aim of the present paper is to study this situation with the emphasis on the following properties: sequentiality, metrizability, being a Moore space, being an Eberlein bicompactum. The results are applied to the investigation of properties of the remainders of metrizable spaces.

Key words: Bicompact space, sequential space, Moore space, Eberlein compact, space of countable type, uniform base.

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We consider the following general problem. Let $\mathcal{P}$ be a class of topological spaces and let X be a bicompact Hausdorff space such that $X = X_1 \cup X_2$, where $X_1, X_2 \in \mathcal{P}$. What can be said in this situation about properties of X ? This question is aimed at clarifying what kind of bicompacta we can get when constructing them by joining together two spaces belonging to a certain "basic" class of spaces. The Alexandroff's "double circumference" is a classical example of a non-metrizable bicompactum which is the union of two metrizable subspaces. In particular, we have in mind the following special question. Is it possible to construct a non-sequential bicompactum of countable tightness as the union of two rather simple spaces?

But the general problem referred to above is interesting not only in connection with examples. The following general question is a special case of it. When a space X has a remainder of the same type? There are different ways to make the question more concrete. First, find classes $\mathcal{P}$ of spaces such that every $X \in \mathcal{P}$ has a remainder in $\mathcal{P}$. When X has a remainder which is homeomorphic to X ? (A remainder of X is any space of the form $bX \setminus X$ where bX is a bicompact Hausdorff extension of X.) Given a class $\mathcal{P}$ of spaces, how to characterize $X \in \mathcal{P}$ such that some remainder of X belongs to $\mathcal{P}$? When a metrizable space X has a metrizable remainder? When a Moore space has a remainder which is a Moore space? When a symmetrizable space has a symmetrizable remainder? The same question can be formulated for σ-spaces, for semi-stratifiable spaces etc.

All the spaces considered in this paper are assumed to be completely regular. $c\ell_X \ldots$ means"closure in X ". If we write $c\ell \ldots$, it is to be understood that the closure is taken in the largest of all the spaces under consideration. N^+ is the set of all positive integers; w(X) - the weight of X; t(X) - the tightness of X; nw(X) - the networkweight of X; c(X) - the Suslin number of X; $\psi(F,X)$ - the pseudocharacter of F in X; $\chi(F,X)$ - the character of F in X; d(X) - the density of X; s(X) - the spread of X. Definitions of these notions can be found in [6].

Let us remind some known results related to the general problem under consideration. Let X be a bicompactum and $X = Y \cup Z$. Then:

1) If $w(Y) \leq \aleph_0$ and $w(Z) \leq \aleph_0$ then $w(X) \leq \aleph_0$ (Ju. Smirnov [17])

2) If $w(Y) \leq \tau$ and $w(Z) \leq \tau$ then $w(X) \leq \tau$ (A. Arhangel'skiǐ, see [6])

3) If Y and Z are perfect spaces then $t(X) \leq \aleph_0$ (A. Arhangel'skiǐ [3]) (perfect means that every closed set is G_δ)

4) If Y and Z are metrizable then X is a Fréchet-Urysohn space [3]

5) If Y and Z are metrizable then X is an Eberlein bicompactum (M.E. Rudin, E.A. Michael [9]).

Below we formulate and prove some new results closely related to 3),4) and 5).

Theorem 1. Let $\mathcal{P}$ be a class of spaces such that the following four conditions are satisfied:
1) every $X \in \mathcal{P}$ is sequential; 2) if $X \in \mathcal{P}$, $Y \subset X$ and Y is closed in X then $Y \in \mathcal{P}$; 3) if $X \in \mathcal{P}$ and X is countably compact then X is bicompact; 4) if $X \in \mathcal{P}$ and X is bicompact then X is first countable at a dense set of points. Further, let Z be a bicompact space such that $Z = X_1 \cup X_2$ where $X_1 \in \mathcal{P}$ and $X_2 \in \mathcal{P}$. Then Z is sequential.

