Aplikace matematiky

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Aplikace matematiky, Vol. 35 (1990), No. 3, 192–208

Persistent URL: http://dml.cz/dmlcz/104403

COMPENSATED COMPACTNESS AND TIME-PERIODIC SOLUTIONS TO NON-AUTONOMOUS QUASILINEAR TELEGRAPH EQUATIONS

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(Received February 1, 1989)

Summary. In the present paper, the existence of a weak time-periodic solution to the nonlinear telegraph equation

$$U_{tt} + dU_t - \sigma(x, t, U_x)_x + aU = f(x, t, U_x, U_t, U)$$

with the Dirichlet boundary conditions is proved. No "smallness" assumptions are made concerning the function f.

The main idea of the proof relies on the compensated compactness theory.

Keywords: Telegraph equation, compensated compactness, vanishing viscosity method.

AMS Classification: 35B10, 35L70, 35Q20.

1. INTRODUCTION

With DiPerna's results [6] concerning the convergence of approximate solutions to conservation laws, the compensated compactness theory developed by Ball, Murat and Tartar embraced truly nonlinear hyperbolic systems in one space dimension. Subsequent progress represented, for instance, by the papers of DiPerna [5], Serre [18], or Rascle [17] has resulted in successfully solving the Cauchy problem for a vast class of nonlinear equations.

The present paper attempts to illustrate the power of this method when applied to boundary value problems of mathematical physics. To put it more exactly, for $U = U(x, t)$ consider the equation

$$\text{(E)} \qquad U_{tt} + dU_t - \sigma(x, t, U_x)_x + aU = f(x, t, U_x, U_t, U)$$

for $x \in (0, l)$, $t \in \mathbb{R}^1$ along with the conditions

$$\text{(B)} \qquad U(0, t) = U(l, t) = 0\,,$$

$$\text{(P)} \qquad U(x, t + \omega) = U(x, t)$$

for all x, t.

To begin with, it is worth dwelling on the mathematical tools one became accustomed to employ when solving the problem in question.

The most frequent method, and to be sure the only one, leans on linearizing the equation and making use of a suitable iteration scheme.

The first group of results associated with this technique comprises those of Matsumura [10], Nishida [13], Milani [12], or Štědrý [20] based on the classical mathematical tools as the Banach or Schauder fixed point principle.

The second branch was opened with the remarkable research of Nash and Moser related to the hard implicit function theorems. Beginning with a truly pioneering work of Rabinowitz [16] we could go on through a relatively long list of papers represented, for instance, by Petzeltová [14], Craig [4], Krejčí [9], or Petzeltová-Štědrý [15].

Using either of these methods, however, we are bound to deal with "small" solutions corresponding to "small" data. This is the major shortcoming associated with all approaches referenced above. To our best knowledge, there seem to be no results concerning the large data problem unless some very restrictive assumptions are made.

To fill this gap, we intend to prove the existence theorem for (E), (B), (P) under the following assumptions:

(A_1) The constants $a, d > 0$ are supposed to satisfy

(1.1) $$d^2 - 4a \geqq 0 .$$

(A_2) $\sigma = \sigma(x, t, u): \mathbb{R}^3 \to \mathbb{R}^1$ is a smooth function, the growth of which is restricted as follows:

(1.2) $$|\sigma_x|, \quad |\sigma_t|, \quad |\sigma_{xu}|, \quad |\sigma_{tu}|, \quad |\sigma_{xxu}| \leqq c_1 ,$$

(1.3) $$\sigma_u(x, t, u) \geqq c_2 > 0$$

for all x, t, u, and

(1.4) $$\lim_{u \to \pm\infty} \sigma_u(x, t, u) = +\infty \text{ uniformly in } x, t.$$

Besides, we require

(1.5) $$\sigma(x, t, u) = \sigma(-x, t, u), \quad \sigma(x + 2l, t, u) = \sigma(x, t, u),$$

(1.6) $$\sigma_{uu}(x, t, u)\, u > 0 \quad \text{whenever} \quad u \neq 0,$$

and (of course)

(1.7) $$\sigma(x, t + \omega, u) = \sigma(x, t, u)$$

for all x, t, u.

(A_3) The function $f = f(x, t, u_1, u_2, u_3): [0, l] \times \mathbb{R}^4 \to \mathbb{R}^1$ is smooth with

(1.8) $$f(x, t + \omega, u_1, u_2, u_3) = f(x, t, u_1, u_2, u_3),$$

(1.9) $$|f(x, t, u_1, u_2, u_3)| \leqq c_3$$

for all x, t, $u_i, i = 1, 2, 3$.

The only justification for the above conditions is that they give rise to the existence of at least one weak solution (see Section 2) along with a relatively comprehensible proof of this fact (cf. Sections 3–6). The actual significance as well as possible improvements of each of our requirements will be discussed in the relevant parts of the paper.

The nature of the method employed makes it necessary for us to transform the original equation to a hyperbolic system (see Section 3).

Having the subsequent application of compensated compactness in mind we approach the problem via the vanishing viscosity method. When slightly adapted the procedure of Amann [1] yields a sequence of approximate solutions provided that we are able to ensure certain a priori estimates (Section 4).

Consequently, a question of primary importance arises concerning the extension of the concept of invariant regions for parabolic systems (cf. [3]) to a non-autonomous case, i.e. when σ does actually depend on x, t. Such a problem was studied in [7] and we adopt here the results.

To pass to a limit in the sequence of approximate solutions, the method of compensated compactness is used; more precisely, the lemma of DiPerna [6], related to the corresponding Young measure (see Sections 5, 6), plays the decisive role.

Here again, the explicit presence of the variables x, t in σ entangles the situation and prevents us from following the arguments of [6] in a direct fashion (Section 6).

To conclude with, let us agree upon the notation used in the text. In our opinion, it is superfluous to repeat here all familiar denominations of the Sobolev spaces, Lebesgue spaces etc. If in doubt, the reader may consult, for example, the monograph [22].

Throughout the whole text, the symbols c or c_i, $i = 1, \dots$ stand for all strictly positive real constants.

