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ON NECESSARY OPTIMALITY CONDITIONS IN A CLASS OF OPTIMIZATION PROBLEMS

Jiří V. Outrata

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Summary. In the paper necessary optimality conditions are derived for the minimization of a locally Lipschitz objective with respect to the constraints $x \in S$, $0 \in F(x)$, where S is a closed set and F is a set-valued map. No convexity requirements are imposed on F. The conditions are applied to a generalized mathematical programming problem and to an abstract finite-dimensional optimal control problem.

Keywords: Set-valued map, tangent cone, generalized gradient, regular point.

AMS Subject Classification: Primary: 49B34, Secondary: 90C99.

1. INTRODUCTION

Let us consider an optimization problem

$$(1.1) \qquad \begin{aligned} & f(x) \to \inf \\ \text{subject to} \quad & \\ & 0 \in F(x) \\ & x \in S\,, \end{aligned}$$

where $x \in \boldsymbol{R}^n$, $f[\boldsymbol{R}^n \to \boldsymbol{R}]$ is a locally Lipschitz objective, $F[\boldsymbol{R}^n \to 2^{\boldsymbol{R}^m}]$ is a closed-valued set-valued map and S is a nonempty closed subset of $\boldsymbol{R}^n$. The relation $0 \in F(x)$ may represent various constraint structures or even optimality conditions for certain internal optimization problems. Optimization problems of the type (1.1) have been already investigated in many works, e.g. [2], [8], [11], but in accordance with most applications the authors assume that F enjoys some kind of convexity properties. The aim of this contribution is to derive necessary optimality conditions for (1.1) in absence of any convexity requirements. To avoid the difficulties with the evaluation of Clarke's tangent cone to the set $Q = \{x \in S \mid 0 \in F(x)\}$ and its polar, we apply the reduction technique of Ioffe [6] and an important result of Hiriart-Urruty [5]. One could alternatively exploit the expression for the above mentioned cone given in [4] in terms of support functions; however, as our map F is not convex-valued, we do not find this characterization suited to our purpose. Besides the theory developed in the next section, we give also an application proposal in Sect. 3 as an illustration.

As we do not want to deal with existence problems, we assume throughout the whole paper that Q is nonempty and compact. Then clearly (1.1) possesses a solution and the corresponding objective value is finite.

For the understanding of the paper a certain basic knowledge of nonsmooth analysis is necessary. For the reader's convenience, we provide here at least the definitions of the basic conical approximations.

Definition 1.1. Let $\Omega \subset \boldsymbol{R}^n$ and $x \in \Omega$. Then a vector $h \in \boldsymbol{R}^n$ belongs to the *contingent cone* $T_\Omega(x)$ to Ω at x if there exist a sequence of (positive) scalars $\lambda_i \downarrow 0$ and a sequence of directions $h_i \to h$ such that $x + \lambda_i h_i \in \Omega$ for all i.

A vector $k \in \boldsymbol{R}^n$ belongs to *Clarke's tangent cone* $C_\Omega(x)$ to Ω at x provided for all sequences of points $x_i \to x$ and all sequences of scalars $\lambda_i \downarrow 0$ there exists a sequence of directions $k_i \to k$ such that $x_i + \lambda_i k_i \in \Omega$ for all i.

As is shown e.g. in [1], $C_\Omega(x)$ is closed and convex. Its negative polar cone $-C^*_\Omega(x)$ is called *normal*, denoted by $N_\Omega(x)$ and for Ω convex it coincides with the normal cone in the sense of convex analysis. The convexity of $C_\Omega(x)$ enables us to generalize the concept of the adjoint set-valued map introduced by Pschenichnyi in [7].

Definition 1.2. Let $\Phi[\boldsymbol{R}^n \to 2^{\boldsymbol{R}^m}]$ be a set-valued map, $y \in \Phi(x)$ and $y^* \in \boldsymbol{R}^m$. Then the map $\Phi^*[\boldsymbol{R}^m \to 2^{\boldsymbol{R}^n}]$ which assigns to y^* the set

$$\Phi^*(y^*; x, y) = \{x^* \in \boldsymbol{R}^n \mid (-x^*, y^*) \in C^*_{\text{graph}\Phi}(x, y)\} \tag{1.2}$$

is termed the *adjoint set-valued map* to Φ at (x, y).