Proof. We have: $Z = X_1 \cup X_2$, $X_1 \in \mathcal{P}$, $X_2 \in \mathcal{P}$. Consider any $A \subset Z$ sequentially closed in Z. Then the set $A_i = A \cap X_i$ is sequentially closed in X_i and hence A_i is closed in X_i (condition 1)). Let us assume that A is not closed in

Z. We fix $z \in cl(A) \setminus A$. It does not matter whether $z \in X_1$, or $z \in X_2$. Let $z \in X_2$. Since A_2 is closed in X_2, we have $z \notin cl(A_2)$. Let Oz be a neighbourhood of z in Z such that $cl(Oz) \cap cl(A_2) = \Lambda$. We put $A_1^* = cl(Oz) \cap A$. Clearly $A_1^* = cl(Oz) \cap A_1$, the set A_1^* is closed in A_1 and $z \in cl(A_1^*) \setminus A_1^*$. Consider a set $M \subset A_1^*$ such that M is discrete and closed in A_1^*. We shall show that the set M is finite. Suppose that M is infinite. We can assume then that $|M| = \aleph_0$. Obviously, M is closed in X_1. Hence the set $F = cl(M) \setminus M$ is contained in X_2. As M is discrete, F is closed in Z. It follows that F is bicompact. Since M is infinite, we have $F \neq \Lambda$. It follows from 4) that $\chi(x,F) \leq \aleph_0$ for some $x \in F$. On the other hand, $\psi(F, cl(M)) \leq |M| = \aleph_0$. Thus $\chi(F, cl(M)) \leq \aleph_0$ and from $\chi(x,F) \leq \aleph_0$ it follows that $\chi(x, cl(M)) \leq \aleph_0$. From this we infer that there exists a sequence ξ in M converging to x. Since A is sequentially closed in Z, we have $x \in A$. From $x \in F \subset X_2$ it follows that $x \in A_2$. But this contradicts $x \in cl(M) \subset cl(Oz) \subset Z \setminus cl(A_2)$. Hence M is finite and the space A_1^* is countably compact. It follows from the conditions 2) and 3) that A_1^* is bicompact. Hence A_1^* is closed in Z and this contradicts $z \in cl(A_1^*) \setminus A_1^*$. The proof is complete.

If the Martin's Axiom MA is assumed (see [7]), then condition 4) in Theorem 1 can be dropped.

<u>Theorem 1'</u>. Assume MA. Let $\mathcal{P}$ be a class of spaces such that: 1) each $X \in \mathcal{P}$ is sequential; 2) if $X \in \mathcal{P}$, $Y \subset X$ and Y is closed in X then $Y \in \mathcal{P}$; 3) if $X \in \mathcal{P}$ and X is countably compact then X is bicompact. Let Z be a bi-

compact space such that $Z = X_1 \cup X_2$ where $X_1 \in \mathcal{P}$ and $X_2 \in \mathcal{P}$. Then Z is sequential.

Proof. We begin the argument as in the proof of Theorem 1. To get a sequence ξ in M converging to some point of F we use the following theorem of D.V. Ranchin [11]: under MA every bicompactum which can be represented as the union of a countable family of sequential bicompact subspaces is sequential. Since $F = c\ell(M) \setminus M$ is sequential bicompactum and M is countable, the theorem of Ranchin can be applied to $c\ell(M)$. Hence $c\ell(M)$ is sequential. Since M is not closed in $c\ell(M)$, it follows that there exists a sequence ξ in M converging to some point in $c\ell(M) \setminus M = F$.

Now we can complete the proof of Theorem 1′ exactly in the same way as we have completed the proof of Theorem 1.

Corollary 1. Let X be a k-space and $X = Y \cup Z$, where Y and Z are both sequential and the diagonals in $Y \times Y$ and $Z \times Z$ are G_δ-sets. Then X is sequential.