2. MAIN RESULTS

There exist sound reasons (cf. Slemrod [19], Milani [11]) for us to deal with the class of weak solutions related to (E), (B), (P).

Remembering all possible difficulties of taking care of boundary conditions we prefer to view all functions satisfying (B), (P) as double-periodic, i.e. defined, in fact, on a torus $T^2 = \{(x, t) \mid x \in S^1, t \in S^2\}$ where $S^1 = [-l, l]/\{-l, l\}$, $S^2 = [0, \omega]/\{0, \omega\}$.

Eventually, consider the cylinder

$$Q = \{(x, t) \mid x \in [0, l], t \in S^2\} .$$

Definition 1. *A function U belonging to the Sobolev space $W^1_\infty(Q)$ is said to be a weak solution of the problem* (E), (B), (P) *if the condition* (B) *holds and, for all* *test functions* $\varphi \in C^\infty(Q)$ *satisfying* (B), *we have*

(2.1) $$\iint_Q -U_t\varphi_t + \sigma(x, t, U_x)\,\varphi_x + (dU_t + aU)\,\varphi \,\mathrm{d}x\,\mathrm{d}t = \\ = \iint_Q f(x, t, U_x, U_t, U)\,\varphi\,\mathrm{d}x\,\mathrm{d}t\,.$$

Note that, according to the embedding relation $W^1_\infty(Q) \subset C(Q)$ (cf. [22]), all above assertions are fully justified.

As claimed in Section 1, our main aim is to establish the following existence result.

Theorem 1. *Let the assumptions* (A_1), (A_2), (A_3) *hold.*
Then there exists at least one weak solution to the problem (E), (B), (P).

3. A HYPERBOLIC SYSTEM

The only reason for assuming (1.1) is the existence of two strictly positive constants $a_1, a_2 > 0$ making the following decomposition possible:

$$d = a_1 + a_2\,, \quad a = a_1 a_2\,.$$

After the change of variables $u = U_x$, $v = U_t + a_1 U$, the equation (E) takes the form of a hyperbolic system

(S_1) $$u_t + a_1 u - v_x = 0\,,$$

(S_2) $$v_t + a_2 v - \sigma(x, t, u)_x = f\,.$$

As shown in [7], a suitable parabolic regularization is provided by adding the terms

$$\mathscr{A}_1 u = u_{xx} + \Psi(x, t, u)_x\,, \quad \mathscr{A}_2 v = v_{xx}$$

where

$$\Psi(x, t, u) = \int_0^u \frac{\sigma_{xu}(x, t, z)}{\sigma_u(x, t, z)}\,\mathrm{d}z\,.$$

Thus, we are led to a perturbed problem

(S_1^ε) $$u_t + a_1 u - v_x = \varepsilon \mathscr{A}_1 u\,,$$

(S_2^ε) $$v_t + a_2 v - \sigma(x, t, u)_x = f^\varepsilon + \varepsilon \mathscr{A}_2 v\,, \quad \varepsilon > 0\,.$$

As we have already remarked in Section 2, we are primarily interested in classical solutions determined on T^2, f^ε being, for the present, a function belonging to the class $C(T^2)$.

The relationship between the original function U and a solution of the system (S_1^ε), (S_2^ε) may be clarified with help of the following assertion.

Lemma 1. *Consider a classical solution* (u, v) *of the equation* (S_1^ε) *on* T^2.
Then there is a unique function $U \in C^1(T^2)$ *satisfying*

(3.1) $$U_x = u\,, \quad U_t + a_1 U = v + \varepsilon(u_x + \Psi(x, t, u))\,.$$

Proof. Multiplying the equation by $e^{a_1 t}$ gives rise to

$$(e^{a_1 t}u)_t - (e^{a_1 t}(v + \varepsilon u_x + \varepsilon\Psi(x, t, u)))_x = 0 \quad \text{on} \quad \mathbb{R}^2 .$$

Now, there is a function V on $\mathbb{R}^2$ satisfying

$$V_x = e^{a_1 t}u , \quad V_t = e^{a_1 t}(v + \varepsilon u_x + \varepsilon\Psi(x, t, u)) ,$$

determined uniquely by the value $V_0 = V(0, 0)$.

Consider a function $U = e^{-a_1 t}V$, $U(0, 0) = V_0$. As the relation (3.1) is easy to verify for U, we have only to choose the constant V_0 so that U may be double – periodic.

To this end, V_0 will be determined uniquely by the requirement $U(0, t + \omega) = U(0, t)$, i.e. the function $U(0, \cdot)$ is to be a unique ω-periodic solution to the ordinary differential equation

(a) $$U_t(x, \cdot) + a_1 U(x, \cdot) = v(x, \cdot) + \varepsilon[u_x(x, \cdot) + \Psi(x, \cdot, u(x, \cdot))]$$

for $x = 0$.

With the relation $U(x, t) = U(0, t) + \int_0^x u(z, t)\,dz$ in mind, we deduce that U is, in fact, ω-periodic in t.

To show periodicity with respect to x, we have only to realize that, firstly, the right-hand side of (a) is $2l$-periodic in x, and, secondly, $U(x, \cdot)$ is determined as the unique solution of (a) which is ω-periodic in t. ■

4. APPROXIMATE SOLUTIONS

To find ω-periodic in t solutions of (S_1^ε), (S_2^ε), an indirect method will be used. It means that we are going to solve the initial value problem given by (S_1^ε), (S_2^ε) with

(I) $$u(x, 0) = u^0(x) , \quad v(x, 0) = v^0(x) , \quad u^0 , \quad v^0 \in C(S^1)$$

and hope to succeed in finding a fixed point of the corresponding evolution operator.

Whenever speaking about a solution (u, v) of (S_1^ε), (S_2^ε), (I) on a certain time interval $[0, t_0)$, we tacitly assume that

$$u, v \in C(S^1 \times [0, t_0)) , \quad u_t, v_t, u_x, v_x, u_{xx}, v_{xx} \in C(S^1 \times (0, t_0)) ,$$

and the equations together with (I) are fulfilled for $x \in S^1$, $t \in (0, t_0)$.

