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A NOTE TO MATHEMATICAL ASPECTS OF THE POLITICAL DECISION THEORY

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This paper deals with the mathematical model of such a human society that is living in dynamic economical equilibrium under simple-commodity-production-like conditions. In this situation the Marx labour value of goods can be defined (and namely in a unique way), the price of each good is equal to it and socially necessary labour is equal to one really employed. The paper is a mathematical one (i.e. men, goods, market, production etc. are mathematical objects and processes with only those properties which are given in axioms). Following G. Debreu's methods (see [1]), O. Lange's ideas (see [7]), studies in collective activity of finite automata (see [4]) and axiomatic studies in racional behavior, this paper is a part of "mathematical politology" or "mathematical theory of political decisions" (see [10]) [1]) and it has been inspired directly by the fruitful summer session on "Methods of mathematical optimization in economics" of Centro internationale matematico estivo in L'Aquila (september 1965) (see [2]). Many of economical ideas of this paper are given in [6] by K. Kříž to whom the author is obliged for the useful consultations[2]).

Let $S = \{1, 2, \ldots, u\}$ be a finite set of finite automata (called people or men) index ed by ι, μ an automaton (called a market) not being in S. Each of automat on of a system (S, μ) (called society) acts (does an action α_{m+1}) in any unite time interval (t_m, t_{m+1}) (called a period m of a cycle t), where $m \in \{0, 1, \ldots, 10\}$, $t \in \{0, 1, 2, \ldots\}$ and a time t_m means $t \,.\, 10 + m$ time units (hence $t_{10} = (t + 1)_0$, α_0 does not exist and $\alpha_{11} = \alpha_1$). In each t_m ι is in state $\sigma_\iota(t_m) = : (\pi_\iota(t_m), \varkappa_\iota(t_m), \varrho_\iota(t_m), g_\iota(t_m), v_\iota(t_m), i_\iota(t_m))$ and μ in $\sigma_\mu(t_m) = : (\pi_\mu(t_m), \varkappa_\mu(t_m), p(t_m)$, $(\sigma(t) = : (\sigma_1(t), \ldots, \sigma_\iota(t), \ldots; \sigma_\mu(t))$ is called a state of society (S, μ) in a time $t)$, where parameters $\pi_\iota(t_m), \ldots$ and actions $\alpha_1, \ldots$ are defined as follows: Consider two integers n, n_1, $1 \leqq n_1 < n$, and denote $N = : \{1, 2, \ldots, n\}$, $\mathrm{I} = : \{1, 2, \ldots, n_1\}$, $\mathrm{II} = : \{n_1 + 1, \ldots, n\}$. Let E^n be a Euclidean n-space and A and n by n matrix[3]) [4]) such that

(1) for each its row $A^i \geq {}^T\sigma$,[5])

(2) $A_j = o$ holds only if $j \in \mathrm{II}$,[6])

(3) for every $i \in \mathrm{I}$ there exists a sequence $i = i_1 < i_2 < \ldots < i_k$ so that $a_{i_{r+1}i_r} > 0$ for all $r = 1, \ldots, k - 1$ and $i_k \in \mathrm{II}$.[7])

Let subsets T_ι (being not nullsets) of N be given (T_ι is called a set for ι possible professions)[8]) and let $k^{\min}, k^{\max} \in \mathsf{E}^n$ be given, $k^{\min} \leqq k^{\max}$, $[k^{\min}]^{\mathrm{I}} = [k^{\max}]^{\mathrm{I}} = o$, $[k^{\min}]^{\mathrm{II}} > o$.[9])

Let it be $o \leqq \pi_\iota(t_m) \in \mathsf{E}^n$, $[\pi_\iota(t_m)]^{\mathrm{II}} = o$ (called production means which ι has in t_m in his possession), $o \leqq \varkappa_\iota(t_m) \in \mathsf{E}^n$, $[\varkappa_\iota(t_m)]^{\mathrm{I}} = o$ (called consumer goods possessed by ι in t_m), $o \leqq \pi_\mu(t_m) \in \mathsf{E}^n$, $[\pi_\mu(t_m)]^{\mathrm{II}} = o$ (called production means possessed by μ in t_m), $o \leqq \varkappa_\mu(t_m) \in \mathsf{E}^n$, $[\varkappa_\mu(t_m)]^{\mathrm{I}} = o$ (called consumer goods possessed by μ in t_m), $o \leqq \varrho_\iota(t_m) \in \mathsf{E}^n$, $o < p(t_m) \in \mathsf{E}^n$ (called price vector), ${}^{T}ep(t_m) = 1$, and at least $g_\iota(t_m)$ (called money), $v_\iota(t_m)$, $i_\iota(t_m)$ be functions having their values in non-negative reals, or $\{0, 1\}$ or in T_ι, respectively.

