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SET FUNCTORS III - MONOMORPHISMS,EPIMORPHISMS,ISOMORPHISMS

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Abstract: Given a functor F from the category of sets into itself, we state necessary and sufficient conditions on a mapping f in order that Ff be a monomorphism (epimorphism, isomorphism). Some corollaries concerning the behaviour of functors are given.

Key words: set-functor, congruence, monomorphism, epimorphism, isomorphism.

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In the present paper, we consider functors (covariant or contravariant) from the category S of sets into itself.

Given a functor F , we state necessary and sufficient conditions on a mapping $f: X \longrightarrow Y$, $X \neq \emptyset$, in order that Ff be a monomorphism (epimorphism, isomorphism). It is shown that they depend only on the congruence (on S) created by F (in the sense $f \sim g$ iff $Ff = Fg$).

Further, we compare congruences, created by functors, in connection with the morphisms which are mapped by these functors on monomorphisms, epimorphisms, isomorphisms.

Conventions and definitions. Let X, Y be sets.

Then $X < Y$ means $\mathrm{card}\, X < \mathrm{card}\, Y$; analogously for $X \leq Y$. $X \simeq Y$ means $\mathrm{card}\, X = \mathrm{card}\, Y$. X^+ is the follower of the $\mathrm{card}\, X$. Every cardinal is regarded as a set.

Let $f, g : X \longrightarrow Y$ be mappings. Put

$\mathcal{U}_{fg} = f[D] \cup g[D]$ where $D = \{x \in X ; f(x) \neq g(x)\}$,

$C_f = \bigcup_{y \in C} f^{-1}(y)$ where $C = \{y \in Y ; f^{-1}(y) > 1\}$,

$\mathrm{Im}\, f = \{f(x) ; x \in X\}$.

A congruence on a category is an equivalence $\sim$ on the class of its morphism such that if $f \sim g$ then f and g have a common domain and common range and

$$f \sim g\ ,\ f_1 \sim g_1 \Longrightarrow f \circ f_1 \sim g \circ g_1$$

provided the composition makes sense.

If $f \sim g$ for every f, g with a common domain and common range then $\sim$ is called the trivial congruence.

If $\sim$ is a congruence on a category K then $K/\sim$ is the factor-category of K with respect to $\sim$. The objects of $K/\sim$ are the same as those of K . Morphisms are equivalence classes and the composition in $K/\sim$ is defined by $[f] \circ [g] = [f \circ g]$ where $[f]$ denotes the class containing f .

The category of sets is denoted by S . The word functor denotes a functor (covariant or contravariant) from S to S . Let F be a functor, α a cardinal, $\alpha > 0$. Denote F^{α} the subfunctor of F defined by

$F^{\alpha}X = \bigcup Ff[FY]$ for every X, the union being taken over all $Y < \alpha$ and all $f : Y \longrightarrow X$ (or $f : X \longrightarrow Y$) in covariant (or contravariant, respectively) case.

P^- denotes the contravariant power-functor:

$P^-X = \exp X$, $P^-f(A) = f^{-1}(A)$, for $f : X \to Y$, $A \in \exp Y$.

A functor is said to reflect monomorphisms if f is a monomorphism provided Ff is a monomorphism. Analogously for epi- and isomorphisms.

<u>Note</u>: Let $f : X \longrightarrow Y$ be a mapping, $X \neq 0$. If f is a monomorphism (an epimorphism) in S then it is a coretraction (a retraction). Thus, every covariant functor (from S to S) preserves monomorphisms and epimorphisms i.e.

f is a monomorphism $\Longrightarrow Ff$ is a monomorphim ,

f is an epimorphism $\Longrightarrow Ff$ is an epimorphism.

The contravariant case is analogous: every contravariant functor turns monomorphism (with non-empty domain) to epimorphism and vice versa. Finally, every covariant faithful functor reflects monomorphisms and epimorphisms. If F is contravariant faithful and Ff is a monomorphism (epimorphism) then f is an epimorphism (monomorphism), see [1]. These facts will be used later without any reference.