For later purposes, we introduce symmetry classes

$$\Gamma_1 = \{w \mid w \in L_2(S^1), w(-x) = w(x)\} ,$$
$$\Gamma_2 = \{w \mid w \in L_2(S^1), w(-x) = -w(x)\} .$$

As to the function f^ε, it is supposed to satisfy

(4.1) $$f^\varepsilon \in C^\nu(T^2) , \quad |f^\varepsilon| \leqq c_3 , \quad \nu \in (0, 1)$$

and, for each $t \in S^2$,

(4.2) $$f^\varepsilon(\cdot, t) \in \Gamma_2$$

(the symbol C^ν stands for the class of ν-Hölder continuous functions – cf. Amann [2]).

The main ideas mentioned in this section can be traced back to Amann [1] we will quote systematically from.

Let us start with a short review of basic properties of a linear operator related to our problem. For $w \in C^\infty(S^1)$ we consider

$$\mathscr{L}w = -\varepsilon w_{xx} + a_2 w .$$

Rather than the operator $\mathscr{L}$ itself, its self-adjoint extension to the space $L_2(S^1)$ is of interest, namely,

$$\mathscr{L}w = \sum_{k\in Z} \lambda_k\, b_k(w)\, e_k$$

where

$$e_k(x) = \begin{cases} \cos(\mu_k x), & k \leqq 0, \\ \sin(\mu_k x), & k > 0, \end{cases}$$

$0 = \mu_0 < \mu_1 = \mu_{-1} < \ldots < \mu_k = \mu_{-k} \ldots,\ \mu_k \approx k,\ \lambda_k = \varepsilon\mu_k^2 + a_2$, and b_k are the Fourier coefficients of the function w with respect to e_k.

Consequently, we may determine a scale of spaces $X_\alpha = D(\mathscr{L}^\alpha)$ with a Hilbert norm

$$\|w\|_\alpha = [\sum_{k\in Z} \lambda_k^{2\alpha}\, b_k^2(w)]^{1/2} .$$

Let the symbol $\{T_t\}_{t\geqq 0}$ denote the semigroup of linear operators on $L_2(S^1)$ generated by $-\mathscr{L}$, i.e.

$$T_t w = \sum_{k\in Z} e^{-\lambda_k t}\, b_k(w)\, e_k . \tag{4.3}$$

Note in passing that

$$\mathscr{L}\Gamma_i \subset \Gamma_i , \quad i = 1, 2$$

and, consequently, the same is true for T_t:

$$T_t\Gamma_i \subset \Gamma_i , \quad i = 1, 2, \quad t \geqq 0 . \tag{4.4}$$

The list of properties of $\{T_t\}$ continues as follows:

Lemma 2. *Given $\alpha, \beta \in [0, 1]$, we have the inequalities*

$$\|T_t w\|_\alpha \leqq c(\alpha, \beta)\, t^{\beta-\alpha} \|w\|_\beta \quad \textit{for} \quad \alpha \geqq \beta , \tag{4.5}$$

$$\|T_t w - w\|_\alpha \leqq c(\alpha, \beta)\, t^{\beta-\alpha} \|w\|_\beta \quad \textit{for} \quad \beta \geqq \alpha . \tag{4.6}$$

Moreover, if $0 < \lambda < 2\alpha - \frac{3}{2}$, there is an embedding relation

$$X_\alpha \hookrightarrow C^{1+\lambda}(S^1) . \tag{4.7}$$

As to the proof, we quote (see Amann [2]) the same result associated with the Dirichlet boundary conditions ([2, Proposition 4.1]). There seem to be no essential difficulties when the periodic case is involved, particularly when taking the explicit expression (4.3) into account.

At this stage, performing a well-known procedure from the theory of evolution equations, we rewrite (S_1^ε), (S_2^ε), (I) to the integral form

$$(I_1) \qquad u(t) = T_t u^0 + \int_0^t T_{t-s}[v_x(s) + (a_2 - a_1)\, u(s) + \varepsilon \Psi(\cdot, s, u(s))_x]\, ds\,,$$

$$(I_2) \qquad v(t) = T_t v^0 + \int_0^t T_{t-s}[\sigma(\cdot, s, u(s))_x + f^\varepsilon(s)]\, ds\,.$$

As to the system (I_1), (I_2), a standard fixed point technique provides the local existence result:

Lemma 3. *Given the initial data $u^0, v^0 \in X_\beta$, $\beta \in (\frac{3}{4}, 1)$, $u^0 \in \Gamma_1$, $v^0 \in \Gamma_2$, $\|u^0\|_\beta$, $\|v^0\|_\beta \leqq \varrho$, we are able to find a positive constant $t_0 = t_0(\beta, \varepsilon, \varrho)$ such that the system (I_1), (I_2) possesses a unique solution $u, v \in C([0, t_0], X_\beta)$, $u(t) \in \Gamma_1$, $v(t) \in \Gamma_2$ for all $t \in [0, t_0]$.*

Proof. Take a set

$$\mathscr{B}(\delta) = \{(u, v) \mid u(t) \in \Gamma_1, v(t) \in \Gamma_2, \|u(t)\|_\beta + \|v(t)\|_\beta \leqq \delta, t \in [0, t_0]\}$$

along with a mapping $K = (K_1, K_2)$,

$$K_1(u, v)(t) = T_t u^0 + \int_0^t T_{t-s}[v_x(s) + (a_2 - a_1)\, u(s) +$$
$$+ \varepsilon \Psi(\cdot, s, u(s))_x]\, ds\,,$$
$$K_2(u, v)(t) = T_t v^0 + \int_0^t T_{t-s}[\sigma(\cdot, s, u(s))_x + f^\varepsilon(s)]\, ds\,.$$

To begin with, observe that $u(t) \in \Gamma_1$, $v(t) \in \Gamma_2$ combined with (1.5) bring forth $\sigma(\cdot, t, u(t)) \in \Gamma_1$ and, consequently, $\sigma(\cdot, t, u(t))_x \in \Gamma_2$. Thus, the relations (4.2), (4.4) ensure that $K_2(u, v)(t) \in \Gamma_2$ whenever $v^0 \in \Gamma_2$.

