Jiří Jarušek

On the regularity of solutions of a thermoelastic system under noncontinuous heating regimes. III

Applications of Mathematics, Vol. 37 (1992), No. 4, 275–288

Persistent URL: http://dml.cz/dmlcz/104510

ON THE REGULARITY OF SOLUTIONS OF A THERMOELASTIC SYSTEM UNDER NONCONTINUOUS HEATING REGIMES. PART III

JIŘÍ JARUŠEK

(Received August 13, 1991)

Summary. In this part we weaken the sufficient condition to obtain the stresses continuous and bounded in the threedimensional case, and we treat a certain coupled system.

Keywords: Spatial isotropy and anisotropy, heat equation, Lamé system, coupled system, viscoelasticity

AMS classification: 73U05 (35M05, 35R05)

0. INTRODUCTION

The goal of this part of the series of papers is to improve the results of [2] for a threedimensional bounded domain Ω and to give some results also for a certain coupled case, where the solution of the Lamé system is included into the heat equation, too. Understanding under the "spatial isotropy" or the "spatial anisotropy" of the heating regime T the same or the different regularity of T in both space variables, respectively, we remove in Sec. 1, devoted to the isotropic case, the gap between the result for the linear heat equation with Ω being a strip (Thm. 3 of [1]) and the general nonlinear case (Thm. 1 of [2]). In Sec. 2 we find a less restrictive condition for the "regular space direction" provided T is noncontinuous both in time and in the other space direction. T is still supposed to be non-decreasing in time or (in a more general setting) having a bounded variation in time uniformly with respect to $x \in \partial\Omega$. In Sec. 3 we extend our results to a certain coupled thermoelastic system with $\operatorname{div} \dot{v}$ in the heat equation and the Lamé system including the acceleration and a viscosity term.

In Secs. 1 and 2 we still consider the heat equation in the form

$$\beta_0 \frac{\partial u}{\partial t} = \Delta u \quad \text{on } Q = (0,\mathcal{T}) \times \Omega, \tag{1}$$

$$\frac{\partial u}{\partial \nu} = g(T) - g(u) \quad \text{on } S = (0,\mathcal{T}) \times \partial\Omega, \quad u(0,\cdot) = 0 \text{ on } \Omega$$

and the Lamé system e.g. in its homogeneous and isotropic version

$$(1-2\sigma)\Delta v + \nabla \operatorname{div} v = (2+2\sigma)\nabla\gamma(u) \quad \text{on } \Omega,\ t \in \langle 0,\mathcal{T}\rangle, \tag{2}$$

$$(1-2\sigma)(\frac{\partial v}{\partial \nu} + ((\nu, \nabla_i v)_i)) + 2\sigma\nu \operatorname{div} v = (2+2\sigma)\gamma(u)\nu \quad \text{on } \partial\Omega,\ t \in \langle 0,\mathcal{T}\rangle,$$

but the results concerning a sufficiently smooth $\partial\Omega$ are valid also for the linear non-homogeneous and anisotropic Lamé system. Our aim is to find sufficient conditions ensuring that all components of the stress tensor

(3)

$$\tau_{ij} = \frac{E(u)}{(2+2\sigma)(1-2\sigma)} \left((1-2\sigma)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + \delta_{ij} 2\sigma \sum_{k=1}^{3} \frac{\partial v_k}{\partial x_k} \right), \quad i,j = 1,2,3,$$

are continuous and bounded on $\bar{Q}$. Here δ denotes the Kronecker symbol.

1. A sharper result for the quasi-coupled threedimensional case with the "spatially isotropic" heating regimes

In this section we shall suppose the above described behaviour of T in time and, moreover, $g(T) \in L_2(0,\mathcal{T}; H^{\alpha}(\Omega))$, $\alpha = 1+\eta$ with $0 < \eta$ arbitrarily small. We shall prove

Theorem 1. *Let $\Omega \subset \mathbf{R}^3$ be a bounded domain with a $C_{\frac{5}{2}+\eta}$-smooth boundary for some $\eta > 0$, let g from (1) be C_2-smooth and non-decreasing on $\mathbf{R}_+ = \langle 0, +\infty)$, let $g(0) = 0$. Let γ from (2) be C_2-smooth, let E from (3) be continuous. Let the heating regime T be non-negative with $T(0) = 0$ and let it satisfy all the above stated requirements. Then the corresponding stress tensor belongs to $C_0(\bar{Q}; \mathbf{R}^9)$.*

Remark . As in (22) of [2], to obtain $g(T) \in L_2(0,\mathcal{T};\ H^{1+\eta}(\Omega))$, $\eta \in (0,1)$, for nonlinear g, we need $T \in L_2(0,\mathcal{T}; H^{2+\eta}(\Omega))$ or $\nabla T \in L_2(0,\mathcal{T};\ C_{\eta}(\Omega))$.

Proof of Thm. 1. From Sec. 1 of [2] we know that $u \in H^{1,2}(Q)$, $\nabla u \in H^{\frac{5}{8}-\varepsilon}(0,\mathcal{T};\ L_2(\Omega;\mathbf{R}^3))$. Extending the function u onto $\mathbf{R}^4$ (e.g. by the local method described in [2], taken from [3], Chapter 1), we prove for the Fourier transform $\hat{u}$ of this extended u and for some constant $c > 0$ independent of $\hat{u}$

(4)

$$\frac{\tau^2}{(1+\tau^2)^{\frac{3}{16}+\varepsilon_0}} |\xi|^{1+\varepsilon} |\hat{u}|^2 \leqslant c(|\tau|^{\frac{5}{4}-\varepsilon}|\xi|^2 + \tau^2)|\hat{u}|^2 \in L_1(\mathbf{R}^4), \quad \varepsilon_0(\varepsilon) \searrow 0 \text{ for } \varepsilon \searrow 0.$$