Let one have

(4) $v_\iota(0_0) = 1$, $\varrho_\iota(0_0) = o$ for each ι and $\pi_\mu(0_0) = o$, $\varkappa_\mu(0_0) = o$ and

(5) $v_\iota(t_m) = 0$ if $v_\iota(t_{m-1}) = 0$ and in this case $\sigma_\iota(t_m) = \sigma_\iota(t_{m-1})$. Define $S(t_m) = \{\iota : \iota \in S,\ v_\iota(t_m) = 1\}$ [10]) From (4) and (5) one has $S = S(0_0) \supset S(t_{m-1}) \supset S(t_m)$ for each m.

Actions α's are defined by transition $\sigma(t_{m-1}) \rightarrow \sigma(t_m)$ of states (and we notice in α'definitions only those parameters, which are eventually changing):

(6) $\pi_\iota(t_1) = \pi_\iota(t_0) - \varepsilon_\iota {}^{T}A^{i(t_0)}$, where ε_ι is in $[0, 1]$ such maximal number, for which the right side is $\geqq o$, $\varrho_\iota(t_1) = \varrho_\iota(t_0) + \varepsilon_\iota e_{i_\iota}$.[11])

(7) $\varrho_\iota(t_2) = o$, $g_\iota(t_2) = g_\iota(t_1) + {}^{T}p(t_1)\,\varrho_\iota(t_1)$, $\pi_\mu(t_2) = \pi_\mu(t_1) + \sum \varrho_\iota(t_1)$ (where sum operates on all ι's with $i_\iota(t_1) \in \mathrm{I}$), $\varkappa_\mu(t_2) = \varkappa_\mu(t_1) + \sum \varrho_\iota(t_1)$ (where ι's are all such that $i_\iota(t_1) \in \mathrm{II}$).[12])

(8) $\varkappa_\iota(t_4) = \varkappa_\iota(t_3) - k^{\min}$ if the right side is $\geqq o$ (in this case it is $v_\iota(t_4) = 1$), in other case we have $\varkappa_\iota(t_4) = \varkappa_\iota(t_3)$ and put $v_\iota(t_4) = 0$.[13])

Let u vector functions $d_\iota(p, g)$ be given (for each $\iota \in S$), where $p \in \mathsf{E}^n$, $p > o$, ${}^{T}ep = 1$, g real number, $g \geqq 0$, $d_\iota(p, g) \in \mathsf{E}^n$, $d_\iota(p, g) > o$.[14])

One says i^* suits for ι in t_0 if $d(t_0) =: (E - A)\,p(t_0)$ is $> o$ (in this case $d(t)$ is called K. Marx's socially evaluated labour) and i^* maximizes $[d(t_0) - d_\iota(p(t_0), g_\iota(t_0))]^i$ on $i \in T_\iota$.[15])

(9) $i_\iota(t_5)$ is such an integer i that maximizes on T_ι $[d(t_4)]^i - [d_\iota(p(t_0), g_\iota(t_0))]^i$ and i is by ι chosen arbitrarily if $i_\iota(t_4)$ is not among them, in other case $i = i_\iota(t_4)$.[16])

(10) $[\varkappa_\iota(t_8)]^i = [\varkappa_\iota(t_7)]^i - [k^{\max} - k^{\min}]^i$ if on the right side is a non-negative number, in other case we put $[\varkappa_\iota(i_8)]^i = 0$.[17])

Let a vector function $\psi(x, \xi) \in \mathsf{E}^n$ ge given, $x \in [0, \infty)$, $\xi \in \mathsf{E}^n$, $\xi \geqq o$ such that $\psi(x, \xi') \geqq \psi(x, \xi'')$ for $\xi' \geqq \xi''$, $\psi(x, \xi) \leqq x \cdot e$, $\psi(0, \xi) = o$ and $\psi(x, o) = o$ for each x and ξ, and $[\psi(x, \xi)]^i > 0$ for $x > 0$, $\xi^i > 0$.

(11) $\pi_\mu(t_9) = o$, $\varkappa_\mu(t_9) = o$, $g_\iota(t_9) = g_\iota(t_8) - [\psi(g_\iota(t_8), \pi_\mu(t_8) + \varkappa_\mu(t_8))]^{i_\iota(t_0)}$. [18])

Let a vector function $\Phi(\sigma_\mu) \in \mathsf{E}^n$ be given such that $\Phi > o$, ${}^T e\Phi = 1$ and $\Phi(\sigma_\mu) = p$ for $\sigma_\mu = (\pi_\mu = o, \varkappa_\mu = o, p)$.