We employ the following notation: $|\cdot|_n$ is a norm in $\boldsymbol{R}^n$, $B^\varepsilon_n(\hat{x}) = \{x \in \boldsymbol{R}^n \mid |x - \hat{x}|_n \leqq \varepsilon\}$. If $\varepsilon = 1$ and $\hat{x} = 0$, we write simply B_n. $\partial f(x_0)$ is the generalized gradient of Clarke of a function f at a point x_0. For a function φ defined in $\boldsymbol{R}^n \times \boldsymbol{R}^m$ the symbols $\partial_x\varphi(x, y)$, $\partial_y\varphi(x, y)$ denote the partial generalized gradients with respect to the first and second variables, respectively. gfF is the graph of a set-valued map F, $\text{dist}_S(x)$ is the distance of a vector x from a set S and $\text{Proj}_S(x)$ means the projection of x onto S.

The properties of the above defined conical approximations as well as all other necessary background are collected e.g. in [1] or [3].

2. OPTIMALITY CONDITIONS

We denote

$$Q = \{x \in S \mid 0 \in F(x)\}$$

(2.1) $$f_1 \colon x \mapsto \operatorname{dist}_{F(x)}(0)$$

and

(2.2) $$A \colon x \mapsto \operatorname{Proj}_{F(x)}(0)\,.$$

Using this notation, we may rewrite (1.1) in the form

$$f(x) \to \inf$$

subject to

$$f_1(x) = 0$$

$$x \in S\,.$$

To be able to apply the reduction theorem of Ioffe, we have to impose some fundamental assumptions. To this purpose we recall that a point $z \in Q$ is said to be a *regular* point (in the sense of Ioffe) for the equality constraint $f_1(x) = 0$ relative to S if there are $k > 0$ and a neighbourhood O of z such that for all $x \in O \cap S$

$$\operatorname{dist}_Q(x) = \inf_{v \in Q} |v - x|_n \leqq k f_1(x) = k \inf_{y \in F(x)} |y|_m\,.$$

Thus, we will assume that for a local solution $\hat{x}$ of (1.1)

(i) f_1 is Lipschitz near $\hat{x}$;

(ii) $\hat{x}$ is a regular point (Ioffe) for f_1 relative to S.

The validity of hypothesis (i) depends on the nature of the "distance" problem

$$J(y) = |y|_m \to \inf$$

(2.3) subject to

$$y \in F(\hat{x})$$

with 0 being its unique global solution. For example, if gfF is given by means of equalities and inequalities, the standard Mangasarian-Fromowitz constraint qualification at $y = 0$ implies that (i) holds, see [10]. Hypothesis (ii) is termed in [4] as *nondegeneracy* of F on S at $\hat{x}$. In [4] and [11] we can find various "regularity" conditions implying this nondegeneracy and hence the validity of (ii).

The application of the result from [5] requires to impose still another assumption which, however, does not seem to be so restrictive as the previous two.

(iii) The set-valued map A possesses a selection α which is continuous at $\hat{x}$.

If gfF is given by equalities and inequalities, the sufficient second-order optimality condition of [9] applied to (2.3) at $y = 0$ (with $J(y)$ replaced by $|y|_m^2$) together with a constraint qualification suffices for hypothesis (iii) to hold, cf. [9].

Proposition 2.1. *Assume that $\hat{x}$ is a local solution of* (1.1) *and that assumptions* (i)–(iii) *hold. Then there exists a vector $y^* \in \boldsymbol{R}^m$ such that the pair $(\hat{x}, y^*)$ satisfies the relation*

(2.4) $$0 \in \partial f(\hat{x}) + F^*(y^*; \hat{x}, 0) + N_S(\hat{x})\,.$$

Proof. For $r > 0$ we denote

$$M_r(x) = f(x) + r(f_1(x) + \text{dist}_S(x)) . \tag{2.5}$$

The reduction theorem from [6] implies that under assumptions (i), (ii) and for sufficiently large r, M_r attains its local minimum at $\hat{x}$. It implies furthermore that due to (i) and Clarke's calculus

$$0 \in \partial f(\hat{x}) + r[\partial f_1(\hat{x}) + \partial(\text{dist}_S(\hat{x}))] . \tag{2.6}$$