The reason of the following formal procedure lies in the possibility of approximating our u e.g. by a solution of (1) with a more regular T obtained e.g. by the time mollification of the original T. Then we obtain $\dot{u}$ in some better space for $\dot{u}/_{\partial\Omega} \in L_2(\Omega)$ to have sense. The following procedure will be carried out for such a solution, all the estimates will be uniform with respect to this approximation and by the limit procedure we obtain the required estimate for the original u, too. We put $\dot{u}_{-\ell} - \dot{u}$ into the variational formulation of (1) (see e.g. (34) of [1]; here we include Λ into g). Here ℓ is a shift in the time argument. For this regular u it is not difficult to prove that the term

$$(5) \qquad \int_{-\delta}^{\delta} \|\dot{u}_{-\ell} - \dot{u}\|^2_{H^{-\frac{3}{16}-\varepsilon_0}(0,T;L_2(\partial\Omega))} |\ell|^{-1+\varepsilon} \mathrm{d}\ell, \quad \varepsilon > 0 \text{ arbitrarily small, } \delta > 0 \text{ arbitrary,}$$

can be estimated by means of the norm on the left hand side of (4), cf. e.g. [5]. On the left hand side of the resulting variational inequality we estimate in fact the terms

$$(6) \quad \beta_0 \int_{-\delta}^{\delta} \int_Q (\dot{u}_{-\ell} - \dot{u})^2 |\ell|^{-1-2\alpha} \mathrm{d}\ell \,\mathrm{d}x \mathrm{d}t + \frac{1}{2} \int_{-\delta}^{\delta} \int_\Omega (\nabla(u_{-\ell} - u))^2 (x, t_0) |\ell|^{-1-2\alpha} \mathrm{d}\ell \,\mathrm{d}x, \ t_0 \in \langle 0, T \rangle,$$

on the right hand side we have the boundary terms which will be estimated as follows:

$$(7) \quad \int_{-\delta}^{\delta} \int_S (g(u_{-\ell}) - g(u))(\dot{u}_{-\ell} - \dot{u}) |\ell|^{-1-2\alpha} \mathrm{d}\ell \,\mathrm{d}x \mathrm{d}t \leqslant c\Big(\int_{-\delta}^{\delta} |\ell|^{-1+\varepsilon} \|\dot{u}_{-\ell} - \dot{u}\|^2_{H^{-\frac{3}{16}-\varepsilon_0}(0,T;\, L_2(\Omega))} \mathrm{d}\ell + \sup_{\langle 0, \bar{B} \rangle} g' \int_{-\delta}^{\delta} \|u_{-\ell} - u\|^2_{H^{\frac{3}{16}+\varepsilon_0}(0,T;\, L_2(\Omega))} |\ell|^{-1-4\alpha-\varepsilon} \mathrm{d}\ell \Big),$$
$$\varepsilon, \varepsilon_0 \text{ arbitrarily small,}$$

with $\bar{B} = \sup_S T$, $c > 0$ a constant independent of u. Using the extension and renormation technique described in Lemma 1 of [1] and [5], we estimate for $\alpha < \frac{5}{16}$ the second term on the right hand side of (7) by $\|\dot{u}\|_{L_2(Q)}$ via (4). Repeating the same procedure for $g(T)$, we must restrict ourselves to $\alpha < \frac{5}{32} = \frac{1}{2}(\frac{1}{2} - \frac{3}{16})$. Thus we have estimated $\|\dot{u}\|_{H^{\frac{5}{32}-\varepsilon}(0,T;L_2(\Omega))}$ for every $\varepsilon \in (0, \frac{5}{32})$. As the estimation does not depend on the above mentioned approximation of T, the relation $\dot{u} \in \bigcap_{\varepsilon > 0} H^{\frac{5}{32}-\varepsilon}(0,T;\, L_2(\Omega))$ holds also for our original T due to the weak convergence.

But now we can take $|\tau|^{2+\frac{5}{16}-\varepsilon}$ instead of τ^2 on the right hand side of (4) and repeat the procedure with the exponent $\frac{3}{16}-\frac{5}{64}-\varepsilon_0$ instead of $\frac{3}{16}-\varepsilon_0$ at the term $(1+\tau^2)$. By such an iterative procedure we finally obtain

$$\dot u \in \bigcap_{\varepsilon>0} H^{\frac{5}{24}-\varepsilon}(0,T;L_2(\Omega)) \tag{8}$$

and we remark that we need only $T \in \bigcap_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon}(S)$ in the whole procedure.

By virtue of (8), using the extension of u onto $\mathbf{R}^4$ again, we can prove by the Hölder inequality applied to the $H^{\frac{29}{24}-\varepsilon,2-\varepsilon}$-norm of u that $\dot u \in \bigcap_{\varepsilon>0} L_2(0,T;\, H^{\frac{10}{29}-\varepsilon}(\Omega))$ (this is valid even for $T \in \bigcap_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon}(S)$). For our more regular T, however, the result together with the interpolation theorem for the elliptic equations (cf. [3], Chapter 1 and [1], Prop. 4) yields $u \in \bigcap_{\varepsilon>0} L_2(0,T;\, H^{2+\frac{10}{29}-\varepsilon}(\Omega))$. Iterating this procedure, we prove finally that $u \in \bigcap_{\varepsilon>0} L_2(0,T;\, H^{\frac{5}{2}-\frac{1}{12}-\varepsilon}(\Omega))$, because $\frac{5}{2}-\frac{1}{12} = 2+\frac{10}{29}\sum_{n=0}^{+\infty}(\frac{5}{29})^n$. Now, using the Hölder inequality $|\tau|^{\frac{23}{12}-\varepsilon_0(\varepsilon)}|\xi|^{1+\varepsilon} \leqslant c(|\tau|^{\frac{29}{12}-\varepsilon}+|\xi|^{\frac{29}{6}-\varepsilon})$, $\tau \in \mathbf{R}^1$, $\xi \in \mathbf{R}^3$, with $\varepsilon_0(\varepsilon) \searrow 0$ for $\varepsilon \searrow 0$ ($c>0$ is a suitable constant independent of τ, ξ and small $\varepsilon>0$), we prove formally $\dot u \in \bigcap_{\varepsilon>0} H^{-\frac{1}{24}-\varepsilon}(0,T;L_2(\partial\Omega))$ and thus by the above described time-regularization procedure $\dot u \in \bigcap_{\varepsilon>0} H^{\frac{11}{48}-\varepsilon}(0,T;L_2(\Omega))$. But then, using this result, we obtain by the above described space-regularization procedure $\dot u \in \bigcap_{\varepsilon>0} L_2(0,T;H^{\frac{1}{2}-\frac{35}{708}-\varepsilon}(\Omega))$ and $u \in \bigcap_{\varepsilon>0} L_2(0,T;H^{\frac{5}{2}-\frac{35}{708}-\varepsilon}(\Omega))$.