(12) $p(t_{10}) = \Phi(\sigma_\mu(t_8))$.[18])

Let λ and a be n by u matrices, all $\geqq O$, λ_μ, p, g vectors, $\lambda_\mu \geqq o$, $\lambda_\mu \in \mathsf{E}^n$, $p > o$, $p \in \mathsf{E}^n$, ${}^T ep = 1$, $g \in \mathsf{E}^u$, $g \geqq o$, $p(\lambda)$, $g(\lambda)$, $\lambda_\mu(\lambda)$ vector functions, $p(\lambda) \in \mathsf{E}^n$, $p(\lambda) > o$, ${}^T e(p(\lambda)) = 1$, $o \leqq g(\lambda) \in \mathsf{E}^u$, $o \leqq \lambda_\mu(\lambda) \in \mathsf{E}^n$, $\Lambda = \Lambda(\lambda, \lambda_\mu, g, p)$ n by u matrix $\geqq O$ and $\lambda(0)$ a fixed n by u matrix $\geqq O$. Let L be such that $\emptyset \neq L \subset S$, $p(\lambda) = p(\lambda(0))$ for all λ, $\lambda_\mu(\lambda) =: \lambda_\mu(\lambda(0)) - \sum_{\iota \in L} [\lambda - \lambda(0)]_\iota$, $[g(\lambda)]^\iota =: [g(\lambda(0))]^\iota - {}^T[\lambda - \lambda(0)]_\iota p(\lambda)$.

We call (λ, Λ, a) to be a marked system, if it holds

$$(*) \qquad \Lambda > O \Leftrightarrow \lambda_\iota^i < a_\iota^i,\ [\lambda_\mu]^i > 0,\ [g]^\iota > 0,$$

(**) to every $\lambda(0), \lambda_\mu(\lambda(0)), g(\lambda(0)), p(\lambda(0))$ the system $\mathrm{d}\lambda/\mathrm{d}\tau = \Lambda(\lambda, \lambda_\mu(\lambda), g(\lambda), p(\lambda))$ has only one solution $\{\lambda(\tau)\}_{\tau \in [0,1]}$ starting from a given $\lambda(0)$,

and

(***) $\dot\lambda(1) = O$ holds for the solution from (**).

Hence any market system defines a transition $(\lambda(0), \lambda_\mu(\lambda(0)), g(\lambda(0))) \to (\lambda(1), \lambda_\mu(\lambda(1)), g(\lambda(1)))$.[19])

Let three market systems be given.

(13) Put in the first system $L = S(t_2)$, $\lambda_\iota(0) = \varkappa_\iota(t_2)$, $\lambda_\mu(\lambda(0)) = \varkappa_\mu(t_2)$, $[g(\lambda(0))]^\iota = g_\iota(t_2)$, $p(\lambda(0)) = p(t_2)$, $a_\iota = k^{\min}$ and define $\varkappa_\iota(t_3)$, $\varkappa_\mu(t_3)$, $g_\iota(t_3)$ by means of the system transition.

(14) Put in the second system $L = S(t_5)$, $\lambda_\iota(0) = \pi_\iota(t_5)$, $\lambda_\mu(\lambda(0)) = \pi_\mu(t_5)$, $[g(\lambda(0))]^\iota = g_\iota(t_5)$, $p(\lambda(0)) = p(t_5)$, $a_\iota = {}^T A^{i_\iota(t_5)}$ and define $\pi_\iota(t_6)$, $\pi_\mu(t_6)$, $g_\iota(t_6)$ by means of the system transition.

(15) Put in the third system $L = S(t_6)$, $\lambda_\iota(0) = \varkappa_\iota(t_6)$, $\lambda_\mu(\lambda(0)) = \varkappa_\mu(t_6)$, $[g(\lambda(0))]^\iota = g_\iota(t_6)$, $a_\iota = k^{\max} - k^{\min}$ and define $\varkappa_\iota(t_7)$, $\varkappa_\mu(t_7)$, $g_\iota(t_7)$ by means of the system transition.[20])

It follows from the definition that to every $\sigma(0_0)$ a sequence $\{\sigma(t_0)\}_{t=0}^{\infty}$ is determined (called a life of society (S, μ)), but not uniquely (remember arbitrary choice of $i_\iota(t_4)$ in (9)).