Similarly, $\Psi(\cdot, t, u(t)) \in \Gamma_2$ implies $\Psi(\cdot, t, u(t))_x \in \Gamma_1$. We infer that $K_1(u, v)(t) \in \Gamma_1$ provided $u^0 \in \Gamma_1$.

The next step is to estimate the expression

$$\sum_{i=1}^{2} \|K_i(u^1, v^1)(t) - K_i(u^2, v^2)(t)\|_\beta$$

for (u^j, v^j), $j = 1, 2$ belonging to $\mathscr{B}(\delta)$.

With $g = \sigma$ or $g = \Psi$ the hardest term takes the form

$$\int_0^t \|T_{t-s}[g(\cdot, s, u^1(s))_x - g(\cdot, s, u^2(s))_x]\|_\beta\, ds \leqq$$

(in view of (4.5))

$$\leqq c(\beta) \int_0^t (t - s)^{-\beta} \|g(\cdot, s, u^1(s))_x - g(\cdot, s, u^2(s))_x\|_0\, ds \leqq$$

(according to (4.7))

$$\leqq c(\beta, \delta, \varepsilon) \int_0^t (t - s)^{-\beta} \|u^1(s) - u^2(s)\|_\beta\, ds \leqq$$
$$\leqq c(\beta, \delta, \varepsilon)\, t_0^{1-\beta} \sup_{s \in [0, t_0]} \|u^1(s) - u^2(s)\|_\beta\,.$$

The remaining terms being much more easy to handle, we arrive at the estimate

(a) $$\sup_{s\in[0,t_0]} \sum_{i=1}^{2} \|K_i(u^1, v^1)(s) - K_i(u^2, v^2)(s)\|_\nu \leqq$$
$$\leqq c(\beta, \delta, \varepsilon)\, t_0^{1-\beta} \sup_{s\in[0,t_0]} \|u^1(s) - u^2(s)\|_\beta + \|v^1(s) - v^2(s)\|_\beta .$$

Alternatively, we have

$$\sum_{i=1}^{2} \|K_i(0, 0)(t)\|_\beta \leqq \|T_t u^0\|_\beta + \|T_t v^0\|_\beta +$$
$$+ \int_0^t \|T_{t-s}[\sigma_x(\cdot, s, 0) + f^\varepsilon(s)]\|_\beta \, ds \leqq$$
(according to (1.2), (1.9), (4.5))
$$\leqq c(\beta, \varepsilon)\, [\varrho + t_0^{1-\beta}(c_1 + c_3)] ,$$

which together with (a) implies

(b) $$\sup_{s\in[0,t_0]} \sum_{i=1}^{2} \|K_i(u, v)(s)\|_\beta \leqq c(\beta, \varepsilon)\, [\varrho + t_0^{1-\beta}(c_1 + c_3)] +$$
$$+ c(\beta, \delta, \varepsilon)\, t_0^{1-\beta}\delta .$$

Now, it is a matter of routine to choose δ, $t_0 > 0$ (t_0 small) so that $K: \mathscr{B}(\delta) \to \mathscr{B}(\delta)$ may be a contractive mapping. Thus, a straightforward application of the Banach fixed point theorem completes the proof. ■

As the next step we observe that the mild solution we have just obtained is, in fact, a classical one.

Lemma 4. *Every pair $u, v \in C([0, t_0], X_\beta)$, $\beta > \frac{3}{4}$ satisfying (I_1), (I_2) is a classical (smooth) solution of the Cauchy problem (S_1^ε), (S_2^ε), (I).*

Proof. Taking the well-known regularity results related to the one dimensional heat equation into account we are to verify

(a) $$\sigma(\cdot, \cdot, u)_x,\ \Psi(\cdot, \cdot, u)_x,\ v_x,\ u, f^\varepsilon \in C^\gamma(S^1 \times [0, t_0])$$

for a certain $\gamma > 0$.

In view of (4.1), we may restrict ourselves to the pair (u, v); more specifically, we need

(4.8) $$u_x, v_x \in C^{\gamma_1}(S^1 \times [0, t_0]) , \quad \gamma_1 > 0 .$$

Finally, due to (4.7) it suffices to show

(b) $$u, v \in C^{\gamma_2}([0, t_0], X_\alpha)$$

for $$\alpha \in (\tfrac{3}{4}, \beta) , \quad \gamma_2 \in (0, \beta - \alpha) .$$

To verify (b), take $y \geqq z$ and estimate

$$\|u(y) - u(z)\|_\alpha , \quad \|v(y) - v(z)\|_\alpha .$$

Thus,

$$\|T_y w^0 - T_z w^0\|_\alpha = \|T_z(T_{y-z}w^0 - w^0)\|_\alpha \leqq$$
(according to (4.5), (4.6))
$$\leqq c(\alpha, \beta)\, |y - z|^{\beta-\alpha}\, \|w^0\|_\beta \quad \text{where} \quad w^0 = u^0, v^0.$$

With $g = \sigma$ or Ψ, the most difficult term, as usual, seems to be

$$\|\int_0^y T_{y-s}g(\cdot, s, u(s))_x \, ds - \int_0^z T_{z-s}g(\cdot, s, u(s))_x \, ds\|_\alpha \leqq$$
$$\leqq \|\int_0^z (T_{y-s} - T_{z-s})\, g(\cdot, s, u(s))_x \, ds\|_\alpha + \|\int_y^z T_{y-s}\, g(\cdot, s, u(s))_x \, ds\|_\alpha .$$