To prove the theorem, we use both the regularization procedures simultaneously. We have proved that $u \in \bigcap_{\varepsilon>0} H^{\frac{5}{4}-\beta_1-\varepsilon,\frac{5}{2}-\beta_2-\varepsilon}(Q)$ and we can suppose $0<\frac{20}{11}\beta_1 \leqslant \beta_2 \leqslant 2\beta_1 < \frac{1}{20}$ in the first step of the simultaneous procedure (the time regularization). Using the inequality of the Hölder type (with c_1 a positive constant independent of τ, ξ and small $\varepsilon>0$)

$$|\tau|^{2d_1-\varepsilon_0(\varepsilon)}|\xi|^{1+\varepsilon} \leqslant c_1(|\tau|^{\frac{5}{2}-2\beta_1-\varepsilon}+|\xi|^{5-2\beta_2-\varepsilon}), \tag{9}$$
$$\tau\in\mathbf{R}^1,\ \xi\in\mathbf{R}^3,\ \varepsilon_0(\varepsilon)\searrow 0 \text{ for } \varepsilon\searrow 0$$

$$\text{with } d_1 = \frac{4-2\beta_2}{5-2\beta_2}\left(\frac{5}{4}-\beta_1\right) = 1-\frac{4\beta_1+\frac{\beta_2}{2}-2\beta_1\beta_2}{5-2\beta_2} > 1-\frac{5\beta_1}{5-2\beta_1} > 1-\frac{25}{24}\beta_1,$$

we prove $u \in H^{1-\frac{25}{24}\beta_1+\eta}(0,T;L_2(\partial\Omega))$ for some $\eta>0$ and thus by the above described procedure we prove $u \in H^{\frac{5}{4}-\frac{25}{48}\beta_1}(0,T;\,L_2(\Omega))$. The second step (the space regularization) exploits this result and the Hölder inequality again. Using the corresponding extension of u and its Fourier transform, we prove

$$\dot u \in \bigcap_{\varepsilon>0} L_2(0,T;H^{d_2-\varepsilon}(\Omega))$$

with

$$d_2 = \frac{12 - 25_{\beta_1}}{60 - 25_{\beta_1}} \left(\frac{5}{2} - \beta_2 \right) = \frac{1}{2} - \frac{10\beta_1 + \frac{12}{5}\beta_2 - 5\beta_1\beta_2}{12 - 5\beta_1} > \frac{1}{2} - \frac{316}{475}\beta_2 > \frac{1}{2} - \frac{2}{3}\beta_2 .$$

Using the interpolation theorem, we prove $u \in L_2(0, T; H^{\frac{5}{2} - \frac{2}{3}\beta_2}(\Omega))$. Thus we take $\frac{2}{3}[\beta_1, \beta_2]$ as a new couple and repeat the whole procedure. Using such an iteration process we finally obtain

$$u \in \bigcap_{\varepsilon > 0} H^{\frac{5}{4} - \varepsilon,\, \frac{5}{2} - \varepsilon}(Q). \tag{10}$$

With the help of the local tangential regularization described in [2] Sec. 1, we immediately prove the local tangential regularity of the order $\frac{5}{2} + \eta - \varepsilon$, $\varepsilon > 0$ arbitrarily small. Theorem 1 of [1] yields $u \in \bigcap_{\varepsilon > 0} C_0(0, T; H_{\mathrm{loc}}^{\frac{3}{2} - \varepsilon,\, \frac{3}{2} + \frac{3}{5}\eta - \varepsilon,\, \frac{3}{2} + \frac{3}{5}\eta - \varepsilon}(\Omega))$, where the symbol $H^{\alpha}_{\mathrm{loc}}$ for a vector α indicates the existence of such local coordinates (being sufficiently smoothly equivalent to the original ones) for every point $x \in \partial\Omega$, that in a neighbourhood of x (denoted U_x) $u/_{\Omega \cap U_x} \in H^{\alpha}(\Omega \cap U_x)$ with respect to the local coordinates. Confront the description of the localization technique in [2], Sec. 1. (Of course, for $x \in \Omega$ u is much more regular in some neighbourhood of x.) Via the local coordinate method we can now prove the same assertion for all components of the stress tensor and via the imbedding theorem (cf. Prop. 2 of [2]) we complete the proof. □

2. The "spatially anisotropic" case

In this section we shall suppose again $T \in \langle 0, \bar{B} \rangle$ on S, $T(0) = 0$ and

$$g(T) \in \bigcap_{\varepsilon > 0} H_{\mathrm{loc}}^{\frac{1}{2} - \varepsilon, \frac{1}{2} - \varepsilon, \frac{11}{6} + \eta}(S) \quad \text{for some} \quad \eta > 0, \tag{11}$$

where the first superscript corresponds to the time and the other to the space variables. The index loc denotes that for every $x \in \partial\Omega$ we are able, after a suitable shift of the origin and sufficiently smooth straightening of the boundary, to find local coordinates such that $g(T) \in \bigcap_{\varepsilon > 0} H^{\frac{1}{2} - \varepsilon}(0, T; L_2(V_x)) \cap \bigcap_{\varepsilon > 0} L_2(0, T; H^{\frac{1}{2} - \varepsilon, \frac{11}{6} + \eta}(V_x))$ on a neighbourhood $V_x \subset \{0\} \times \mathbf{R}^2$. Of course, the transformation of the coordinates must be sufficiently smooth (we suppose it to be at least of the class $C_{3+\eta}$ with η from (11)). In such a coordinate system we shall suppose x_1 to be the normal coordinate, x_2 the tangential coordinate in which T is possibly discontinuous and x_3 the tangential coordinate in which T is regular. We shall suppose $g \in C_2(0, \bar{B})$,

$\gamma \in C_{3+\eta}(0, \bar{B})$. $\partial\Omega$ must be globally of the class $C_{\frac{7}{3}+\eta}$, but along the x_3-variable it must be of the class $C_{3+\eta}$.