Define n by n matrix $A^{\min}$ and $s(t_m) \in \mathsf{E}^n$ by $(A^{\min})^i =: A^i + {}^T k^{\min}$, $[s(t_m)]^i = \operatorname{card}\{\iota : i_\iota(t_m) = i,\ v_\iota(t_m) = 1\}$.

We call $\sigma(t_0)$ an equilibrium if it holds:

$$\text{(i)} \qquad S(t_0) \neq \emptyset \text{ and } g_\iota(t_0) = 0,\ \pi_\iota(t_0) = {}^T A^{i_\iota(t_0)},\ \varkappa_\iota(t_0) = o \quad \text{for each } \iota \in S(t_0).$$

(ii) $i_\iota(t_0)$ is for each living ι in t_0 suitable.

(iii) $$[{}^T s(t_0)\,(E - A)]_{\mathrm{I}} = {}^T o.$$

(iv) $${}^T s(t_0)\,(E - A^{\min}) \geqq {}^T o.$$

(v) $$(E - A^{\min})\,p(t_0) \geqq o.$$

(vi) In the transition $(\lambda(0), \lambda_\mu(\lambda(0)), g(\lambda(0))) \to (\lambda(1), \lambda_\mu(\lambda(1)), g(\lambda(1)))$ by means of a third marked system for $\lambda(0) = O$, $\lambda_\mu(\lambda(0)) = {}^T s(t_0)$ $(E - A^{\min})$, $[g(\lambda(0))]^\iota = [(E - A^{\min})\,p(t_0)]^{i_\iota(t_0)}$ it is $g(\lambda(1)) = o$.

Remark: Society (S, μ) with considered properites exists, as we easily see from this example (which is in 0_0 in equilibrium): $n = 2$,

$$n_1 = 1,\ A = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 0 \end{pmatrix},\ k^{\min} = \left(0, \frac{1}{4}\right),\ k^{\max} = (0,1),\ p(0_0) = \left(\frac{1}{2}, \frac{1}{2}\right),$$

$T_1 = \{1\}$, $T_2 = \{2\}$, $v_1(0_0) = v_2(0_0) = 1$, $g_1(0_0) = g_2(0_0) = 0$, $\varkappa_1(0_0) = \varkappa_2(0_0) = o$, $\pi_1(0_0) = \left(\frac{1}{2}, 0\right)$, $\pi_2(0_0) = \left(\frac{1}{2}, 0\right)$, $\varkappa_\mu(0_0) = o$, $\pi_\mu(0_0) = o$, $i_1(0_0) = 1$, $i_2(0_0) = 2$, $\varrho_1(0_0) = \varrho_2(0_0) = o$, $d_1(p, g) = d_2(p, g) = 1$, $\psi(x, \xi) = x \,.\, e$, $\Phi(\sigma_\mu(t_m)) = p(t_m)$ and in all three market systems let it be $\Lambda_\iota^i = [\lambda_\mu]^i .$ (sgn max $(0, [g]^\iota)$) (sgn max $(0, [\lambda_\mu]^\iota)$) (sgn max $(0, a_\iota^i - \lambda_\iota^i)$).

Let us add to each ι's state $\sigma_\iota(t_m)$ other parameter $l_\iota(t_m) \in \mathsf{E}^n$ (called labour vector) with these properties: For each living ι it is

(16) $$[l_\iota(t_0)]^{T_\iota} = e^{T_\iota},\ [l_\iota(t_0)]^{N - T_\iota} = o,$$

(17) $$[l_\iota(t_1)]^{i_\iota(t_0)} = 1 - \varepsilon_\iota,$$

(18) $$[l_\iota(t_4)]^{i_\iota(t_4)} = 1.$$ [21])

One says an abstract labour is done if n linear real functions $\beta_i(y)$, $y \in [0, \infty)$ are given such that $\beta_i(0) = 0$ and $\beta_i(y) > 0$ for $y > 0$ (i.e. $\beta_i(y) = [\beta]^i\, y$, where $\beta \in \mathsf{E}^n$ is any fixed point $> o$). [22])

Notice that

(19) $\delta(y) =: (E - A)\,p(t_0)y$ for $\sigma(t_0)$ being equilibrium is an abstract labour (hence $\delta(1) = d(t_0)$ is K. Marx's socially evaluated labour).

