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REFLECTIVE MAC NEILLE COMPLETIONS OF FIBRE-SMALL CATEGORIES NEED NOT BE FIBRE-SMALL

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Abstract: See Title

Key words: Initial completion, universal initial completion, Mac Neille completion, semi-topological functor, topologically-algebraic functor, fibre-smallness, strong fibre-smallness.

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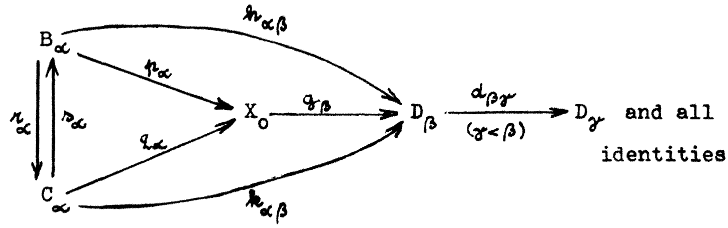
Mac Neille completions have been defined in [2]. Categories having reflective Mac Neille completions, have been characterized by Wischnewsky and Tholen [8], Hoffmann [5], Adámek [1] and by Herrlich and Strecker [3] as those ($\underline{A}$,U), for which U is semi-topological. Categories, having fibre-small Mac Neille completions, have been characterized by Adámek [1] and by Herrlich and Strecker [4] as those ($\underline{A}$,U), which are strongly fibre-small. The title statement provides a negative answer to a problem posed by Adámek [1], p. 22. The example is as follows.

Let $(\Omega, \leq)$ be a large complete lattice. Let $\underline{X}$ be the following category:

Objects: X_0, B_α, C_α, D_α for all $\alpha \in \Omega$

Morphisms:



Composition is uniquely determined by the fact that morphism classes hom (X,Y) contain at most one element.

Let $\underline{A}$ be the subcategory of $\underline{X}$, obtained by removing X_0, id_{X_0}, all r_α, p_α, q_α, g_α, and all $h_{\alpha\beta}$ with $\beta > \alpha$, and let $U:\underline{A} \to \underline{X}$ be the embedding functor. Then U is not only semi-topological, but even topologically-algebraic in the sense of Y.H. Hong [7] and S.S. Hong [6], i.e. any U-source has some (generating, initial)-factorization

$$X \xrightarrow{f_i} UA_i = X \xrightarrow{g} UA \xrightarrow{Um_i} UA_i$$

as indicated by the following table:

	$X \xrightarrow{f_i} UA_i$	g
(1)	$X = B_\alpha$ and $\{f_i \mid i\in I\} \cap (\{r_\alpha\}\cup\{h_{\alpha\beta} \mid \beta>\alpha\}) = \phi$	id_{B_α}
(2)	$X = B_\alpha$ and $\{f_i \mid i\in I\} \cap (\{r_\alpha\}\cup\{h_{\alpha\beta} \mid \beta>\alpha\}) \neq \phi$	r_α
(3)	$X = C_\alpha$ and $\{f_i \mid i\in I\} \cap (\{id_{C_\alpha}\}\cup\{k_{\alpha\beta} \mid \beta>\alpha\}) \neq \phi$	id_{C_α}
(4)	$X = C_\alpha$ and $\{f_i \mid i\in I\} \cap (\{id_{C_\alpha}\}\cup\{k_{\alpha\beta} \mid \beta>\alpha\}) = \phi$	s_α
(5)	$X = X_0$, $\gamma = \sup\{\alpha\in\Omega \mid g_\alpha \in \{f_i \mid i\in I\}\}$	g_γ
(6)	$X = D_\beta$, $\gamma = \sup\{\alpha\in\Omega \mid d_{\beta\alpha} \in \{f_i \mid i\in I\}\}$	$d_{\beta\gamma}$

Hence, by [3], $(\underline{A},U)$ has not only a reflective Mac Neille completion but even a reflective universal initial completion. Since the $g_\beta : X_0 \longrightarrow UD_\beta$ are pairwise non-equivalent semi-final U-morphisms, ($\underline{A}$,U) is not strongly fibre-small. Hence its Mac Neille completion is not fibre-small.

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