From the preceding section and [2], Sec. 1, we know that $u \in \bigcap_{\varepsilon>0} H^{\frac{29}{24}-\varepsilon, 2-\varepsilon}(Q) \cap \bigcap_{\varepsilon>0} H^{\frac{11}{12}-\varepsilon, \frac{3}{2}-\varepsilon}(S)$. Using the last result, we can improve the result of Prop. 5 of [1]. In fact, in the crucial estimate (45) there we can take $2\alpha - \frac{1}{2} < \frac{11}{12}$, i.e. $\alpha < \frac{17}{24}$ and prove $\nabla u \in \bigcap_{\varepsilon>0} H^{\frac{17}{24}-\varepsilon}(0, T; L_2(\Omega; \mathbb{R}^3))$. With this result we can improve the auxiliary formal estimate for $\dot{u}/_S$ (which in fact represents the existence of the corresponding fractional derivative in time and will be only used together with the limit procedure like in Sec. 1—see (4) and what follows). By means of the Hölder inequality for the Fourier transform of the extended u we prove $\dot{u} \in \bigcap_{\varepsilon>0} H^{-\frac{1}{24}-\varepsilon}(0, T; L_2(\partial\Omega))$. Let us suppose in general that we have proved $\dot{u} \in \bigcap_{\varepsilon>0} H^{\frac{1}{4}-\frac{1}{3}\frac{1}{2^k}-\varepsilon}(0, T; L_2(\Omega))$ and $\nabla u \in \bigcap_{\varepsilon>0} H^{\frac{3}{4}-\frac{1}{3}\frac{1}{2^k}-\varepsilon}(0, T; L_2(\Omega; \mathbb{R}^3))$ for some k (in fact we have proved it for $k = 3$). With the help of the Hölder inequality $|\xi|^{1+\varepsilon}|\tau|^{2-\frac{1}{3}\frac{1}{2^{k-1}}-\varepsilon_0(\varepsilon)} \leqslant c(|\tau|^{\frac{5}{2}-\frac{1}{3}\frac{1}{2^{k-1}}-\varepsilon} + |\tau|^{\frac{3}{2}-\frac{1}{3}\frac{1}{2^{k-1}}-\varepsilon}|\xi|^2)$ which is valid for $\tau \in \mathbb{R}^1$, $\xi \in \mathbb{R}^3$, with a suitable constant $c > 0$ (independent of τ, ξ and small $\varepsilon > 0$) and with $\varepsilon_0 \searrow 0$ for $\varepsilon \searrow 0$, we prove $\dot{u} \in \bigcap_{\varepsilon>0} H^{-\frac{1}{3}\frac{1}{2^k}-\varepsilon}(0, T; L_2(\partial\Omega))$. In the estimation similar to that in the proof of Prop. 5 of [1] we can now take $2\alpha - \frac{1}{2} < 1 - \frac{1}{3}\frac{1}{2^k}$, i.e. $\alpha < \frac{3}{4} - \frac{1}{3}\frac{1}{2^{k+1}}$ and prove $\nabla u \in \bigcap_{\varepsilon>0} H^{\frac{3}{4}-\frac{1}{3}\frac{1}{2^{k+1}}-\varepsilon}(0, T; L_2(\Omega; \mathbb{R}^3))$. Simultaneously in our estimate (7) made for $g(T)$ instead of $g(u)$ we can take $2\alpha < \frac{1}{2} - \frac{1}{3}\frac{1}{2^k}$, i.e. $\alpha < \frac{1}{4} - \frac{1}{3}\frac{1}{2^{k+1}}$ and prove $\dot{u} \in \bigcap_{\varepsilon>0} H^{\frac{1}{4}-\frac{1}{3}\frac{1}{2^{k+1}}-\varepsilon}(0, T; L_2(\Omega))$. Using this iterative procedure we can finally prove

$$(12) \quad u \in \bigcap_{\varepsilon>0} H^{\frac{5}{4}-\varepsilon, 2-\varepsilon}(Q) \cap \bigcap_{\varepsilon>0} H^{1-\varepsilon, \frac{3}{2}-\varepsilon}(S), \quad \nabla u \in \bigcap_{\varepsilon>0} H^{\frac{3}{4}-\varepsilon}(0, T; L_2(\Omega; \mathbb{R}^3)).$$

The Hölder inequality $|\tau|^{1+\varepsilon}|\xi|^{\frac{8}{3}-\varepsilon_0(\varepsilon)} \leqslant c(|\tau|^{\frac{3}{2}-\varepsilon}|\xi|^2 + |\xi|^{4-\varepsilon})$, which is valid for $\tau \in \mathbb{R}^1$, $\xi \in \mathbb{R}^3$, a constant $c > 0$ (independent of τ, ξ and small $\varepsilon > 0$) and with $\varepsilon_0(\varepsilon) \searrow 0$ for $\varepsilon \searrow 0$, combined with (12) and Thm.1 of [1] yields

$$(13) \qquad u \in \bigcap_{\varepsilon>0} C_0(0, T; H^{\frac{4}{3}-\varepsilon}(\Omega)).$$

We remark that the time regularity of u as in (12) is valid even for

$$T \in \bigcap_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon}(0, T; L_2(\Omega)).$$

In fact, starting with the inequality analogous to (4) in Sec. 1 and the iterative procedure described there we prove

$$\dot{u} \in \bigcap_{\varepsilon>0} H^{\frac{1}{4}-\frac{1}{18}-\varepsilon}(0,T;L_2(\Omega)) \cap \bigcap_{\varepsilon>0} H^{-\frac{1}{9}-\varepsilon}(0,T;L_2(\partial\Omega)).$$

By the corresponding Hölder inequality we have $\nabla u \in \bigcap\limits_{\varepsilon>0} H^{\frac{3}{4}-\frac{1}{18}-\varepsilon}(0,T;L_2(\Omega;\mathbf{R}^3))$ and the above described iterative procedure yields the time regularity as in (12).

Now, we shall proceed in the tangential regularization as in [2], Sec. 1, but only in the "regular" variable x_3. In the rest of the section we shall denote by the regularization step α (the α-regularization step) the proof of the fact $u \in \bigcap\limits_{\varepsilon>0} H_{\mathrm{loc}}^{\frac{5}{4}-\varepsilon,2-\varepsilon,2-\varepsilon,1+\alpha}(Q)$, $\alpha \geqslant 1$. From (20) of [2] we know that for a regularization step $\alpha \in \langle 1, \frac{3}{2})$ the use of the fractional derivative norm of an order less than 1 for $g(u)$, $g(T)$ is sufficient. For such α we can avoid the estimation like (16) of [2] and the proof of

$$u \in \bigcap_{\varepsilon>0} H_{\mathrm{loc}}^{\frac{5}{4}-\varepsilon,\,2-\varepsilon,\,2-\varepsilon,\,\frac{5}{2}-\varepsilon}(Q) \cap \bigcap_{\varepsilon>0} H_{\mathrm{loc}}^{1-\varepsilon,\,\frac{3}{2}-\varepsilon,2-\varepsilon}(S) \tag{14}$$

is practically straightforward. For the same reason for $\alpha \in \langle \frac{5}{2}, \frac{7}{2})$ we need (using Lemma 1 of [1]) such norms for $\frac{\partial}{\partial x_3} g(u)$. Using the Hölder inequality like in (16) of [2] and (14), we obtain from the imbedding theorem (Prop. 2 of [2])

$$\begin{aligned} &\left(1+\frac{2}{3}+\frac{1}{2+a}\right)\left(\frac{1}{2}-\frac{1}{2p}\right)+\frac{1}{2+a}<1 \Longrightarrow p<\frac{13+5a}{7-a}, \\ &\frac{1}{2p}\left(1+\frac{2}{3}+\frac{1}{2+a}\right)+\frac{a'}{2+a}<1 \Longrightarrow a'<\frac{5}{6}+\frac{7}{6}a \end{aligned} \tag{15}$$

and put $a = 0$. Thus we make the regularization step for $\alpha < 2 + a'$, $a' = \frac{5}{6}$.