We say $[w]^i$ is the labour value of a good i, if $(E - A)\,w = \beta$ has unique and positive solution $w \in \mathsf{E}^n$. [23])

We say a point $s \in \mathsf{E}^n$ is reproductive if $s > o$ and ${}^T s(E - A) \geqq {}^T o$. One says K is a fundament of s, if $\emptyset \neq K \subset N$, if $a_{ij} = 0$ for $i \notin K$, $j \in K$ and if $[{}^T s(E - A)]^K = {}^T o$. [24])

In [9] it is shown that

(20) $E - A$ is regular and it holds $(E - A)^{-1}\, l \geq o$ for $l \geq o$ (with $(E - A)^{-1}\, l > o$ for $l > o$) for A with our properties (1), (2), (3) if reproductive vector exists and has no fundament.

Theorem. *Let $\{\sigma(t_0)\}_{\bar{t}=0}^{\infty}$ be a life of a society (S, μ) and $\sigma(\bar{t}_0)$ be an equilibrium. Then it is*

(a) $\sigma(\bar{t}_0) = \sigma[(\bar{t} + 1)_0] = \ldots,$

(b) $s(\bar{t}_0) = s[(\bar{t} + 1)_0] = \ldots$ *is reproductive and has no fundament,*

(c) *to every abstract labour $\beta(y)$ the labour value w of goods is well-defined (in every cycle)),*

(d) *for $\delta(y)$ as abstract labour (in equilibrium) labour value of goods is equal to their prices.*

Proof: From $S(\bar{t}_0) \neq \emptyset$ [see (i)] we have $s(\bar{t}_0) \geq o$. It results $[s(\bar{t}_0)]^{\mathrm{II}} > o$ (from (iv), (2) and $[k^{\min}]^{\mathrm{II}} > o$) which implies $[s(\bar{t}_0)]^{\mathrm{I}} > o$ (from (iv)) it follows ${}^{T}s(\bar{t}_0)\,(E - A) \geqq o$ which results $[s(\bar{t}_0)]^{i_r} = 0$ for all i_r in (3), where $i \in \mathrm{I}$ is such that $[s(\bar{t}_0)]^{i} = 0$, but $i_k \in \mathrm{II}$ — a contradiction). Hence $s(\bar{t}_0)$ is reproductive. We have $K \subset \mathrm{I}$ for any fundament K (because of $[{}^{T}s(\bar{t}_0)\,(E - A)]^{K} = {}^{T}o$ but ${}^{T}s(\bar{t}_0)\,(E - A) = \{{}^{T}o^{\mathrm{I}}, {}^{T}[s(\bar{t}_0)]^{\mathrm{II}}\}$). Let $i \in K$ and $i = i_1 < i_2 < \ldots < i_k$ be a sequence from (3). Since $a_{i_2 i_1} > 0$ there is $i_2 \in K$. For the same reason there is $i_3, \ldots, i_k \in K$. But $i_k \in \mathrm{II}$ and we get a contradiction. Hence $s(\bar{t}_0)$ has no fundament.[25]) It follows now immediately from (20) and (19) that (c) and (d) are true. For the proof of the whole theorem it suffices to prove $\sigma(\bar{t}_0) = \sigma[(\bar{t} + 1)_0]$: Because of $\pi_\iota(\bar{t}_0) = {}^{T}A^{i_\iota(\bar{t}_0)}$ for $\iota \in S(\bar{t}_0)$ [see (i)] we have from (6) $\pi_\iota(\bar{t}_1) = o$ and $\varrho_\iota(\bar{t}_1) = e_{i_\iota(\bar{t}_0)}$ (because of $\varrho(\bar{t}_0) = o$ [see (i)] and $i_\iota(\bar{t}_1) = i_\iota(\bar{t}_0)$). Hence according to (7) $\varrho_\iota(\bar{t}_2) = o$ (and hence $\varrho_\iota[(\bar{t} + 1)_0] = o$), $g_\iota(\bar{t}_2) = [p(\bar{t}_0)]^{i_\iota(\bar{t}_0)}$ (because of $g_\iota(\bar{t}_0) = 0$ [see (i)], $p_\iota(\bar{t}_1) = p_\iota(\bar{t}_0)$ and $\varrho_\iota(\bar{t}_1) = e_{i_\iota(\bar{t}_0)}$), ${}^{T}\pi_\mu(\bar{t}_2) = ({}^{T}[s(\bar{t}_0)]^{\mathrm{I}}, {}^{T}o^{\mathrm{II}})$ and ${}^{T}\varkappa_\mu(\bar{t}_2) = ({}^{T}o^{\mathrm{I}}, {}^{T}[s(\bar{t}_0)]^{\mathrm{II}})$ (because of $\pi_\mu(\bar{t}_0) = \pi_\mu(\bar{t}_1) = o$, $\varkappa_\mu(\bar{t}_0) = \varkappa_\mu(\bar{t}_1) = o$ [see (4), (11), (6)] and $\varrho_\iota(\bar{t}_1) = e_{i_\iota(\bar{t}_0)}$). We have $\varkappa_\iota(\bar{t}_3) = k^{\min}$, ${}^{T}\varkappa_\mu(\bar{t}_3) = ({}^{T}o^{\mathrm{I}}, {}^{T}[s(\bar{t}_0)]^{\mathrm{II}} - (\sum_{i \in N} [s(\bar{t}_0)]^{i})\, {}^{T}[k^{\min}]^{\mathrm{II}})$, $g_\iota(\bar{t}_3) = [p(\bar{t}_0)]^{i_\iota(\bar{t}_0)} - {}^{T}k^{\min} p(\bar{t}_0)$ (Because $\{\lambda_\mu[\lambda(0)]\}^{\mathrm{I}} = o$ [from (13)] we have from (*) $[\lambda(1)]^{\mathrm{I}} = o$, i.e. from (13) $[\varkappa_\iota(\bar{t}_3)]^{\mathrm{I}} = o$. Let $i' \in \mathrm{II}$, ι' exist such that $\lambda^{i'}_{\iota'}(1) < a^{i'}_{\iota'}$. From (*) it must be either $[\lambda_\mu(\lambda(1))]^{i'} = 0$ or $[g(\lambda(1))]^{i'} = 0$. The first equality does not hold because of (iv), the second is not true for (v). Hence for each ι $a^{i}_{\iota} = \lambda^{i}_{\iota}(1)$ and we have finished.). From this by (8) we receive $\varkappa_\iota(\bar{t}_4) = o$, $v_\iota(\bar{t}_4) = 1$ and from (16), (17), (18) $l_\iota(\bar{t}_0) = l_\iota(\bar{t}_4)$ (hence $v_\iota((\bar{t} + 1)_0) = v_\iota(\bar{t}_0)$, $S((\bar{t} + 1)_0) = S(\bar{t}_0)$ and $l_\iota(\bar{t} + 1)_0) = l_\iota(\bar{t}_0)$). From (ii) and (9) we have $i_\iota(\bar{t}_5) = i_\iota(\bar{t}_0)$ (and hence $i_\iota((\bar{t} + 1)_0) = i_\iota(\bar{t}_0)$). One receives $\pi_\iota(\bar{t}_6) = {}^{T}A^{i_\iota(\bar{t}_0)}$, $\pi_\mu(\bar{t}_6) = o$, $g_\iota(\bar{t}_6) = [p(\bar{t}_0)]^{i_\iota(\bar{t}_0)} - {}^{T}k^{\min} p(\bar{t}_0) - A^{i_\iota(\bar{t}_0)}\, p(\bar{t}_0)\, (\pi_\iota(t_6) =$