Proof. It suffices to consider the case when X is bicompact. The class $\mathcal{P}$ of all sequential spaces with G_δ-diagonal trivially satisfies the conditions 1) and 2) in Theorem 1. From a theorem of J. Chaber (see [6]) it follows that the conditions 3) and 4) are also satisfied by $\mathcal{P}$. Hence X is sequential by Theorem 1.

Corollary 2. If X is a k-space and $X = Y \cup Z$ where Y and Z are both symmetrizable (see [10]) then X is sequential.

Proof. We can assume that X is bicompact. For the class $\mathcal{P}$ of all symmetrizable spaces the conditions 1) and

2) of Theorem 1 are obviously true. It was shown by S.I. Nedev [10] that $\mathcal{P}$ satisfies the condition 3) as well. It is known also that every symmetrizable bicompactum is metrizable. Thus we can apply Theorem 1 and the space X is sequential.

If X is a Moore space, or σ-space [6], or semi-stratifiable space (see [8]) or there exists a one-to-one continuous mapping of X onto a Moore space, then the diagonal in $X \times X$ is a G_δ-set. Hence we have

Corollary 3. Let X be a k-space and $X = Y \cup Z$. Then in each of the following four cases the space X is sequential:

a) Y and Z are semi-stratifiable sequential spaces;

b) Y and Z are sequential σ-spaces;

c) Y and Z are Moore spaces;

d) Y and Z are sequential and each of them can be mapped onto a Moore space by a one-to-one continuous mapping.

Corollary 4. If Martin's Axiom holds then every k-space which is the union of two realcompact sequential spaces is sequential.

Proof. It is easy to check that the class $\mathcal{P}$ of all realcompact spaces satisfies all the four conditions of Theorem 1.

If the summands Y and Z in $X = Y \cup Z$ are such that every bicompact subspace of Y and every bicompact subspace of Z satisfies the first axiom of countability at a dense set of points, there is no need to assume the Martin's Axiom. Thus we have

Corollary 5. Let X be a bicompactum and $X = Y \cup Z$ where Y and Z are realcompact sequential spaces such that if $F \subset Y$ or $F \subset Z$ and F is bicompact then F is first countable at a dense set of points. Then X is sequential.

A space X is called metalindelöf if every open covering of X can be refined by an open point-countable covering. G. Aquaro proved (see [6]) that every metalindelöf countably compact space is bicompact.

Corollary 6. Assume Martin's Axiom. If X is bicompact and $X = Y \cup Z$, where Y and Z are metalindelöf sequential spaces then X is sequential.

Again we can drop the Martin's Axiom if all bicompact subspaces of Y and Z are first countable at a dense set of points. In particular, we have

Corollary 7. If X is a k-space and $X = Y \cup Z$ where Y and Z are spaces with a point-countable base then X is sequential.

Corollary 8. Assume Martin's Axiom and the inequality $2^{\aleph_0} > \aleph_1$. Let X be a k-space and $X = Y \cup Z$ where Y and Z are perfect. Then X is sequential.

Proof. Let us consider the class $\mathcal{P}$ of all perfect spaces ($X \in \mathcal{P}$ iff every closed set in X is a G_δ-set). It is clear that $\mathcal{P}$ satisfies the conditions 1),2) and 4). It follows from MA and $2^{\aleph_0} > \aleph_1$ that the condition 3) also holds for $\mathcal{P}$: this remarkable theorem was proved by W.A.R. Weiss [15]. Theorem 1 now yields that X is sequential.

Definition 1 [2]. A space X is of countable type if for each bicompact $F \subset X$ there exists a bicompact $F^* \subset X$ such that $F \subset F^*$ and $\chi(F^*, X) \leq \aleph_0$.

The boundary Fr(A) of a set $A \subset X$ in X is the set $c\ell(A) \cap c\ell(X \setminus A)$. We consider the following general problem. Let X be a bicompactum and $A \subset X$. Assume that a class $\mathcal{P}$ of spaces is specified and $A \in \mathcal{P}$, $X \setminus A \in \mathcal{P}$. What can be said then about Fr(A) ?