Having performed the regularization step $\alpha = \frac{7}{3} + \eta$ corresponding to (11) we have proved

$$u \in \bigcap_{\varepsilon>0} C_0(0,T;H_{\mathrm{loc}}^{\frac{4}{3}-\varepsilon,\frac{4}{3}-\varepsilon,2+\frac{3}{5}\eta-\varepsilon}(\Omega)) \tag{16}$$

due to (12) and (13). All components of the corresponding stress tensor are in the same space. Using Prop. 2 of [2] we have proved the following theorem.

Theorem 2. *Let Ω be a bounded domain in $\mathbf{R}^3$, let $g(T)$ satisfy* (11) *and let all the other suppositions concerning $\partial\Omega$, g, γ, T stated at the beginning of the section be satisfied. Then the corresponding stress tensor belongs to $C_0(\bar{Q};\mathbf{R}^9)$.*

Remark 1. In general, to obtain $g(T)$ in the required space in (11) for a nonlinear g it is sufficient e.g. $T \in \bigcap\limits_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon,\frac{1}{2}-\varepsilon,\frac{17}{6}+\eta}(S)$ or T of the class $C_{\frac{11}{6}+\eta}$ along the variable x_3.

2. In general, if $g(T) \in \bigcap_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon,\alpha_1-\varepsilon,\alpha_2-\varepsilon}(S)$, $\alpha_1 \in \langle 0, \frac{1}{2})$, then we need $\alpha_2 > \frac{13-4\alpha_1}{3(1+2\alpha_1)}$ to prove the assertion of Theorem 2. (The proof of such an assertion requires, naturally, the use of a longer regularization procedure with respect to the regular space variable—the third or even fourth derivatives of the composed function emerging from the nonlinear boundary terms must be estimated. To do it, a suitably more regular g is needed.)

3. Of course, the method of this section can be combined with the results valid for nonsmoothnesses of the boundary of the type of edges and for supports, at least if these edges or the boundary of the contact part of $\partial\Omega$ are sufficiently smooth. As the calculation is quite cumbersome we avoid the details.

4. With the help of the procedures of this section we can prove for $T \in \bigcap_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon,\alpha}(S)$, $\alpha \in \langle 0, \frac{1}{2})$ that u and all components of the corresponding stress tensor belong to $\bigcap_{\varepsilon>0} C_0(0,T; H^{\frac{7}{6}+\frac{1}{3}\alpha-\varepsilon}(\Omega))$, which for $\dim\Omega = 2$ improves quantitatively the result of Thms.4 and 5 of [1] (and yields the $(\frac{\alpha}{3}+\frac{1}{6}-\varepsilon)$-Hölder continuity of the components of the stress tensor in the space variables for each $\varepsilon \in (0, \frac{1}{6})$—their $(\frac{1}{4}-\varepsilon)$-Hölder continuity follows from (12)). The result for the case of the coefficient β independent of u is a straightforward consequence of the above mentioned procedures. If β depends on u in the way supposed in Thms. 4 and 5 of [1], the proof of the better time regularity of u done in Sec. 1 remains practically without changes: the additional term

$$\int_{\mathbf{R}^2} |\ell|^{-1-2\alpha_0} \int_\Omega (\beta(u_{-\ell}) - \beta(u)) \frac{\partial u}{\partial t}\Big(\frac{\partial u_{-\ell}}{\partial t} - \frac{\partial u}{\partial t}\Big) \mathrm{d}x\mathrm{d}t\mathrm{d}\ell$$
$$\leqslant \eta_1 \int_{\mathbf{R}^2} |\ell|^{-1-2\alpha_0} \int_\Omega \Big(\frac{\partial u_{-\ell}}{\partial t} - \frac{\partial u}{\partial t}\Big)^2 \mathrm{d}x\mathrm{d}t\mathrm{d}\ell$$
$$+ \Big(\sup_{\langle 0,\bar{B}\rangle} \beta'\Big)^2 \eta_1^{-1} \int_{\mathbf{R}^2} |\ell|^{-1-2\alpha_0} \int_\Omega (u_\ell - u)^2 \Big(\frac{\partial u}{\partial t}\Big)^2 \mathrm{d}x\mathrm{d}t\mathrm{d}\ell$$

(with $\alpha_0 = \alpha$ in (6), (7) and η_1 sufficiently small) will be further estimated in such a way that we use the Hölder continuity of u in time whose exponent is $\alpha_1 - \frac{1}{2} - \varepsilon$, where α_1 is the fractional derivative (in the Sobolev sense) of ∇u whose existence was proved, and $\varepsilon > 0$ is arbitrarily small. E.g. for $\alpha = 0$ the relation $\nabla u \in \bigcap_{\varepsilon>0} H^{\frac{7}{12}-\varepsilon}(0,T; L_2(\Omega;\mathbf{R}^2))$ immediately implies that $\dot{u} \in \bigcap_{\varepsilon>0} H^{\frac{1}{12}-\varepsilon}(0,T; L_2(\Omega))$. Now the proof of the time regularity as in (12) works in the same way as above and the end of the proof is the same as for β independent of u.

5. The above mentioned results promise regular stresses also for the case when the heating regime is a sufficiently regular distribution. The study of such a situation is a little bit beyond the framework of our research and for our nonlinear model

requires a different approach. It does not seem, however, that such a regularity can be obtained if T is of the Dirac type.

3. A COUPLED VISCOELASTIC CASE WITH NONLINEAR BOUNDARY VALUE CONDITION IN THE HEAT EQUATION

In this section we shall study the regularity of stresses for a coupled model supposing again the noncontinuity of the heating regime. The model employed is not much realistic from the physical point of view. Its treatment and the results derived, being close to the quasi-coupled case, indicate the arising difficulties when more realistic models are to be treated. For the sake of simplicity we restrict ourselves to a bounded domain Ω in $\mathbb{R}^2$ with a C_3-smooth boundary. Our system is

$$(17)\quad \begin{cases} \left.\begin{aligned} \beta_0 \dot{u} &= \triangle u + \delta_0 \operatorname{div} \dot{v} \\ \ddot{v} &= (1-2\sigma_1)\triangle \dot{v} + (1-2\sigma)\triangle v + \nabla \operatorname{div} v - (2+2\sigma)\gamma \nabla u \end{aligned}\right\} \text{ on } Q = (0,T)\times\Omega, \\ \left.\begin{aligned} \frac{\partial u}{\partial \nu} &= g(T) - g(u) \\ (1-2\sigma_1)\frac{\partial \dot{v}}{\partial \nu} + (1-2\sigma)\Big(\frac{\partial v}{\partial \nu} + ((\nu, \nabla_i v)_i)\Big) &+ 2\sigma\nu \operatorname{div} v = (2+2\sigma)\gamma u \nu \end{aligned}\right\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } S = (0,T)\times\partial\Omega, \\ u(0,\cdot) = 0, \;\; \dot{v}(0,\cdot) = 0, \;\; v(0,\cdot) = 0 \qquad\qquad \text{on } \Omega. \end{cases}$$

We suppose σ, σ_1, γ, δ_0, E to be positive constants and we preserve the suppositions concerning g from Theorem 1. Moreover we suppose that g is bounded from below and its growth at infinity is polynomially bounded.

