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THE CONSISTENCY OF SOME THEOREMS CONCERNING LEBESGUE MEASURE

Preliminary communication

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Let I_0 be the set of all constructive real numbers of the unit segment $I = \langle 0; 1 \rangle$. Thus, $I_0 = I \cap L$, where L is the class of constructive sets (see [G]). μ is the Lebesgue measure on I.

The proof of the following theorem is similar to the Vitali's construction:

If $I_0 \neq I$ and I_0 is Lebesgue measurable, then $\mu(I_0) = 0$. If $I_0 \neq I$ and I_0 has the Baire property, then I_0 is of first category.

Using this theorem, the author can construct a model of the set theory Σ^* in which the following assertions hold:

$$(\alpha)(2^{\aleph_\alpha} = \aleph_{\alpha+1}),$$

all cardinals of Δ -model are absolute i.e. the cardinals of Δ are precisely those of the whole thery Σ^*,

$$\mu(I_0) = 0.$$

The results of Vopěnka (see [V]) and Hajnal (see [H]) imply a metatheorem:

Let φ be a formula for which $\vdash_{\Sigma^*} (\alpha)(\varphi(\alpha) \rightarrow \alpha \in On) \& (\exists ! \alpha)\varphi(\alpha)$. Then there is a model of the theory Σ^* in which the following assertions hold:

$$(\alpha)(\varphi(\alpha) \rightarrow 2^{\aleph_0} = \aleph_{\alpha+1})$$

all cardinals of Δ -model are absolute,

$$(\alpha)(cg(\alpha) \rightarrow (x)(x \subseteq I \;\&\; \mathrm{card}\; x \leq \aleph_\alpha . \rightarrow (u(x) = 0)).$$

R e f e r e n c e s

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