Theorem 2. Let X be a bicompactum and $Y \subset X$. Then the following statements are true:
a) if Y and $X \setminus Y$ are semi-stratifiable spaces of countable type then the bicompactum Fr(Y) is perfectly normal and hereditarily separable; b) if Y and $X \setminus Y$ are σ-spaces (see [6]) of countable type then Fr(Y) is a metrizable bicompactum; c) if Y and $X \setminus Y$ are Moore spaces then Fr(Y) is a metrizable bicompactum. Furthermore, in each of the cases a),b) and c), the space $X \setminus Fr(Y)$ is locally bicompact and locally metrizable, and $X \setminus Fr(Y)$ belongs to the same class as Y and Z.

Proof. First we shall prove the last assertion. We have: $X \setminus Fr(Y) = (Y \setminus Fr(Y)) \cup ((X \setminus Y) \setminus Fr(Y))$, where $Y \setminus Fr(Y)$ and $(X \setminus Y) \setminus Fr(Y)$ are disjoint, open and closed sets in $X \setminus Fr(Y)$. Besides, $Y \setminus Fr(Y)$ is open in Y and $(X \setminus Y) \setminus Fr(Y)$ is open in $X \setminus Y$. To prove the last assertion of Theorem 2 it suffices now to remind that every semi-stratifiable bicompactum is metrizable [8]. Now let us prove a).

Since Y is of countable type and $c\ell(Y)$ is bicompact, it follows from a theorem of Henriksen and Isbell [13] that $Y_1 = c\ell(Y) \setminus Y$ is Lindelöf. Since $Y_1 \subset Z = X \setminus Y$ and Z is semi-stratifiable, Y_1 is also semi-stratifiable. Applying the results from [8] we conclude that Y_1 is hereditarily Lindelöf and hereditarily separable. By the same argument we show that the space $Z_1 = c\ell(Z) \setminus Z$ (where $Z = X \setminus Y$) is hereditarily Lindelöf and hereditarily separable. Hence $Fr(Y) = Y_1 \cup Z_1$ is a hereditarily Lindelöf and hereditarily separable space as well.

b) Every σ-space is semi-stratifiable. Hence the argument in a) shows that $Y_1 = c\ell(Y) \setminus Y$ and $Z_1 = c\ell(X \setminus Y) \setminus (X \setminus Y)$ are Lindelöf spaces. Since each Lindelöf σ-space has a countable network, $nw(Y_1) \le \aleph_0$ and $nw(Z_1) \le \aleph_0$. It follows that $Fr(Y) = Y_1 \cup Z_1$ has a countable network. Now $Fr(Y)$ is a bicompactum. Hence $w(Fr(Y)) = nw(Fr(Y)) \le \aleph_0$ (see [6]) and $Fr(Y)$ is metrizable.

c) Every Moore space is a σ-space [1]. Besides, every Moore space is a p-space. Since N.V. Veličko [4] has shown that every p-space is a space of countable type, it follows that every Moore space is a space of countable type. It remains to apply b). With the help of Theorem 2 we get the following generalization of a theorem of M.E. Rudin and E. Michael [9].

<u>Theorem 3</u>. If X is a bicompactum and $X = Y \cup Z$ where Y and Z are spaces with uniform bases then X is an Eberlein bicompactum.

Proof. From Theorem 2 b) it follows that $Fr(Y)$ is a bicompactum with countable base. We put $X_1 = X \setminus Fr(Y)$, $Y_1 = Y \setminus Fr(Y)$ and $Z_1 = (X \setminus Y) \setminus Fr(Y)$. Then (see the proof of Theorem 2) $X_1 = Y_1 \cup Z_1$, $Y_1 \cap Z_1 = \Lambda$, $Y_1 \subset Y$, $Z_1 \subset Z$ and Y_1, Z_1 are open and closed in X_1. It follows that the space X_1