In the model we must include the term $\frac{E}{2+2\sigma_1}\big(\frac{\partial \dot{v}_i}{\partial x_j} + \frac{\partial \dot{v}_j}{\partial x_i}\big)$ into the stress tensor, whose boundedness and continuity is the goal of our investigation (cf. (3)). Therefore we should prove the boundedness and continuity both of ∇v and of $\nabla \dot{v}$.

Taking the usual variational formulation of (17) for a test function $w = [w_0, w_1] = [w_0, [w_1^1, w_1^2]] : \Omega \to \mathbb{R}^3$, we obtain with the help of the Green formula

$$(18)\quad \int_Q \beta_0 \dot{u} w_0 + (\nabla u, \nabla w_0) + (1-2\sigma_1)(\nabla \dot{v}, \nabla w_1) + (1-2\sigma)(\nabla v, \nabla w_1)$$
$$+ \operatorname{div} v \;\operatorname{div} w_1 + \ddot{v} w_1 \mathrm{d}x\mathrm{d}t + \int_S g(u) w_0 \;\mathrm{d}x\mathrm{d}t$$
$$= \int_Q (2+2\sigma)\gamma u \operatorname{div} w_1 - \delta_0 \operatorname{div} \dot{v} w_0 \mathrm{d}x\mathrm{d}t + \int_S g(T) w_0 \mathrm{d}x\mathrm{d}t.$$

Putting $w_0 = u$, $w_1 = \dot{v}$ into (18) we immediately obtain the energy estimate

$$(19) \qquad \|\nabla\dot{v}\|^2_{L_2(Q;\mathbf{R}^4)} + \|\dot{v}\|^2_{L_\infty(0,T;L_2(\Omega;\mathbf{R}^2))} + \|\nabla v\|^2_{L_\infty(0,T;L_2(\Omega;\mathbf{R}^4))} + \|u\|^2_{L_\infty(0,T;\,L_2(\Omega))} + \|\nabla u\|^2_{L_2(\Omega;\mathbf{R}^2)} \leqslant \text{const}$$

applying the Gronwall lemma as usual. From (19) we prove the existence and uniqueness of the solution of (18) via the usual Galerkin method and Gronwall-lemma type argument.

Now we employ the time-shift method for extended u, v as in [1], Sec. 3 ($u, v = 0$ on $((-\infty, 0) \cup (4T, +\infty)) \times \Omega$, $f = f(T, \cdot)$ on $(T, 2T) \times \Omega$ for $f = u, v$ etc.). For $T \in L_2(0, T; H^\alpha(\partial\Omega)) \cap \bigcap_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon}(0, T; L_2(\partial\Omega)), \alpha > 0$ we can prove in the same way

$$(20) \qquad \|\nabla\dot{v}\|^2_{H^{\alpha_1}(0,T;L_2(\Omega;\mathbf{R}^4))} + \|\nabla u\|^2_{H^{\alpha_1}(0,T;L_2(\Omega;\mathbf{R}^2))} \leqslant \text{const}, \quad \alpha_1 \in (0, \tfrac{1}{2}) \text{ arbitrary}.$$

We put, moreover, $w_1 = 0$, $w_0 = \dot{u}$. Then for every $t_0 \in (0, T)$ we obtain the inequality

$$(21) \qquad \int_{Q_{t_0}} \beta_0 \dot{u}^2 \mathrm{d}x\mathrm{d}t + \frac{1}{2}\int_\Omega |\nabla u(x, t_0)|^2 \mathrm{d}x = -\int_{S_{t_0}} g(u)\dot{u}\, \mathrm{d}x\mathrm{d}t + \int_{S_{t_0}} g(T)\dot{u}\, \mathrm{d}x\mathrm{d}t + \int_{Q_{t_0}} \delta_0 \operatorname{div}\dot{v}\dot{u}\, \mathrm{d}x\mathrm{d}t,$$

$$\text{where } Q_{t_0} = (0, t_0) \times \Omega,\ S_{t_0} = (0, t_0) \times \partial\Omega.$$

With the additional supposition that $g(y) \geqslant \tilde{c}$ for a constant $\tilde{c} < 0$ and every $y \in \mathbf{R}^1$, the first integral on the right hand side of (21) can be estimated by $-\tilde{c}(\text{mes}\partial\Omega + \|u\|_{L_\infty(0,T;L_2(\Omega))})$, for the third integral we use the obvious Schwartz inequality with a sufficiently small $\eta > 0$ at $\int_{Q_{t_0}} \dot{u}^2 \mathrm{d}x\mathrm{d}t$. The second integral on the right hand side of (21) will be estimated by $\frac{1}{\eta_0}\|g(T)\|_{H^{\frac{1}{2}-\varepsilon}(0,T;\,L_2(\Omega))} + \eta_0\|\dot{u}\|_{H^{-\frac{1}{2}+\varepsilon}(0,T;L_2(\Omega))}$ with an arbitrarily small $\eta_0 > 0$. Using the extension method for u onto $\mathbf{R}^3$ and the Hölder inequality

$$(22) \qquad \int_{\mathbf{R}^3} |\tau|^{2\beta_1}|\xi|^{1+\varepsilon}|\hat{u}|^2 \mathrm{d}\xi\mathrm{d}t \leqslant c_1 \int_{\mathbf{R}^3} (1 + |\tau|^2 + |\tau|^{1-\varepsilon}|\xi|^2)|\hat{u}|^2 \mathrm{d}\xi\mathrm{d}t$$

which holds with a suitable constant $c_1 > 0$ and for β_1 arbitrarily close to $\frac{3}{4}$ for $\varepsilon > 0$ arbitrarily small, we estimate $\|\dot{u}\|_{H^{-\frac{1}{2}+\varepsilon}(0,T;L_2(\Omega))}$ by $c_2(\|\nabla u\|_{H^{\alpha_1}(0,T;\,L_2(\Omega;\mathbf{R}^2))} +$

$\|\dot{u}\|_{L_2(Q)})$ for a constant $c_2 > 0$. (The reason of this formal procedure is the same as in Sec. 1—cf. (4) and what follows.) Thus we prove $\dot{u} \in L_2(Q)$ without an a priori knowledge of $u \in L_\infty(Q)$.

