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ON FULL EMBEDDINGS OF CATEGORIES OF ALGEBRAS INTO CATEGORIES OF FUNCTORS WITH THIN DOMAIN

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(Preliminary communication)

Following [1], a category $\mathcal{K}$ is said to be binding if every category of universal algebras can be fully embedded into it.

Definition. A small category $\mathcal{k}$ is said to be rich if the category $S^{\mathcal{k}}$ (of all functors from $\mathcal{k}$ into the category of sets) is binding.

In [3],[4], various questions concerning rich monoids are studied. The aim of the present note is to present two theorems concerning rich thin x) categories.

Theorem 1. Let $\mathcal{k}$ be a finite thin category. Let M, a non-trivial monoid without a non-trivial (i.e., non-identical) idempotent be given. Then the following properties of $\mathcal{k}$ are equivalent:

(1) $\mathcal{k}$ is rich.

(2) $S^{\mathcal{k}}$ contains $\aleph_1$ non-isomorphic rigid objects xx).

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x) We recall that a category is said to be thin if there is at most one morphism with given domain and range.

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xx) An object a is called rigid if there is no nonidentical $\alpha : a \rightarrow a$.

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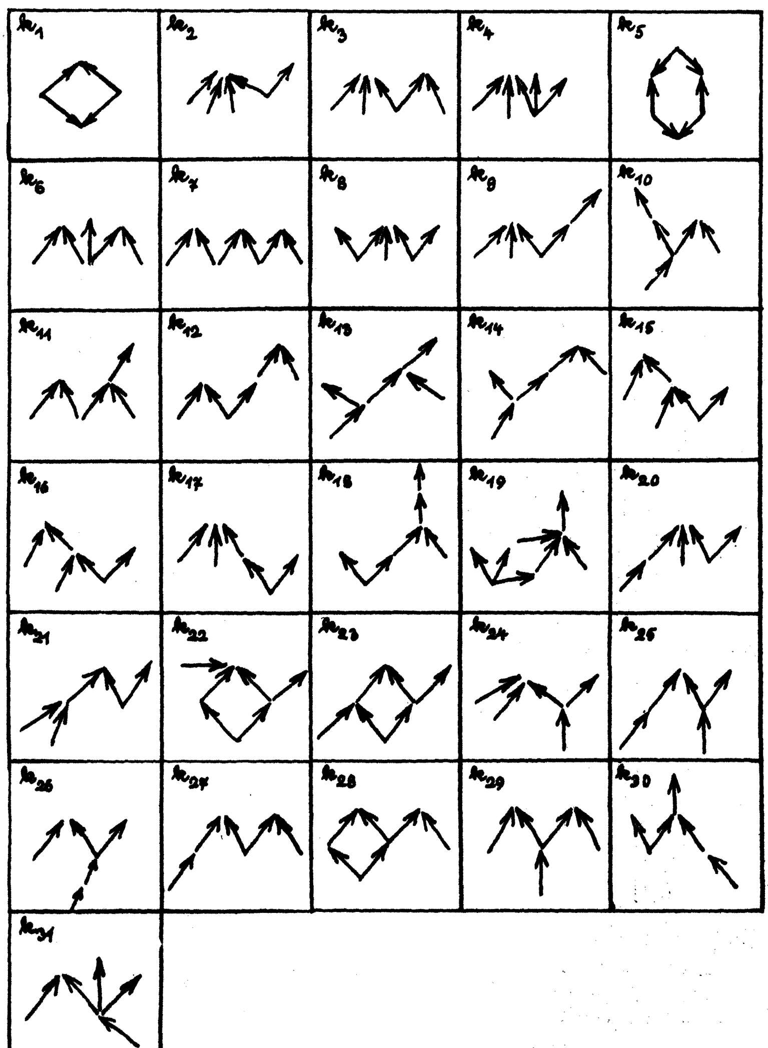
(3) M can be fully embedded into $g^{\mathcal{K}}$.

(4) Some one from the following categories $\mathcal{K}_1, \ldots, \mathcal{K}_{31}$ is a full subcategory of $\mathcal{K}$ (the identities and the composed morphisms are not indicated:



Definition. We say that a category $\mathfrak{K}$ is a category with trivial composition if either α or β is an identity whenever the composition $\alpha \circ \beta$ of morphisms α, β is defined.

Theorem 2. Let $\mathfrak{K}$ be a small thin category with trivial composition. Then the assertions (1) - (4) from the previous theorem are also equivalent. (Now, of course, $\mathfrak{K}_9$ - $\mathfrak{K}_{31}$ in (4) are superfluous.)

References

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