$= A^{i_\iota(\bar{t}_0)}$ follows from (14) and (iv) by the same argument like in α_3 and $\pi_\iota(\bar{t}_0) = o$ follows from (iii)). From (15) and (vi) we have $\varkappa_\iota(\bar{t}_7) \leqq \leqq k^{\max} - k^{\min}$, $g_\iota(\bar{t}_7) = 0$. The last equality results $\varkappa_\mu(\bar{t}_7) = o$ (because of $\sum_{\iota \in S(\bar{t}_6)} g_\iota(\bar{t}_6) = {}^T s(\bar{t}_0)\ (E - A)\ p(\bar{t}_0) - (\sum_{i \in N} [s(\bar{t}_0)]^i)\ ({}^T k^{\min} . p(\bar{t}_0)) = ({}^T o^{\mathrm{I}},\ {}^T[s(\bar{t}_0)]^{\mathrm{II}})\ p(\bar{t}_0) - ({}^T k^{\min}\ p(\bar{t}_0))$ card $S(\bar{t}_0) = {}^T[s(\bar{t}_0)]^{\mathrm{II}}\ [p(\bar{t}_0)]^{\mathrm{II}} - [{}^T k^{\min}\ p(\bar{t}_0))$ card $S(\bar{t}_0) = \sum_{i \in \mathrm{II}} ([s(\bar{t}_0)]^i - [k^{\min}]^i$ card $S(\bar{t}_0))\ [p(\bar{t}_0)^i = {}^T\dot{\varkappa}_\mu(\bar{t}_6)\ p(\bar{t}_6))$. From this it is (by (10)) $\varkappa_\iota(\bar{t}_8) = 0$ and $\pi_\mu(\bar{t}_9) = o$, $\varkappa_\mu(\bar{t}_9) = o$, $g_\iota(\bar{t}_9) = 0$ (from (11) because $g_\iota(\bar{t}_8) = g_\iota(\bar{t}_7) = 0$). This results from (12) $p(\bar{t}_{10}) = p(\bar{t}_0)$ and hence for each ι $i_\iota[(\bar{t} + 1)_0]$ suits for ι in $(\bar{t} + 1)_0$; Q.E.D.