Putting $w_0 = 0$, $w_1 = \ddot{v}$ and using the integration by parts in the time variable for the term $\int_Q u \operatorname{div} \ddot{v} \, dx dt$, we can easily see that $\ddot{v} \in L_2(Q, \mathbf{R}^2)$. Using the monotonicity of g, it is not difficult to prove by the method of local coordinates, the straightening of the boundary and the shift method that $u \in L_2(0, \mathcal{T}; H^{1, \frac{3}{2}+\alpha}_{\text{loc}}(\Omega)), \dot{v} \in L_2(0, \mathcal{T}; H^{1, \frac{5}{2}+\alpha}_{\text{loc}}(\Omega; \mathbf{R}^2))$ for $T \in L_2(0, \mathcal{T}; H^\alpha(\partial\Omega))$ and $\alpha \in (0, \frac{1}{2})$. The first variable in the anisotropic spaces employed is supposed to be normal, the second the tangential one. This result together with the up-to-now proved time regularity of u and the fact that $\operatorname{div} \dot{v} \in L_2(Q)$ yields $u \in \bigcap_{\varepsilon>0} H^{\frac{1}{2}-\varepsilon}(0, \mathcal{T}; H^{\frac{1}{2}-\varepsilon}(\partial\Omega)) \cap \bigcap_{\varepsilon>0} L_2(0, \mathcal{T}; H^{1-\varepsilon}(\partial\Omega))$, which can be imbedded in $L_p(S)$ for every $p \in (1, +\infty)$ (we use e.g. Theorem 1 of [1]). Thus for g with polynomially bounded growth at infinity we can prove $g(u) \in L_2(S)$. The interpolation technique employed just in [1] (the proof of Prop. 4) gives $u \in L_2(0, \mathcal{T}; H^{\frac{3}{2}}(\Omega))$. Like in Prop. 5 of [1] we can now prove $\nabla u \in H^{\frac{1}{2}+\eta}(0, \mathcal{T}; L_2(\Omega; \mathbf{R}^2))$ with some $\eta > 0$ and like in Theorem 4 of [1] (combined with Prop. 2 of [2]) we conclude that $u \in H^{\frac{1}{2}+\eta_1}(0, \mathcal{T}; H^{1+\eta_2}(\Omega))$ with $\eta_1 > 0$, $\eta_2 > 0$ and therefore $u \in C_0(\bar{Q})$. Then we can apply the described technique again. We prove in fact $u \in L_2(0, \mathcal{T}; H^{\frac{3}{2}+\alpha}(\Omega))$, and $\nabla u \in \bigcap_{\varepsilon>0} H^{\frac{7+6\alpha}{12+8\alpha}-\varepsilon}(0, \mathcal{T}; L_2(\Omega; \mathbf{R}^2))$ for an arbitrary $\alpha \in (0, \frac{1}{2})$ in the same way as in [1], Sec. 3.

Like in [1], Sec. 2 we are now able to prove that $C_1 \frac{\partial^2 v}{\partial x_n^2} + C_2 \frac{\partial^2 \dot{v}}{\partial x_n^2} \in L_{2,\text{loc}}(Q)$ for some functions C_1, C_2 dependent on the space variable only and bounded below by a positive constant, because all the other terms of the transformed Lamé system belong to that space as well. Here and in the rest of the section we employ the notation x_n for the normal space variable, x_t for the tangential one and the index loc has the same sense as in the preceding sections. Denoting by Ω_η the intersection of a certain neighbourhood of the straightened part of $\partial\Omega$ with Ω, we have proved

$$(23) \quad c > \int_0^{\mathcal{T}} \int_{\Omega_\eta} C_1^2 \frac{\partial^2 v}{\partial x_n^2} + C_2^2 \frac{\partial^2 \dot{v}}{\partial x_n^2} \, dx dt + \int_{\Omega_\eta} C_1(x) C_2(x) \frac{\partial^2 v}{\partial x_n^2}(x, \mathcal{T}) dx, \quad c \in \mathbf{R}^1,$$

and thus $v, \dot{v} \in \bigcap_{\varepsilon>0} L_2(0, \mathcal{T}; H^{2,3-\varepsilon}_{\text{loc}}(\Omega))$. Now we differentiate the Lamé system of (17) in the time variable and put $w_1 = \ddot{v}$. Thus we obtain, using the Korn inequality, that for some constants $k, k_1 > 0$, $0 < \varepsilon \ll \frac{k}{k_1}$

$$(24)\quad \sup_{t\in(0,T)} \int_\Omega (\ddot v(x,t))^2 \mathrm{d}x + k\int_Q |\nabla \ddot v|^2 \mathrm{d}x\mathrm{d}t \leqslant (2+2\sigma)\gamma \int_Q \dot u \operatorname{div} \ddot v \, \mathrm{d}x\mathrm{d}t$$

$$\leqslant (2+2\sigma)\gamma \int_Q \frac{1}{\varepsilon}\dot u^2 + \varepsilon\, k_1 |\nabla \ddot v|^2 \mathrm{d}x\mathrm{d}t$$

and therefore $\nabla \ddot v \in L_2(Q;\mathbf{R}^4)$. Together with the preceding results, we have proved that the "right hand side" $\operatorname{div}\dot v$ of the heat equation in (17) belongs to $H^1(Q)$. Hence like in the preceding sections we can prove $u \in \bigcap_{\varepsilon>0} H^{\frac{5}{4}-\varepsilon,2-\varepsilon}(Q)$, $\nabla u \in \bigcap_{\varepsilon>0} H^{\frac{3}{4}-\varepsilon,1-\varepsilon}(Q;\mathbf{R}^2)$, $\dot u \in \bigcap_{\varepsilon>0} H^{\frac{1}{4}-\varepsilon,\frac{1}{2}-\varepsilon}(Q)$—the space regularity of $\dot u$ is a consequence of the relations $\dot u \in \bigcap_{\varepsilon>0} H^{\frac{1}{4}-\varepsilon}(0,T;L_2(\Omega))$, $\nabla u \in \bigcap_{\varepsilon>0} H^{\frac{3}{4}-\varepsilon}(0,T;L_2(\Omega;\mathbf{R}^2))$ and can be proved in the usual way via the Fourier transformation and the Hölder inequality. The relation $\nabla \ddot v \in \bigcap_{\varepsilon>0} H^{\frac{1}{4}-\varepsilon,\frac{1}{2}-\varepsilon}(Q;\mathbf{R}^4)$ can be proved via the shift technique in time and in the tangential space variable used to the time-differentiated Lamé system, whereas for the normal space variable we use the interpolation technique together with the up-to-now proved results for $\nabla \dot v$.