Let $F : S \longrightarrow S$ be a functor. Put $f \sim g$ iff $Ff = Fg$ for every f, g with a common domain and common range. Then $\sim$ is a congruence on S, called the congruence created by F. In [3] we show that every congruence on S is created by a functor and we give there the following description of all congruences on S.

<u>Theorem 1</u> [3] : Let $\sim$ be a non-trivial congruence on S. Then one of the following cases takes place.

1) There exists a normal subgroup N, of a symmetric group S_α (of all permutations of a finite cardinal α) such that, for every $f, g : X \longrightarrow Y$, $f \sim g$ iff one of the following holds:

a) $Im\, f,\ Im\, g < \alpha$.

b) There exist $k : Y \longrightarrow \alpha$, $l : \alpha \longrightarrow Y$, $h \in N$ such that
$$l \circ h \circ k \circ f = g .$$

2) There are cardinals $\alpha_1, \alpha_2, \ldots, \alpha_n, \beta_1, \beta_2, \ldots, \beta_n$ where
$$\beta_n < \beta_{n-1} < \ldots < \beta_1 \leq \alpha_1 < \alpha_2 < \ldots < \alpha_n ,$$

α_1, β_{n-1} infinite, β_n either infinite or equal to 1 such that, for every $f, g : X \longrightarrow Y$, $f \sim g$ iff one of the following holds:

a) $Im\, f,\ Im\, g < \alpha_1$.

b) $\alpha_i \leq Im\, f \simeq Im\, g < \alpha_{i+1}$, $\mathcal{U}_{fg} < \beta_i$ for some i.

c) $\alpha_n \leq Im\, f \simeq Im\, g$, $\mathcal{U}_{fg} < \beta_n$.

The congruence described in 1) is called the fine congruence with the characteristics (α, N). The congruence described in 2) is called the coarse congruence with the characteristics $\langle (\alpha_1, \ldots, \alpha_n), (\beta_1, \ldots, \beta_n) \rangle$.

The preceding theorem will now be used for an investigation of a necessary and sufficient condition for $f: X \longrightarrow Y$ in order that Ff be a monomorphism (epimorphism, isomorphism). It turns out that these conditions depend only on the congruence created by F. We may consider only functors creating fine and coarse congruences. In fact, if F creates the trivial congruence then it is, up to natural equivalence, constant on non-empty sets (and so Ff is an isomorphism for any $f: X \longrightarrow Y$, $X \neq \emptyset$).

Lemma 2. Let F be a covariant functor creating the coarse congruence with the characteristics $\langle(\alpha_1, \dots, \alpha_m), (\beta_1, \dots, \beta_m)\rangle$. Let $f: X \longrightarrow Y$ be a mapping, $X \geq \alpha_1$. Let $C_f < \beta_j$, where $j = \max_{X \geq \alpha_i} i$. Assume that either $\beta_j > \aleph_0$ or $(Y - \mathrm{Im}\, f) \geq C_f$. Then there exists a monomorphism $g: X \longrightarrow Y$ such that $Ff = Fg$.

Proof: If $(Y - \mathrm{Im}\, f) \geq C_f$, then we can find a monomorphism $g: X \longrightarrow Y$ such that $g(x) = f(x)$ for $x \in X - C_f$. By Theorem 1, $Ff = Fg$ because $\mathcal{U}_{fg} \subset f(C_f) \cup g(C_f) < \beta_j$. If $\beta_j > \aleph_0$, then there exists $Z \subset X$ such that $Z \supset C_f$, $Z - C_f \simeq Z < \beta_j$. Following the definition of C_f and taking to account that $f(Z) \cap f(X - Z) = \emptyset$, $f/_{X-Z}$ is a monomorphism. Further $f(Z) \simeq Z$ and so there is $g: X \longrightarrow Y$ such that $g/_{X-Z} = f/_{X-Z}$ and $g/_Z$ is a bijection onto $f(Z)$. Obviously, g is a monomorphism; as $\mathcal{U}_{fg} \subset f(Z) < \beta_j$, we have $Ff = Fg$ by Theorem 1.