Denoting the former term on the right-hand side by B_1 we get

$$B_1 \leqq \int_0^z \|T_{z-s}(T_{y-z} - \mathrm{Id})\, g(\cdot, s, u(s))_x\|_\alpha \, ds \leqq$$
(using (4.5), (4.6))
$$\leqq c(\alpha, \beta)\, |y - z|^{\beta-\alpha} \int_0^z \|T_{z-s}\, g(\cdot, s, u(s))_x\|_\beta \, ds \leqq$$
$$\leqq c(\alpha, \beta, \varepsilon, \sup_{s\in[0,t_0]} \|u(s)\|_\beta)\, t_0^{1-\beta} |y - z|^{\beta-\alpha} .$$

The latter term being denoted by B_2, we obtain

$$B_2 \leqq c(\alpha) \int_z^y (y - s)^{-\alpha}\, \|g(\cdot, s, u(s))_x\|_0 \, ds \leqq$$
$$\leqq c(\alpha, \varepsilon, \sup_{s\in[0,t_0]} \|u(s)\|_\beta)\, |y - z|^{1-\alpha} .$$

Estimating the other terms in a similar fashion we complete the proof. ■

Now, we turn to the question of global existence. In view of [1], the L_∞ – a priori estimates would guarantee the results we look for. To this end, consider the Riemann invariants

$$r(x, t, u, v) = v + \int_0^u \sqrt{(\sigma_u(x, t, z))}\, dz ,$$
$$s(x, t, u, v) = v - \int_0^u \sqrt{(\sigma_u(x, t, z))}\, dz$$

along with a set

$$M = M(c_4) = \{(x, t, u, v) \mid -c_4 \leqq r, s \leqq c_4\} \subset \mathbb{R}^4 .$$

From [7] we quote the following result.

Lemma 5. [7, Theorem 1.]

There is a sufficiently large constant c_4, independent of ε, such that any local solution (u, v) of (S_1^ε), (S_2^ε), (I) with

$$[x, 0, u^0(x), v^0(x)] \in M(c_4) \quad \textit{for all} \quad x \in S^1$$

is bound to satisfy

(4.9) $$[x, t, u(x, t), v(x, t)] \in M(c_4) \quad \textit{for all} \quad x \in S^1 , \quad t \in [0, t_0] .$$

To exploit the above information for showing the global existence, we will prove: If (4.9) holds, then the local solution (u, v) admits the estimate

(4.10) $$\|u(t)\|_\beta + \|v(t)\|_\beta \geqq h(t)$$

where h: $[0, +\infty) \to [0, +\infty)$ is bounded on bounded sets. In other words, (u, v) may be prolonged to become a global solution, i.e. $t_0 = +\infty$.

For this purpose, we need the following generalization of the Gronwall lemma.

Lemma 6. [8, Lemma 7.1.1.]
Let $w \geqq 0$, $w \in L_1(0, t_0)$ *satisfy*

$$w(t) \leqq c_5 t^{-\beta} + c_6 \int_0^t (t - s)^{-\beta} w(s) \, ds \,, \quad \beta \in (0, 1) \,.$$

Then

$$w(t) \leqq c_5 \, c(c_6, \beta, t_0) \, t^{-\beta} \,, \quad t \in (0, t_0] \,. \tag{4.11}$$

With (4.9) in mind, we estimate

$$\|u(t)\|_\beta + \|v(t)\|_\beta \leqq c(\beta, \varepsilon) \{\|u^0\|_\beta + \|v^0\|_\beta + \int_0^t (t - s)^{-\beta} [\|v_x(s)\|_0 + \|\sigma(\cdot, s, u(s))_x\|_0 + \|\Psi(\cdot, s, u(s))_x\|_0 + \|u(s)\|_0 + c_3] \, ds\} \leqq$$

(according to (4.7), (4.9))

$$\leqq c(\beta, \varepsilon, c_4) \{t_0(\|u^0\|_\beta + \|v^0\|_\beta) \, t^{-\beta} + \int_0^t (t - s)^{-\beta} (\|u(s)\|_\beta + \|v(s)\|_\beta) \, ds\} \,.$$

Consequently, the conclusion of Lemma 6 implies (4.10).

As the final step, we establish the existence of time-periodic solution to (S_1^ε), (S_2^ε). Consider the set

$$\mathscr{M} = \{(u, v) \mid u, v \in X_\beta \,, \quad u \in \Gamma_1, v \in \Gamma_2 \,, \quad [x, 0, u(x), v(x)] \in M(c_4) \quad \text{for all} \quad x \in S^1\} \,.$$

One easily observes that $\mathscr{M}$, being regarded as a subset of the space X_β, is a nonempty closed convex set. Note in passing that, in view of the periodicity of σ, the condition

$$[x, 0, u(x), v(x)] \in M(c_4)$$

is equivalent to

$$[x, k\omega, u(x), v(x)] \in M(c_4) \,, \quad k \in \mathbb{Z} \,.$$

Moreover, taking the above results into account we are able to define the **Poincaré** operator

$$\Pi : \mathscr{M} \to \mathscr{M}$$

where $\Pi(u^0, v^0) = (u(\omega), v(\omega))$, u, v being the unique solution of (I_1), (I_2).

Lemma 7. *Π is a mapping continuous and compact with respect to the X_β-topology induced on $\mathscr{M}$, where $\beta \in (\frac{3}{4}, 1)$.*

Proof. By virtue of Lemma 6, the continuity of Π may be proved by taking advantage of the procedure which has become standard in this section.

As to the compactness of $\Pi(\mathscr{M})$, it suffices to prove the boundedness of this set in $X_\gamma \times X_\gamma$, $\gamma \in (\beta, 1)$ since the embedding $X_\gamma \hookrightarrow X_\beta$ is compact whenever $\gamma > \beta$. To show this, we compute

$$\|u(t)\|_\gamma + \|v(t)\|_\gamma \leqq c(\gamma, \varepsilon)\{(\|u^0\|_0 + \|v^0\|_0)\, t^{-\gamma} + \\ + \int_0^t (t-s)^{-\gamma}\,[\|\sigma(\cdot, s, u(s))_x\|_0 + \|v_x(s)\|_0 + \|u(s)\|_0 + \\ + \|\Psi(\cdot, s, u(s))\|_0 + c_3]\, ds\}\,.$$

Now, (4.7), (4.9) combined with Lemma 6 give finally

$$\|u(t)\|_\gamma + \|v(t)\|_\gamma \leqq c(\gamma, \omega, \varepsilon, c_3, c_4)\,.$$ ■

Making use of the Schauder fixed point theorem we achieve the final result.