Extending all components of $\nabla \dot v$ onto $\mathbf{R}^3$ as usual, the shift method yields that they belong to $\bigcap_{\varepsilon>0} L_2(0,T;H^{1,2-\varepsilon}_{\mathrm{loc}}(\Omega))$ while $\frac{\partial^2 \dot v_i}{\partial x_t^2}$, $\frac{\partial^2 \dot v_i}{\partial x_n \partial x_t}$ belong to $\bigcap_{\varepsilon>0} L_2(0,T;L_2(0,\eta_1;H^{1-\varepsilon}(V_{\eta_1})))$ for every set $(0,\eta_1)\times V_{\eta_1} \subset \Omega_\eta$ such that $\{0\}\times V_{\eta_1} \subset \Omega_\eta \cap \partial\Omega$, V_{η_1} is an open interval in $\mathbf{R}^1$. Using the Lamé system and the supposed smoothness of $\partial\Omega$, we prove the same fact for $\frac{\partial^2 \dot v}{\partial x_n^2}$. Thus for $\psi_{ij} = \frac{\partial \dot v_i}{\partial x_j}$ (in its extension), $i,j = 1, 2$, ξ_n, ξ_t the dual variables to x_n, x_t, respectively, we have proved

$$(25)\quad \int_{\mathbf{R}^3} |\hat\psi_{ij}|^2 (1 + |\tau|^{\frac{5}{2}-\varepsilon} + |\xi_n|^2|\xi_t|^{2-\varepsilon} + |\tau|^2|\xi|^{1-\varepsilon} + |\xi_t|^{4-\varepsilon})\, \mathrm{d}\xi_n \mathrm{d}\xi_t \mathrm{d}\tau < +\infty,$$

$$i,j = 1,2,\ \ \varepsilon \in (0, \tfrac{1}{4}) \text{ arbitrary}.$$

From (25) it is not difficult to find indices α_1, α_2, α_3 all greater than 1 such that

$$(26)\qquad \int_{\mathbf{R}^3} |\hat\psi_{ij}|^2 (1+|\tau|^{\alpha_1})(1+|\xi_n|^{\alpha_2})(1+|\xi_t|^{\alpha_3})\, \mathrm{d}\xi_n \mathrm{d}\xi_t \mathrm{d}\tau < +\infty,$$

and using Thm. 1 of [1] we prove $\psi_{ij} \in C_0(\mathbf{R}^3)$, $i,j = 1, 2$. Thus we have proved

Theorem 3. *Under the above mentioned suppositions concerning Ω, $\partial\Omega$, g, T and all the constants, the unique solution of (17) yields all the components of the stress tensor bounded and continuous on $\bar Q$.*

Remark 1. The quantitative results declared in Remark 4 of the preceding section can be proved for this model, too.

2. We fixed the temperature in the "coupling term" in the heat equation of (17) as the constant δ_0 and we have not supposed the heat equation to contain the term corresponding to the viscosity in the Lamé system. A problem including the terms which we have not considered is studied e.g. in [7]. It is not possible to put $w_0 = u$ into the variational inequality to the heat equation and it is not known how to solve such a problem for $\dim \Omega > 1$. If we avoided the viscosity term in Lamé system, we would face some analogous difficulties with the energy estimations, because for $w_0 = u$ we were not able to estimate the boundary term arising after the use of the Green formula in the heat equation. With $w_0 = 1$ (cf. [7]) it seems to be hardly possible to obtain a sufficient energy estimate to start the proof of the regularity. Therefore it could be more promising to employ a class of coupled models including suitable nonlinearities helping in the energy estimations. A certain type of such models is studied in [4], [6].

Conclusion

The results, the theorems in this part of the series of papers were formulated for the heat equation and the Lamé system with constant coefficients. In fact, this is only a technical supposition simplifying estimations and could be avoided. The assertion holds also for the preceding parts with the exception of Sec. 4 of [1] and Sec. 2 of [2]. The methods of proofs admit also some nonzero right hand sides in the system and nonzero initial conditions, if these input data satisfy some compatibility conditions.

The author hopes that the model investigated in the first two parts and in Secs.1 and 2 of this part of the series of papers neglecting the coupling and acceleration terms should be satisfactory for the technical practice of heating regimes in furnaces at least in the cases, where no phase transition needs to be considered and that the series gives a comprehensive answer to the question when the thermoelastic system is a sufficient description of real processes in furnaces from the point of view of the stresses.

References

[1] *J. Jarušek*: On the regularity of solutions of a thermoelastic system under noncontinuous heating regimes, Apl. Mat. *35* (1990), 426–450.

[2] *J. Jarušek*: On the regularity of solutions of a thermoelastic system under noncontinuous heating regimes. Part II, Appl. Math. *36* (1991), 161–180.

[3] *J. Lions, E. Magenes*: Problèmes aux limites non-homogènes et applications, Dunod, Paris, 1968.

[4] *J. Nečas*: Dynamics of thermoelastic systems with strong viscosity, Proc. conf. part. diff. eq. Holzhau (GDR) 1988 (B. W. Schulze, ed.), Teubner Texte Math. Vol. 112, Leipzig, 1989.

[5] *J. Nečas, J. Jarušek, J. Haslinger*: On the solution of the variational inequality to the Signorini problem with small friction, Bol. Un. Mat. Ital. (5) *17-B* (1980), 796–811.
[6] *J. Nečas, A. Novotný, V. Šverák*: Uniqueness of solutions to the systems for thermoelastic bodies with strong viscosity, Math. Nachr. *149* (1990), 319–324.
[7] *J. Sprekels*: Global solution in onedimensional magneto-thermoviscoelasticity, preprint, Univ. GH. Essen.

Souhrn

REGULARITA ŘEŠENÍ TERMOELASTICKÉHO SYSTÉMU S NESPOJITÝMI REŽIMY OHŘEVU. ČÁST III

JIŘÍ JARUŠEK

V této části zeslabujeme postačující podmínky pro omezenost a spojitost napětí pro trojrozměrnou úlohu a zkoumáme jistý "zkuplovaný" systém.

Author's address: *Jiří Jarušek*, Institute of Information Theory and Automation, Czech. Acad. Sci., Pod vodárenskou věží 4, 182 08 Praha 8, Czechoslovakia.