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ON CERTAIN THEOREMS OF BERRY AND A LIMIT THEOREM OF FELLER
(Preliminary communication$^{x)}$)

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The purpose of this paper is to improve the statements of certain theorems due to A.C. Berry in [1] (our theorems 1,2,3) and of a limit thoerem due to W. Feller in [2] (our theorem 4), in accordance with the results of V.M. Zolotarev [4] .

Let $F_1(x), F_2(x), \dots, F_n(x)$ be the distribution functions of mutually independent random variables

(1) $$X_1, X_2, \dots, X_n ,$$

$F(x)$ the distribution function of the sum

$$X = \sum_{k=1}^{n} X_k$$

and

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt, \quad x \in (-\infty, \infty) .$$

Let a_k $(k = 1, 2, \dots, n)$, $b > 0$ be real numbers and $a = \sum_{k=1}^{n} a_k$.

Let

$$\overline{M} = \sup_x |F(x) - \Phi(\frac{x-a}{b})| , \quad x \in (-\infty, \infty) .$$

For a given real number $\varkappa > 0$ we define the following quantities

$$\varkappa_0 = \sum_{k=1}^{n} P\{|X_k - a_k| > \varkappa b\} ,$$

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x) The complete text will be published in the Matematicko-fyzikálny časopis

$$\bar{\varkappa}_1 = \frac{1}{b}\sum_{k=1}^{n} \left| \int_{a_k - \bar{\varkappa} b}^{a_k + \bar{\varkappa} b} (x - a_k)\, dF_k(x) \right|$$

$$\bar{\varkappa}_2 = \left| 1 - \frac{1}{b^2}\sum_{k=1}^{n} \int_{a_k - \bar{\varkappa} b}^{a_k + \bar{\varkappa} b} (x - a_k)^2\, dF_k(x) \right|$$

Theorem 1 For a given $\bar{\varkappa} > 0$, let $\bar{\varkappa}_i \leq \bar{\varkappa}$, $i = 0, 1, 2$. Then $\bar{M} < 4,647\, \bar{\varkappa}$

Suppose that for $k = 1, 2, \dots, n$ the mean values $E(X_k) = \alpha_k$ and the dispersions $\sigma_k^2 = E(X_k - \alpha_k)^2$ of the random variables (1) are finite. Moreover, let

$$\alpha = \sum_{k=1}^{n} \alpha_k \quad \text{and} \quad \sigma = \sqrt{\sum_{k=1}^{n} \sigma_k^2}\ .$$

Put $M = \sup_x | F(x) - \Phi(\frac{x-\alpha}{\sigma}) |$, $x \in (-\infty, \infty)$.

We define the quantities (for a given real number $\varkappa > 0$)

$$\varkappa_0 = \sum_{k=1}^{n} P\{ |X_k - \alpha_k| > \varkappa \sigma \}$$

$$\varkappa_1 = \frac{1}{\sigma}\sum_{k=1}^{n} \left(\int_{-\infty}^{\alpha_k - \varkappa\sigma} + \int_{\alpha_k + \varkappa\sigma}^{\infty} \right) (x - \alpha_k)\, dF_k(x)$$

$$\varkappa_2 = \frac{1}{\sigma^2}\sum_{k=1}^{n} \left(\int_{-\infty}^{\alpha_k - \varkappa\sigma} + \int_{\alpha_k + \varkappa\sigma}^{\infty} \right) (x - \alpha_k)^2\, dF_k(x)$$

Theorem 2 For a given $\varkappa > 0$, let $\varkappa^2 \varkappa_0 \leq \varkappa_2$, $\varkappa \varkappa_1 \leq \varkappa_2$, $\varkappa_2 \leq \varkappa^3$. Then $M < 3,188\, \varkappa$

Theorem 3 For a given $\varkappa > 0$, let $\varkappa^2 \varkappa_0 \leq \varkappa_2$, $\varkappa \varkappa_1 \leq \varkappa_2$. For random variables X_k $(k = 1, 2, \dots, n)$ let $\mu_{s,k} = E(|X_k - \alpha_k|^s) \leq L < \infty$ for some $s > 2$ (not necessarily integral). Then $M < 3,188\, (\tilde{\varepsilon}^*)^{\frac{1}{s+1}}$, where $\tilde{\varepsilon}^* = \frac{1}{\sigma^s}\sum_{k=1}^{n} \mu_{s,k}$

Theorem 4 Suppose that $X_1, X_2, \dots, X_n$ are mutually independent random variables, which satisfy the following conditions: For $k = 1, 2, \dots, n$, 1) $E(X_k) = 0$ 2) $E(X_k^2) = \sigma_k^2 \leq L < \infty$ 3) $|X_k| < \lambda \sigma$, where $\lambda > 0$, $\sigma^2 = \sum_{k=1}^{n} \sigma_k^2$. Suppose further that $0 < \lambda x < \frac{1}{12}$

For $x \in (-\infty, \infty)$ let

$$F(x) = P\left(\sum_{k=1}^{n} X_k < x\right) \text{ and } \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy .$$

Then we have

$$1 - F(x\sigma) = e^{-\frac{1}{2}x^2 Q(x)} \{1 - \Phi(x) + \theta \lambda e^{-\frac{1}{2}x^2}\} ,$$

where

a) $|\theta| < 7,464$ b) $Q(x) = \sum_{\nu=1}^{\infty} q_\nu x^\nu$,

with

$$|q_1| \leq \frac{1}{3}\lambda , \quad |q_\nu| < \frac{1}{8}(12\lambda)^\nu, \quad \nu = 2, 3, \ldots$$

Furthermore, for every $0 < i < j \leq n$,

$$|Q^{(j)}(x) - Q^{(i)}(x)| < 1,256 \frac{\sigma^{(j)2} - \sigma^{(i)2}}{\sigma^2} ,$$

where $Q^{(k)}(x) = Q(x)$ and $\sigma^{(k)2} = \sigma^2$ for $k = n$.

R e f e r e n c e s

[1] A.C.BERRY: The accuracy of the Gaussian approximation to the sum of independent variates. Trans. Amer.Math.Soc.49(1941),122-136.

[2] W. FELLER: Generalization of a probability limit theorem of Cramér. Trans.Amer.Math.Soc.54(1943), 361-372

[3] K. TAKANO: A remark to a result of A.C. Berry. Res. Mem.Inst.Math.9,6(1951)

[4] В.М. ЗОЛОТАРЕВ: Абсолютная оценка остаточного члена в центральной предельной теореме. Теория вероятностей и ее применения,т.XI,(1966),вып.1, стр.108-119

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