Lemma 8. *Let* (4.1), (4.2) *hold.*

Then there exists at least one classical, double-periodic (*i.e determined on* T^2) *solution to the parabolic system* (S_1^ε), (S_2^ε).

Moreover,

(4.12) $\qquad u(\cdot, t) \in \Gamma_1\,, \quad v(\cdot, t) \in \Gamma_2$ *for all* $t \in S^2$,

and, in view of (4.9), *the estimate*

(4.13) $\qquad \|u\|_{C(T^2)} + \|v\|_{C(T^2)} \leqq C$

holds independently of $\varepsilon > 0$.

Finally, due to (4.8), there is $\mu = \mu(\varepsilon) > 0$ such that

(4.14) $\qquad u_x, v_x \in C^\mu(T^2)\,.$

5. A LIMIT PROCESS

In this section we are going to construct a weak solution to (E), (B), (P) taking advantage of the following limit process.

Let us set

$$\tilde{U}(x, t) = \int_0^x u(z, t)\, dz\,, \quad x \in [0, l]\,, \quad t \in S^2\,.$$

For $\varepsilon_n = (1/n)$, we define

$$f^{\varepsilon_n} = \psi^n(x) f(x, t, u, v - a_1\tilde{U}, \tilde{U}) \quad \text{for} \quad x \in [0, l]\,, \quad t \in S^2$$

with

$$\psi^n(x) = \begin{cases} 0 & x \in \left(-\infty, \frac{1}{n}\right] \cup \left[l - \frac{1}{n}, +\infty\right) \\ \in [0, 1] \quad \text{for} & x \in \left[\frac{1}{n}, \frac{2}{n}\right] \cup \left[l - \frac{2}{n}, l - \frac{1}{n}\right] \\ 1 & x \in \left[\frac{2}{n}, l - \frac{2}{n}\right]. \end{cases}$$

Under such circumstances, the function f^{ε_n} may be prolonged onto T^2 to satisfy (4.2).

In view of (1.9), all results of Section 4 apply to the problem $(S_1^{\varepsilon_n})$, $(S_2^{\varepsilon_n})$ with the right-hand side f^{ε_n} determined above (in fact, it is an integro-differential system). Consequently, Lemma 8 gives rise to the existence of a solution pair (u^n, v^n).

Moreover, according to Lemma 1, there is a function U^n satisfying

(5.1) $$U_x^n = u^n\,, \quad U_t^n + a_1 U^n = v^n + (1/n)\,(u_x^n + \Psi(x, t, u^n))\,.$$

The relation (4.13) induces a very important estimate

(5.2) $$\|u^n\|_{C(T^2)} + \|v^n\|_{C(T^2)} \leqq C \quad \text{for} \quad n = 1, 2, \ldots.$$

According to (4.12), (5.1) we get

(5.3) $$U^n(0, t) = U^n(l, t) = 0\,, \quad t \in S^2\,, \quad n = 1, 2, \ldots,$$

which implies that U^n and $\tilde{U}$ coincide on $[0, l] \times \mathbb{R}^1$.

Being led by analogy with autonomous systems we introduce the concept of entropy-flux pairs.

Definition 2. *A couple of functions* $\eta = \eta(x, t, u, v)$, $q = q(x, t, u, v)$: $T^2 \times \mathbb{R}^2 \to \mathbb{R}^1$ *is called an entropy-flux* $(e-f)$ *pair if* η, q *are of the class* C^1 in x, t, C^2 *in* u, v, *and solve a linear system of equations*

(5.4) $$\begin{aligned} &q_v + \eta_u = 0 \\ &q_u + \sigma_u(x, t, u)\,\eta_v = 0 \end{aligned}$$

for all x, t, u, v.

As a natural example, consider the pair

$$\eta = \mathscr{E}(x, t, u, v) = \Sigma(x, t, u) + \frac{v^2}{2}\,,$$

$$q = \varphi(x, t, u, v) = -v\sigma(x, t, u)\,,$$

where $\Sigma(x, t, u) = \int_0^u \sigma(x, t, z)\,dz$, corresponding to the total energy.

To be apparently short, we adopt the following convention. For an arbitrary function $g = g(x, t, u, v)$, the symbol g^n stands for the superposition $g(x, t, u^n(x, t), v^n(x, t))$. In other words, g^n is understood as a function of the variables x, t only.

After a rather lengthy but straightforward computation, we arrive at the formula:

(5.5) $$(\eta^n)_t + (q^n)_x = \sum_{i=1}^{4} B_i$$

where

$$B_1 = \eta_t^n + q_x^n + \eta_v^n \sigma_x^n - \eta_u^n a_1 u^n - \eta_v^n a_2 v^n + \eta_v^n f^n + \frac{1}{n}\,\eta_u^n \Psi_x^n\,,$$

$$B_2 = \frac{1}{n}\,((\eta_u^n u_x^n)_x + (\eta_v^n v_x^n)_x)\,,$$

$$B_3 = -\frac{1}{n}(\eta^n_{uu}(u^n_x)^2 + 2\eta^n_{uv}u^n_x v^n_x + \eta^n_{vv}(v^n_x)^2),$$

$$B_4 = \frac{1}{n}(-\eta^n_{xu}u^n_x - \eta^n_{xv}v^n_x + \eta^n_u\Psi^n_u u^n_x)$$

on T^2 for each (e–f) pair η, q.