Lemma 2*: Let F be a covariant functor creating the coarse congruence with the characteristics $\langle(\alpha_1,\dots,\alpha_m),(\beta_1,\dots,\beta_m)\rangle$. Let $f: X \longrightarrow Y$ be a mapping, $Y \geq \alpha_1$, $X \neq \emptyset$. Let $(Y - Im\, f) < \beta_j$, where $j = \max_{Y \geq \alpha_i} i$. Assume that either $\beta_j \geq \aleph_0$ or $(Y - Im\, f) \cup f(C_f) \leq C_f$. Then there exists an epimorphism $g: X \longrightarrow Y$ such that $Ff = Fg$.

Proof: If $(Y - Im\, f) \cup f(C_f) \leq C_f$, then there exists $X_1 \supset C_f$ such that $f(C_f - X_1) = f(C_f)$ and $X_1 \simeq Y - Im\, f$. Choose $g: X \longrightarrow Y$ such that $f(x) = g(x)$ for every $x \in X - X_1$ and $g(X_1) = Y - Im\, f$. Clearly, g is an epimorphism and $U_{fg} \subset (Y - Im\, f) \cup f(X_1) < 2(Y - Im\, f)$. Therefore $Ff = Fg$. If $\beta_j > \aleph_0$, then there is $Z \subset Y$ such that $(Y - Im\, f) \subset Z$ and $Z \simeq Z - (Y - Im\, f) = Z \cap Im\, f < \beta_j$. Then $f^{-1}(Z) \geq Z$ and so we can choose $g: X \longrightarrow Y$ such that

$$g/X - f^{-1}(Z) = f/X - f^{-1}(Z) \quad \text{and} \quad g(f^{-1}(Z)) = Z .$$

Obviously, g is an epimorphism; as $U_{fg} = Z < \beta_j$, we have $Ff = Fg$ by Theorem 1.

Theorem 3: Let F be a functor creating the coarse congruence $\sim$ with the characteristics $\langle(\alpha_1,\dots,\alpha_m),(\beta_1,\dots,\beta_m)\rangle$. Let $f: X \longrightarrow Y$ be a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a) Ff is a monomorphism if F is covariant, Ff is an epimorphism if F is contravariant.

b) $[f]$ is a monomorphism in $S/\sim$.

c) $[f]$ is a coretraction in $S/\sim$.

d) $C_f < \beta_i$ as soon as $X \geq \alpha_i$.

Proof: First we shall prove this theorem for a covariant functor.

a) $\Longrightarrow$ b) is obvious.

b) $\Longrightarrow$ d) Choose a bijection $g : X \longrightarrow X$ such that $g[f^{-1}(y)] = f^{-1}(y)$ for every $y \in Y$ and $g(x) \neq x$ provided $x \in C_f$. We have $f \circ g = f$ and so $[f][g] = [f][1_X]$. If $[f]$ is a monomorphism, $[g] = [1_X]$, i.e. $g \sim 1_X$. Now, apply Theorem 1 to g and 1_X . Using the fact that

$$\mathcal{U}_{g, 1_X} = C_f \ , \ Im\, g = Im\, 1_X = X \ ,$$

we get d) immediately.

d) $\Longrightarrow$ c) If $X < \alpha_1$ then $Fg \circ Ff = F(g \circ f) = F1_X$ for any $g : Y \longrightarrow X$ by Theorem 1. Thus $[g] \circ [f] = [1_X]$ and so c) holds.

Let $X \geq \alpha_1$. Put $h = i \circ f$ where $i : Y \longrightarrow Y \vee C_f$ is the canonical injection. Then h fulfils the assumptions of Lemma 2 and therefore there exists a monomorphism $g : X \longrightarrow Y \vee C_f$ such that $Fh = Fg$. Choose $\kappa : Y \vee C_f \longrightarrow X$ with $\kappa \circ g = 1_X$. Then $F(\kappa \circ i) \circ Ff = F\kappa \circ F(i \circ f) = F\kappa \circ Fg = F1_X$. Thus $[\kappa \circ i][f] = [1_X]$ and so $[f]$ is a coretraction.

c) $\Longrightarrow$ a) is evident.