Notices:

[1] Summary of [10]: For the reliable application of operation research branches in political economy, sociology of politics and politology a new branch (interlinking them) has been appeared (the author offers a name "the mathematical theory of political decisions" for its destination, or a name "mathematical politology" as analogon with mathematical economy) completely rebuilding the economical, sociological and political theories in an axiomatic way (being "experients" in political "laboratory"). It is a new instrument of the Marxist analysis.

[2]) From K. Kříž's paper these economical ideas are used here: A partition $N = \mathrm{I} + \mathrm{II}$, definition of $d(t)$ using it for a choice of a profession, consideration of $d(t)$ as an abstract labour and its resulting equality $w = p$.

[3] For a matrix A we denote A^i or A_j its i-th row or j-the column (and in general for $S_1, S_2 \subset N$ one denotes $A^{S_1}_{S_2}$ a matrix (a_{ij}), $i \in S_1$, $j \in S_2$) ${}^T A$ its transpose and AB the matrix multiplication row by column. If A, B are of the same type we write $A > B$ or $A \geq B$ or $A \geqq B$ if for corresponding elements it holds always $>$ or $\geqq$ but not always $=$ or $\geqq$. Points of Euclidean space E^k are matrices k-by-1 and x^i is the i-th coordinate of $x \in \mathsf{E}^n$ (hence for $x \in \mathsf{E}^n$, $S = \{i_1, \ldots, i_t\} \subset N$, x^S is such a point of E^t, that ${}^T(x^S) = (x^{i_1} \ldots, x^{i_t})$, ${}^T o = : (0, \ldots, 0)$, ${}^T e = : (1, \ldots, 1)$, $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)$ and $E = : (\delta_{ij})$ (for $\delta_{ii} = 1$, $\delta_{ij} = 0$ $(i \neq j)$, $i, j \in N$). $\emptyset$ is nullset and card K is the number of elements of the set K. For real x, sgn $x = : 1$ if $x > 0$, $= 0$ if $x = 0$, $= -1$ if $x < 0$.

[4] Our society can be interpreted in this manner (the interpretation of our model (S, μ) is given in this and other notices): There exist n different kinds of goods. Each good is measured in fixed units. For

$A = (a_{ij})$ a_{ij} is the quantity of a good j which vanishes during the production of the unit of the i^{th} one. The production takes place in production branches. Each good is produced in one branch only (namely, the good i in the branch i). Each branch produces one kind of goods only. Hence a Leontief production is considered — see e.g. [5] or[3].

5) No good can be produced from nothing.

6) Goods with indices of II (called consumer goods) are never used in any production.

7) Any good with an index in I (called production means) is needed for production of at least one consumer good.

8) I.e. the set of branches in which ι is able to work.

9) ι's consume only consumer goods and namely ι must consume at least $k^{\min}$ for each period but at most $k^{\max}$. The assumption $[k^{\min}]^{\mathrm{II}} > o$ is not for a loss of generality, for we substitute the fact "in society one car is bought in one period by 10% of people" to the fact "each buys $\frac{1}{10}$ of car in one period".

10) If we say ι lives in t_m if $v_\iota(t_m) = 1$, then $S(t_m)$ is a set of living men and a man once died never becomes living again.

11) Action α_1 defined in (6) is a production. Each living ι works in a branch $i_\iota(t_0)$ and works with an intensity ε_ι. In this case he produces $\varepsilon_\iota e_{i_\iota(t_0)}$ goods. Each ι can work with the intensity ε_ι and he works with the maximal intensity he can. Each ι can produce in α_1 at most one unit of a good $i_\iota(t_0)$. Hence $\pi_\iota(t_0)$ is such production means, which ι has for his production (he needs an input $\varepsilon_\iota A^{i_\iota(t_0)}$). From (6) it follows that production (given in A) is technologically the same for all periods—hence no progress is considered. Hence in t_1 each ι has these kinds of goods in his possession: production means $\pi_\iota(t_1)$, consumer goods $\varkappa_\iota(t_1)$ and $\varrho_\iota(t_1)$ (with only one component eventually positive—his produced good).