has a uniform base. Since X is normal and the space $Fr(Y)$ has a countable base, it is not difficult to construct a countable family γ of open F_σ-sets in X such that γ separates the points of $Fr(Y)$ (i.e. if $x', x'' \in Fr(Y)$ and $x' \neq x''$ then there exists $U \in \gamma$ such that $x' \in U$ and $x'' \notin U$). We can fix a uniform base $\mathcal{B}$ in the space X_1 such that $[U] \cap Fr(Y) = \Lambda$ for each $U \in \mathcal{B}$. Then each $U \in \mathcal{B}$ is an open F_σ-set in X and the family $\mathcal{B}$ is σ-point-finite. We put $\widetilde{\mathcal{B}} = \mathcal{B} \cup \gamma$. Then $\widetilde{\mathcal{B}}$ is the union of a countable family of point-finite systems of open F_σ-sets in X. One can easily check that $\widetilde{\mathcal{B}}$ T_0-separates the points of X - i.e. for any $x', x'' \in X$ there exists $U \in \widetilde{\mathcal{B}}$ such that $U \cap \{x', x''\}$ is a singleton. Applying the Rosenthal's Theorem [12], we conclude that X is an Eberlein bicompactum.

Example 1. Let us consider the well known Franklin's bicompactum X (see [3]). We have: $X = X_1 \cup X_2 \cup X_3$ where X_1, X_2 and X_3 are discrete spaces, X_3 is a singleton, X_1 is countably infinite and open in X and X_2 is uncountable. We put $Y = X_1 \cup X_2$ and $Z = X_3$. Then Y and Z are Moore spaces and $X = Y \cup Z$. Nevertheless, X is not an Eberlein bicompactum - it is not even a Fréchet-Urysohn space. The same example shows that Theorem 2 is no longer true when X is decomposed into three metrizable summands. Note that it is not a coincidence that X is sequential - see Corollary 3, c).

Let us consider the spaces $Y' = X_1 \cup X_3$ and $Z' = X \setminus Y' = X_2$. The space Y' is countable so that Y' is a σ-space. The space Z' is discrete so that Z' is metrizable. Hence Z' is a

σ-space of countable type. On the other hand, $c\ell(Y') = X$, $c\ell(Z') = X_2 \cup X_3$ and $Fr(Y') = c\ell(Y') \cap c\ell(Z') = X_2 \cup X_3$. Thus $Fr(Y')$ is a non-metrizable bicompactum which is not even perfectly normal. The reason (see Theorem 2 b)) for non-metrizability of $Fr(Y')$ is that Y' is not a space of countable type - all other conditions are satisfied by X, Y' and Z'. This shows that Theorem 2 cannot be significantly improved.

Theorem 2 permits to get particularly strong conclusions when the summands do not have points of local bicompactness - or there are not too many such points.

Corollary 9. Let X be a bicompactum and $X = Y \cup Z$, where Y and Z are semi-stratifiable spaces of countable type without points of local bicompactness. Then X is a perfectly normal hereditarily separable bicompactum.

Proof. Clearly both Y and Z are everywhere dense in X. Hence $Fr(Y) = X$. From Theorem 2 a) it follows that the bicompactum X is perfectly normal and hereditarily separable.

Corollary 10. Let X be a bicompactum and $X = Y \cup Z$ where Y and Z are σ-spaces of countable type such that $Y \cap Z = \Lambda$. Suppose also that Y and Z do not have points of local bicompactness. Then the space X is metrizable.

Proof. We argue as in the proof of Corollary 9 and then refer to Theorem 2 b).

Since every Moore space is a space of countable type and every subspace of a Moore space is a Moore space, we have:

Corollary 11. Let X be a bicompactum and $X = Y \cup Z$, where Y and Z are Moore spaces without points of local bicompactness. Then X is metrizable.

We can somewhat weaken the restrictions on Y and Z in Corollaries 10 and 11 - it will suffice to assume that the sets of all points of local bicompactness in Y and Z form Lindelöf spaces.

Our considerations naturally lead to some curious statements about the remainders.