Thus the special choice $\eta = \mathscr{E}$, $q = \varphi$ combined with integration by parts of the relation (5.5) leads to

$$0 = \frac{1}{n}\iint_{T^2} \sigma^n_u(u^n_x)^2 + (v^n_x)^2\,dx\,dt + \frac{1}{n}\iint_{T^2} \sigma^n_x u^n_x + \sigma^n\Psi^n_u u^n_x + \ldots$$

$$\ldots \text{ bounded terms.}$$

Evoking the estimate (5.2) one obtains

$$\|\sigma^n_x\|_{L_\infty}, \quad \|\sigma^n\Psi^n_u\|_{L_\infty} \leqq c_6$$

which, with help of (1.3) and the Cauchy-Schwarz inequality, yields

$$\text{(5.6)} \qquad \frac{1}{n}(\|u^n_x\|^2_{L_2} + \|v^n_x\|^2_{L_2}) \leqq c_7\,.$$

The key for obtaining a weak solution to our problem is contained in the rather surprising conjecture, the proof of which we postpone to the next section:

(H) $\qquad u^n \to u\,, \quad v^n \to v \quad$ *for a.e.* $(x, t) \in Q$

passing to subsequences as the case may be.

One easily observes that, due to (4.12), (H) holds, in fact, on T^2.

As a direct consequence of (5.2), (H), we deduce

(5.7) $\qquad u^n \to u\,, \quad v^n \to v \quad$ strongly in $\quad L_p(T^2)$ for all $p < +\infty$.

According to (5.2), (5.6),

(5.8) $\qquad \frac{1}{u}u^n_x\,, \quad \frac{1}{n}v^n_x\,, \quad \frac{1}{n}\Psi^n \to 0 \quad$ strongly in $\quad L_2(T^2)$.

Seeing that the functions U^n_t, U^n are orthogonal in $L_2(T^2)$ we draw from (5.1) that

(5.9) $\qquad U^n \to U \quad$ strongly in $\quad W^1_2(T^2)$

with

(5.10) $\qquad U_x = u\,, \quad U_t + a_1 U = v\,.$

Moreover, by (5.3), (5.9) we obtain

(5.11) $\qquad U(0, \cdot) = U(l, \cdot) = 0 \quad$ on $\quad S^2$

at least in the sense of traces.

Finally,

(5.12) $\qquad \sigma(\cdot, \cdot, u^n) \to \sigma(\cdot, \cdot, u)$

strongly in, say, $L_1(T^2)$, and

(5.13) $\qquad f^{\varepsilon_n} \to \tilde{f}$ strongly in, say, $L_1(T^2)$

where $\tilde{f}(\cdot, t) \in \Gamma_2$, $t \in S^2$, and

(5.14) $\qquad \tilde{f}(x, t) = f(x, t, U_x, U_t, U)$ for a.e. $(x, t) \in Q$.

Being multiplied by a test function $\varphi \in C^\infty(T^2)$, $\varphi(\cdot, t) \in \Gamma_2$ and integrated by parts the equation $(S_2^{\varepsilon_n})$ takes the form

(5.15) $\qquad \iint_{T^2} -v^{n+1}\varphi_t - (1/n)\, v^{n+1}\varphi_{xx} + a_2 v^{n+1}\varphi + \sigma(x, t, u^{n+1})\,\varphi_x - f^{\varepsilon_n}\varphi \,\mathrm{d}x\,\mathrm{d}t = 0,$

which, combined with the symmetry properties (4.12), gives rise to

(5.16) $$\iint_Q -v^{n+1}\varphi_t - (1/n)\, v^{n+1}\varphi_{xx} + \sigma(x, t, u^{n+1})\,\varphi_x + \\ + a_2 v^{n+1}\varphi - f^{\varepsilon_n}\varphi \,\mathrm{d}x\,\mathrm{d}t = 0\,.$$

Taking advantage of the aforementioned relations concerning the convergence of (u^n, v^n) we are able to pass to the limit in (5.16) to obtain (2.1.)

In the conclusion, note that (5.10) gives successively $U_t = v - a_1 U \in L_p(Q)$. $p \in [1, +\infty)$, $U \in C(Q)$ and, finally, $U \in W^1_\infty(Q)$, U satisfying (B) due to (5.11).

Theorem 1 has been proved.

6. THE PROOF OF THE CONJECTURE (H)

As already remarked, the compensated compactness theory along with the concept of the Young measure proved to be very useful when dealing with passage to the limit in weakly convergent sequences.

Following the line of arguments presented in [6], we intend to prove the conjecture (H) claimed in Section 5. However, note that some differences appear as a consequence of the explicit dependence of σ on the variables x, t.

We start with the Young measure related to our system, the basic reference material being represented by Tartar's work [21].

Consider the sequences $\{u^n\}$, $\{v^n\}$ viewed as functions defined on $\mathbb{R}^2$. By virtue of (5.2), there are subsequences (not relabelled for convenience) such that

(6.1) $\qquad u^n \to u\,,\quad v^n \to v$ weakly-star in $L_\infty(Q)$.

We determine two auxiliary sequences $\{w_1^n\}$, $\{w_2^n\}$ as $w_1^n(x, t) = x$, $w_2^n(x, t) = t$ on Q We have (obviously!)

(6.2) $\qquad w_1^n \to w_1\,,\quad w_2^n \to w_2$ uniformly on $C(Q)$

where $w_1 \equiv x$, $w_2 \equiv t$.

It can be shown (cf. [21]), passing to subsequences if necessary, that the limit

$$\lim_{n\to\infty} g(w_1^n, w_2^n, u^n, v^n) = \bar{g}$$

does exist for all $g \in C(\mathcal{O})$, $\mathcal{O} = Q \times [-C, C]^2$ in the sense of the weak-star topology on $L_\infty(Q)$.

Moreover, there is a family of probability measures $\nu_{x,t}$ (the Young measures) on the set $\mathcal{O}$ satisfying

(6.3) $$\langle \nu_{x,t}, g \rangle = \bar{g}(x, t) \quad \text{for a.e.} \quad (x, t) \in Q .$$

It is easy to see (cf. [21]) that (H) holds if, (and only if) $\nu_{x,t}$ reduces to a Dirac mass (centered at the point $[x, t, u(x, t), v(x, t)]$) for a.e. $(x, t) \in Q$.

To prove the last assertion, we desire to minimize the possible support of $\nu_{x,t}$.