Now, let F be contravariant. Then $P^- \circ F$ is covariant and it creates the same congruence as F (as P^- is faithful); further, $(P^- \circ F)f$ is a monomorphism iff Ff is an epimorphism. This concludes the proof.

<u>Theorem 3*</u>: Let F be a functor creating the coarse congruence $\sim$ with the characteristics $\langle(\alpha_1,\dots,\alpha_n), (\beta_1,\dots,\beta_n)\rangle$. Let $f: X \longrightarrow Y$ be a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a*) Ff is an epimorphism if F is covariant, Ff is a monomorphism if F is contravariant.

b*) $[f]$ is an epimorphism in $S/\sim$.

c*) $[f]$ is a retraction in $S/\sim$.

d*) $(Y - Im\, f) < \beta_i$ as soon as $Y \geq \alpha_i$.

Proof: We may again assume that F is covariant (if F is contravariant then use $P^- \circ F$ as above).

a*) $\Longrightarrow$ b*) is obvious.

b*) $\Longrightarrow$ d*) Choose $y_0 \in Im\, f$ and define $g: Y \longrightarrow Y$ by

$$g(x) = x \quad \text{for } x \in Im\, f, \quad g(x) = y_0 \quad \text{for } x \in Y - Im\, f.$$

Thus, $g \circ f = f$ and so $[g] \cdot [f] = [1_Y] \circ [f]$. If $[f]$ is an epimorphism, then $[g]$ $[1_Y]$, i.e. $g \sim 1_Y$. In case that $Y - m\, f \neq \emptyset$ we have $U_{g1_Y} = (Y - Im\, f) \cup \cup \{y_0\}$ else $U_{g1_Y} = Y - Im\, f$. Further $Im\, 1_Y = Y$ and d*) follows almost immediately from Theorem 1.

d*) $\Longrightarrow$ c*) If $Y < \alpha_1$ then $Ff \cdot Fg = F(f \circ g) = F1_Y$ for any $g: Y \longrightarrow X$ by Theorem 1. Thus $[f] \circ [g] = [1_Y]$ and so c*) holds. Let $Y \geq \alpha_1$. Put $h = f \circ p$, where p is the projection from $X \times (Y - Im\, f)^+$ to X. If $Y - Im\, f = \emptyset$ then f is a retraction and so is $[f]$. If $Y - Im\, f \neq \emptyset$ then h fulfils the assumptions of Lemma 2* and therefore there exists an epimorphism $g : X \longrightarrow Y$ such that $Fg = Fh$. Choose $j : Y \to X$ with $g \circ j = 1_Y$. Then $Ff \circ F(p \circ j) = F(f \circ p) \circ Fj = Fg \circ Fj = F1_Y$ and so $[f] \circ [p \circ j] = [1_Y]$. Thus, $[f]$ is a retraction.

c*) $\Longrightarrow$ a*) is evident.

The following corollary is obtained almost immediately from the preceding theorems. To prove it, just note that only two of all the combinations of the conditions in d) (Theorem 3) and d*) (Theorem 3*) can take place for a given mapping $f : X \longrightarrow Y$.

<u>Corollary</u>. Let F be a functor creating the coarse congruence $\sim$ with the characteristics $\langle (\alpha_1, \dots, \alpha_n), (\beta_1, \dots, \beta_n) \rangle$. Let $f : X \longrightarrow Y$ be a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a) Ff is an isomorphism.

b) $[f]$ is an isomorphism in $S/\sim$.

c) Either $X, Y < \alpha_1$ or $X \simeq Y$ and $C_f \times (Y - \operatorname{Im} f) < \beta_i$ as soon as $X \geq \alpha_i$.