Proposition 1. Let Y be a Moore space and bY - a bicompactification of Y such that the remainder $bY \setminus Y$ is a space of countable type. Then $R(Y) = \{ y \in Y : c\ell(V)$ is not bicompact for every neighbourhood V of y in $Y\}$ is a space with countable base.

Proof. We put $Z = bY \setminus Y$ and $bZ = c\ell(Z)$. Clearly $R(Y) = bZ \setminus Z$. Since the space Z is of countable type, by the theorem of Henriksen and Isbell [13] the space $bZ \setminus Z$ is Lindelöf. Since $bZ \setminus Z \subset Y$, $R(Y) = bZ \setminus Z$ is a Moore space. Hence R(Y) is a space with countable base.

Theorem 4. Let X be a space metrizable by a complete metric. Let us also assume that X is periferally bicompact - i.e. there exists a base $\mathcal{B}$ in X such that Fr(U) is bicompact for every $U \in \mathcal{B}$. Then the following conditions are pairwise equivalent: a) X has a remainder which is a space of countable type; b) X has a remainder which is a p-space; c) X has a remainder which is a Moore space; d) X has a metrizable remainder; e) X has a remainder with countable base; f) X has a remainder which is a countable space with countable base; g) X has a countable remainder.

Proof. Clearly, f) $\Rightarrow$ e) $\Rightarrow$ d) $\Rightarrow$ c) $\Rightarrow$ b) $\Rightarrow$ a). From a) it follows by means of Proposition 1 that R(X) is a space with countable base. Applying a theorem of T. Hoshina [14], we can now conclude that X has a countable remainder. Thus a) $\Longrightarrow$ g). It remains to show that g) $\Longrightarrow$ f) - this result belongs to G. Dimov [5]. For the sake of completeness we prove it below. Let $P = bX \setminus X$. Since X is metrizable and P is countable, from a result of T. Hoshina [14] it follows that the space $c\ell(P) \cap X$ is Lindelöf. Hence the space $c\ell(P) \cap X$ has a countable base. Then the space $c\ell(P) = (c\ell(P) \cap X) \cup P$ has a countable network. Since $c\ell(P)$ is bicompact, $w(c\ell(P)) = nw(c\ell(P)) \leq \aleph_0$ (see [6]).

The following problems remain unsolved.

1) Can one generalize Theorem 1, or any of the Corollaries 1 - 11, to the case of arbitrary finite number of summands?

2) Can one prove Theorem 1′ and Corollaries 4 and 6 without the Martin's Axiom?

3) Will the Corollary 8 remain true if we do not assume the Martin's Axiom and the negation of continuum-hypothesis?

4) Is it true (without additional hypotheses) that every non-empty sequential bicompactum is first countable at some point?

The problem 4 was for the first time formulated in [16]. It is proved in [16] that if $2^{\aleph_0} < 2^{\aleph_1}$ then the answer to the question 4) is positive. From positive answer to the question 4) a positive answer to the question 2) would follow.

5) When a countable space has a metrizable remainder?

6) Let X be a perfectly normal bicompactum such that $X = Y \cup Z$ where Y and Z are symmetrizable. Is it true then that X is metrizable?

7) Let X be a bicompactum and $X = Y \cup Z$ where Y and Z are semi-stratifiable. Is it true then that X is sequential?

8) Let X be a bicompactum and $X = Y \cup Z$ where Y and Z are σ-spaces. Is it true then that X is sequential?

9) Can one construct (not using $\diamondsuit$, CH or other additional set-theoretic principles) a bicompactum X such that $X = Y \cup Z$ where Y and Z are perfect spaces and X is not sequential?

Ostaszewski constructed a bicompactum as in 9) under the principle $\diamondsuit$ (see [18]).

It is worth noting that all the bicompacta involved in 7),8) and 9) have countable tightness [3]. Hence the negative answer to 7) or positive answer to 9) would yield an absolute example of a non-sequential bicompactum of countable tightness.

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