Lemma 9. *Under the hyhotheses* (6.1), (6.2), *the Young measure* $\nu_{x,t}$ *is supported by the set* N,

$$N = \{[w_1(x, t), w_2(x, t), u, v] \mid (u, v) \in [-C, C]^2\} .$$

In other words, for our particular choice of w_1^n, w_2^n, *there is a probability measure* $\bar{\nu}_{x,t}$ *on* $[-C, C]^2$ *such that*

(6.4) $$\langle \nu_{x,t}, g \rangle = \langle \bar{\nu}_{x,t}, g(x, t, \cdot, \cdot) \rangle .$$

Proof. Take a continuous function g such that $\operatorname{supp}(g) \cap N = \emptyset$.

We are to show $\langle \nu_{x,t}, g \rangle = \bar{g}(x, t) = 0$.

According to (6.2), there is a neighbourhood $\mathcal{N}$ of the point (x, t) and an index n_0 such that

$$[w_1^n, w_2^n, u^n, v^n] \cap \operatorname{supp}(g) = \emptyset$$

for all $(y, s) \in \mathcal{N}$, $n \geqq n_0$.

Consequently, $\bar{g} = 0$ on $\mathcal{N}$. ∎

At this stage, let us turn to the relation (5.5). We set $\Omega = \{(x, t) \mid x \in (-2l, 2l), t \in (-2\omega, 2\omega)\}$.

With help of the estimates (5.2), (5.6), one deduces:

(a) B_1 is bounded in $L_\infty(\Omega)$,
(b) B_2 belongs to a compact set of $W_2^{-1}(\Omega)$,
(c) B_3 is bounded in $L_1(\Omega)$,
(d) B_4 is bounded in $L_2(\Omega)$,
(e) $\{\eta^n\}$, $\{q^n\}$ are bounded in $L_\infty(\Omega)$,

for any (e–f) pair η, q and independently of n. In view of the above relations, Murat's lemma [21] offers the following conclusion:

(6.5) $$(\eta^n)_t + (q^n)_x \quad \text{belongs to a compact set of} \quad W_2^{-1}(Q)$$

independently of n.

For any (e–f) pair η_i, q_i, $i = 1, 2$, denote by

$$\eta_i^n \to \bar{\eta}_i , \quad q_i^n \to \bar{q}_i$$

the corresponding weak-star limits on $L_\infty(Q)$.

The estimate (6.5) enables us to evoke the classical result of the compensated compactness theory – the "div-curl" lemma in order to obtain

$$\eta_1^n q_2^n - \eta_2^n q_1^n \to \bar{\eta}_1 \bar{q}_2 - \bar{\eta}_2 \bar{q}_1 .$$

In another form:

$$(6.6) \qquad \langle \nu_{x,t}, \eta_1 q_2 - \eta_2 q_1 \rangle = \langle \nu_{x,t}, \eta_1 \rangle \langle \nu_{x,t}, q_2 \rangle - \langle \nu_{x,t}, \eta_2 \rangle \langle_{x,t}, q_1 \rangle .$$

For fixed $(x, t) \in Q$, consider a pair of functions $\eta = \eta(u, v)$, $q = q(u, v)$ solving (5.4). It is a matter of routine to construct an (e−f) pair $\tilde{\eta}$, $\tilde{q}$ (in the sense of Definition 2) satisfying

$$\tilde{\eta}(x, t, \cdot, \cdot) = \eta , \quad \tilde{q}(x, t, \cdot, \cdot) = q .$$

Note in passing that such an extension is by no means uniquely determined.

Thus, the relation (6.4) together with (6.6) gives finally

$$(6.7) \qquad \langle \bar{\nu}_{x,t}, \eta_1 q_2 - \eta_2 q_1 \rangle = \langle \bar{\nu}_{x,t}, \eta_1 \rangle \langle \bar{\nu}_{x,t}, q_2 \rangle - \langle \bar{\nu}_{x,t}, \eta_2 \rangle \langle \bar{\nu}_{x,t}, q_1 \rangle$$

for each pair η_i, q_i, $i = 1, 2$ satisfying (5.4) for fixed (x, t).

The relation (6.7) is nothing else than the Tartar equation for the Young measure $\bar{\nu}_{x,t}$ appearing when dealing with autonomous hyperbolic systems of nonlinear elasticity, the functions η_i, q_i representing some entropy-flux pair in the classical sense (see DiPerna [6]).

However, by virtue of the remarkable result of DiPerna [6, Section 5], $\bar{\nu}_{x,t}$ is bound to be a Dirac mass whenever (1.3), (1.6) hold.

Thus, the conjecture (H) is proved.

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Souhrn

METODA KOMPENSOVANÉ KOMPAKTNOSTI A ČASOVĚ PERIODICKÁ ŘEŠENÍ NEAUTONOMNÍ KVASILINEÁRNÍ TELEGRAFNÍ ROVNICE

EDUARD FEIREISL

V práci je dokázána existence slabého časově-periodického řešení nelineární telegrafní rovnice

$$U_{tt} + dU_t - \sigma(x, t, U_x)_x + aU = f(x, t, U_x, U_t, U)$$

s Dirichletovými okrajovými podmínkami. Pravá strana rovnice nemusí být nutně „malá". Idea důkazu je založena na metodě kompensované kompaktnosti.

Резюме

МЕТОД КОМПЕНСИРОВАННОЙ КОМПАКТНОСТИ И ПЕРИОДИЧЕСКИЕ ВО ВРЕМЕНИ РЕШЕНИЯ НЕОДНОРОДНОГО КВАЗИЛИНЕЙНОГО ТЕЛЕГРАФНОГО УРАВНЕНИЯ

EDUARD FEIREISL

В работе доказано существование по крайней мере одного слабого периодического во времени решения для уравнения

$$U_{tt} + dU_t - \sigma(x, t, U_x)_x + aU = f(x, t, U_x, U_t, U)$$

с граничными условиями Дирихле. Отметим, что на функцию f не налагаются никакие условия „малости".

Основная идея доказательства — метод компенсированной компактности.

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