<u>Theorem 4:</u> Let F be a functor creating the fine congruence $\sim$ with the characteristics (α, N). Let $f : X \longrightarrow Y$ be a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a) Ff is a monomorphism if F is covariant, Ff is an epimorphism if F is contravariant.

b) $[f]$ is a monomorphism in $S/\sim$.

c) $[f]$ is a coretraction in $S/\sim$.

d) Either $X < \alpha$ or f is a monomorphism.

Proof: We may consider the covariant case only (see the proof of Theorem 3).

a) $\Longrightarrow$ b) is obvious.

b) $\Longrightarrow$ d) Assume that $X \geq \alpha$ and that f is not a monomorphism. Then there are $x_0, y_0 \in X$ such that $f(x_0) = f(y_0)$. Define $g : X \longrightarrow X$ by $g(x_0) = g(y_0) = y_0$, $g(x) = x$ otherwise. Then $\alpha \leq Im\, 1_X \neq Im\, g$ and so $g \not\sim 1_X$ i.e. $[g] \neq [1_X]$. On the other hand, $f \circ g = f$; hence $[f][g] = [f][1_X]$ and $[f]$ is not a monomorphism.

d) $\Longrightarrow$ c) If $X < \alpha$ then $F(g \circ f) = F1_X$ for any $g : Y \longrightarrow X$. Thus $[g][f] = [1_X]$ and so c) holds. If f is a monomorphism then it is a coretraction and so is $[f]$.

c) $\Longrightarrow$ a) is evident.

<u>Theorem 4*</u>: Let F be a functor creating the fine congruence $\sim$ with the characteristics (α, N). Let $f : X \longrightarrow Y$ be a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a*) Ff is an epimorphism if F is covariant, Ff is a monomorphism if F is contravariant.

b*) $[f]$ is an epimorphism in $S/\sim$.

c*) $[f]$ is a retraction in $S/\sim$.

d*) Either $Y < \alpha$ or f is an epimorphism.

Proof: Again, we may consider F covariant (see the proof of Theorem 3*).

$a^*) \Longrightarrow b^*)$ is clear.

$b^*) \Longrightarrow d^*)$ Assume that $Y \geq \alpha$ and that f is not an epimorphism. Then there is $g : Y \longrightarrow Y$ such that $Im\, g = Im\, f$ and $g \circ f = f$. Thus $[g][f] = [1_Y][f]$ but $[g] \neq [1_Y]$ by Theorem 1. Hence $[f]$ is not an epimorphism.

$d^*) \Longrightarrow c^*)$ If $Y < \alpha$ then $F(f \circ g) = F_{1_Y}$ for any $g : Y \longrightarrow X$ by Theorem 1. Thus $[g][f] = [1_Y]$ and so $c^*)$ holds. If f is an epimorphism, then it is a retraction and so is $[f]$.

$c^*) \Longrightarrow a^*)$ is evident.

<u>Corollary</u>. Let F be a functor creating the fine congruence $\sim$ with the characteristics $(\alpha, \mathcal{N})$. Let $f : X \longrightarrow Y$ be a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a) Ff is an isomorphism.

b) $[f]$ is an isomorphism in $S/\sim$.

c) Either $X, Y < \alpha$ or f is an isomorphism.

<u>Theorem 5</u>: Let F be a functor, $f : X \longrightarrow Y$ a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a) Ff is a monomorphism if F is covariant, Ff is an epimorphism if F is contravariant.

b) Either $Ff = Fg$ for some monomorphism $g : X \to \longrightarrow Y$ or Ff is an isomorphism.

c) $F(i \circ f) = Fg$ for some monomorphisms i, g .

Proof: We may assume that F is covariant and that it creates a nontrivial congruence. It suffices to prove a) $\Longrightarrow$ b) and a) $\Longrightarrow$ c). Thus let Ff be a monomorphism.