12) In action α_2 defined in (7) each living ι sells the whole result of his work [i.e. $\varrho_\iota(t_1)$] to μ and receives from μ money according to the prices $p(t_1) = p(t_0)$. We consider that μ has unlimited lot of money.

13) In action α_4 each living ι consumes the minimal consumption $k^{\min}$ (if he has it in his possession). In this case the reproduction of his working ability is done and hence he remains to be living i.e. $v_\iota(t_4) = 1$. If he has not minimal consumption of his possession, he died ($v_\iota(t_4) = 0$) and lives the living society for ever [see (5)].

[14] $[d_\iota(p, g)]^i$ can be interpreted as ι's subjective avaluation of labour in a branch i if ι works with full intensity (for instance the income which ι has to receive for production of one unit of a good i). Evidently this evaluation depends on prices p on the marked and on his money wealth g.

[15] K. Kříž's socially evaluated labour is social evaluation of labour, i.e. an income which a man receives for production of one unit of a good i if he can sell this good and if he can buy all he needs for the next production.

[16] Each ι believes that he sells all he will produce (in the next production) and that he will work with the intensity $\varepsilon = 1$. Hence he chooses such profession which yields him a greatest income (according to his expectations and his individual evaluation). It is better to consider d_ι in t_0 rather than in t_4, because $p(t_4) = p(t_0)$ but $g(t_4)$ is deformed by marked activity (ι received money for $\varrho_\iota(t_1)$ and as we shall see gives money in α_3 for $k^{\min}$).

[17] Each living ι in α_8 consumes all he can.

[18] If anyone cannot sell the result of his production, he loses. This fact we substitute by this one: Each sells all his production to μ, μ destroys all goods unsold, but losses in corresponding branches are in depreciation Ψ of money wealth (action α_9) and transition $p(t_0) \to p[(t + 1)_0]$ of prices (in α_{10}) becomes as a result.

[19] Each ι has $\lambda_\iota^i(0)$ of a good i in the time $\tau = 0$ and $[g(\lambda(0))]^\iota$ units od money. μ has $[\lambda_\mu(\lambda(0))]^i$ units of a good i on stock. Then each ι starts to suck up each good i with a speed Λ_ι^i from μ's stock $[\lambda_\mu]^i$ and simultaneously μ sucks up from each ι the corresponding amount of money under current (constant) prices $p = p[\lambda(0)]$. The suction stops if either ι has no money, or μ has no quantity of a good i or if ι has a good i in to him satisfactory quantity a_ι^i. The suction finished during unit time interval and the speed of suction in time $\tau \in [0, 1]$ depends on the whole situation.

[20] We consider μ as a market with fully information for all. Therefore we assume that in α_2 ι sells to μ all its production, in α_3 each living ι tries to buy so much consumer goods from μ to have $k^{\min}$, in α_4 consumes it (i.e. each wants to live and does not want to die). In α_6 he buys from μ production means for the input of the next production. In α_7 he buys luxury goods.

[21] Let us assume there exist n different kinds of labour. Any kind of labour is measured in fixed units. For the production of a unit of the good i ι needs to give one unit of labour i and no quantity of labour j, $j \neq i$. For ε units one emits ε of it. Hence each ι can give in α_1 at most one unit of labour i for each $i \in T_\iota$.

[22]) It can be interpreted as the evaluation of labour: y units of labour i is $\beta_i(y)$ units of abstract labour.

[23]) K. Marx has defined (see [8]) the labour value $[w]^i$ of a good i as a quantity of abstract "living" labour (i.e. of the labour (measured in the units of abstract one) emiting during the production) plus the quantity of labour "objectivized" (i.e. the quantity of abstract labour, which is contained as a labour value in goods that were vanished during the production). Hence $[w]^i = [\beta]^i + \sum_{j=1}^{n} a_{ij}[w]^j$ for each $i \in N$, where $\beta_i(y) = [\beta]^i \, . \, y$ is an abstract labour.

[24]) If we consider $[s]^i$ as a sum of all intensities ε_ι with which ι's working during α_1 in a branch i work i.e. the amount of units of good i the whole branch i produces in α_1), then the fact of nonexistence of K is equivalent to this property: no set of branches forms "selfpurposed" production.

[25]) Hence the aim of the whole production is to produce consumer goods only and no branch is indespensable.

Mathematical politology reguires its axiomes to be true and considers the society as a system.

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