1) Let F create the coarse congruence with the characteristics $\langle(\alpha_1, \dots, \alpha_m), (\beta_1, \dots, \beta_m)\rangle$. Let $X < \alpha_1$; if $Y \geq X$ then $Ff = Fg$ for any monomorphism $g : X \longrightarrow Y$; if $Y < X$ then, by Corollary to Theorem 3, Ff is an isomorphism and $F(i \circ f) = Fg$ for any monomorphisms $i : Y \longrightarrow \alpha_1$, $g : X \longrightarrow \alpha_1$ (see Theorem 1). Let $X \geq \alpha_1$. We can suppose $Y - Im\, f < C_f$ (or else we use Lemma 2). But then $Y - Im\, f < \beta_j$ where $j = \max\limits_{X \geq \alpha_i} i$ and Ff is an epimorphism by Theorem 3*. Thus, Ff is an isomorphism. Further, for any monomorphism $i : Y \longrightarrow Z$ $i \circ f$ fulfils the assumptions of Lemma 2 provided Z is sufficiently large and so c) holds.

2) Let F create the fine congruence with the characteristics (α, N). If $X < \alpha$ we proceed as above in case $X < \alpha_1$. If $X \geq \alpha$ then f is a monomorphism by Theorem 4 and b), c) are obvious.

<u>Theorem 5*</u>: Let F be a functor, $f : X \longrightarrow Y$ a mapping, $X \neq \emptyset$. Then the following conditions are equivalent:

a*) Ff is an epimorphism if F is covariant, Ff is a monomorphism if F is contravariant.

b*) Either $Ff = Fg$ for some epimorphism $g : X \to$ $\to Y$ or Ff is an isomorphism.

c*) $F(f \circ j) = Fg$ for some epimorphisms j, g.

Proof is quite analogous to that of Theorem 5.

Let $\sim$, $\approx$ be the congruences on S; $\sim$ is finer than $\approx$ if always $f \sim g \Rightarrow f \approx g$. Clearly, every congruence is finer than the trivial one; further, every fine congruence is finer than every coarse one. The fine congruence with the characteristics (α, N) is finer than the fine one with the characteristics (α', N') iff either $\alpha < \alpha'$ or $\alpha = \alpha'$ and $N \subset N'$. Finally, if $\sim$, $\approx$ are coarse congruences with the characteristics $\langle(\alpha_1, \dots, \alpha_m), (\beta_1, \dots, \beta_m)\rangle$, $\langle(\alpha'_1, \dots, \alpha'_m), (\beta'_1, \dots, \beta'_m)\rangle$ then $\sim$ is finer than $\approx$ iff $\alpha'_1 \geq \alpha_1$ and, for every i, $\beta'_i \geq \beta_j$ where $j = \max_{\alpha'_i \geq \alpha_k} k$.

Using Theorems 3, 3*, 4, 4* and their corollaries we get immediately the following

<u>Theorem 6:</u> Let F, G be functors of the same variance. Then the following conditions are equivalent:

1) For every $f : X \longrightarrow Y$, $X \neq \emptyset$, if Ff is a monomorphism then so is Gf.

2) For every $f : X \longrightarrow Y$, $X \neq \emptyset$, if Ff is an epimorphism then so is Gf.

3) For every $f : X \longrightarrow Y$, $X \neq \emptyset$, if Ff is an isomorphism then so is Gf.

4) Either the congruence created by F is finer than that one created by G or F and G create fine congruences with the characteristics (α, N) and (α, N') respectively for some α, N, N'.

<u>Corollary.</u> Let F, G be functors of the same vari-

ance. Then the following conditions are equivalent:

1) For every $f : X \longrightarrow Y$, $X \neq \emptyset$, Ff is a monomorphism iff Gf is.

2) For every $f : X \longrightarrow Y$, $X \neq \emptyset$, Ff is an epimorphism iff Gf is.

3) For every $f : X \longrightarrow Y$, $X \neq \emptyset$, Ff is an isomorphism iff Gf is.

4) Either F and G create the same congruence or F and G create fine congruences with the characteristics (α, N), (α, N') respectively, for some α, N, N'.

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