However, now we need to express $\partial f_1(x)$ or an upper estimate of this set in terms of F. Because of assumptions (i), (iii), we may apply to this purpose the important result from [5] stating that

$$\partial f_1(\hat{x}) \subset \{x^* \in R^n \mid (x^*, -y^*) \in N_{gfF}(\hat{x}, v), y^* \in \partial J(v)\} ,$$

where $v = \alpha(\hat{x}) \subset A(\hat{x})$. As $A(\hat{x}) = \{0\}$, one has $\partial J(v) = B_m$ and we obtain that

$$\partial f_1(\hat{x}) \subset \{x^* \in R^n \mid x^* \in F^*(y^*; \hat{x}, 0), y^* \in B_m\} \tag{2.7}$$

by using the adjoint set-valued map concept introduced in Def. 1.2.

It is well-known from the nonsmooth analysis that for any $\tau > 0$

$$\tau\, \partial(\text{dist}_S(\hat{x})) \subset N_S(\hat{x}) . \tag{2.8}$$

Thus, the assertion is directly implied by (2.6), (2.7), (2.8). □

The method used has enabled us to express the optimality conditions in terms of the normal cone to gfF instead of that to Q and hence to avoid the difficulties with the evaluation of $N_Q(x)$.

We employ now the regularity concept of Clarke to characterize y^*. We recall that a set Ω is *regular* (in the sense of Clarke) at $x \in \Omega$ provided $T_\Omega(x) = C_\Omega(x)$.

Proposition 2.2. *Let all assumptions of Prop.* 2.1 *hold and additionally let gfF be regular* (*Clarke*) *at* $(\hat{x}, 0)$. *Then*

$$y^* \in -N_{F(\hat{x})}(0) . \tag{2.9}$$

Proof. By Prop. 2.1 and due to the regularity hypothesis

$$0 \in \partial f(\hat{x}) + x^* + N_S(\hat{x}) ,$$

and

$$\langle -x^*, \xi \rangle + \langle y^*, \eta \rangle \geqq 0 \quad \text{for all} \quad (\xi, \eta) \in T_{gfF}(\hat{x}, 0) .$$

Let $v \in C_{F(\hat{x})}(0) \subset T_{F(\hat{x})}(0)$. By the definition of the contingent cone it is clear that then $(0, v) \in T_{gfF}(\hat{x}, 0)$. (Indeed, there exist sequences $\lambda_i \downarrow 0$ and $v_i \to v$ such that $\lambda_i v_i \in F(\hat{x})$ for all i. But this implies that $(\hat{x} + \lambda_i 0, 0 + \lambda_i v_i) \in gfF$ for all i.) Thus,

$$\langle y^*, v \rangle \geqq 0 \quad \text{for all} \quad v \in C_{F(\hat{x})}(0) ,$$

and the proof is complete. □

Corollary. *Under the assumptions of Prop.* 2.2 *and provided that* $F(\hat{x})$ *is convex, one has*

$$\langle y^*, y \rangle \geqq 0 \quad \text{for all} \quad y \in F(\hat{x}) . \tag{2.10}$$

Proof. Due to the convexity of $F(\hat{x})$

$$y \in C_{F(\hat{x})}(0) \quad \text{for all} \quad y \in F(\hat{x}) . \qquad \square$$

Relations (2.4), (2.10) are termed in the literature as the *support principle*, cf. [4], [11].

In many real cases the computation of $C_{gfF}(\hat{x}, 0)$ is very difficult and hence the evaluation of the adjoint map F^* is hardly possible. However, from the proof of Prop. 2.1 it is clear that in (2.4) $F^*(y^*; \hat{x}, 0)$ may be replaced by the set

$$\{x^* \in \mathbf{R}^n \mid (-x^*, y^*) \in D^*\} , \tag{2.11}$$

where D is any convex cone with the vertex at the origin and satisfying the inclusion $D \subset C_{gfF}(\hat{x}, 0)$. The optimality conditions are then correspondingly less selective.

Assumption (ii) concerns only the problem constraints $x \in Q$. It may be replaced by a "calmness" requirement concerning, however, the whole problem (1.1). We denote

$$\psi_\varepsilon(s) = \inf \{f(x) \mid x \in S \cap B_n^\varepsilon(\hat{x}), s \in F(x)\} \tag{2.12}$$

and replace (ii) by the hypothesis

(ii)′ there exists an $\varepsilon > 0$ such that

$$\liminf_{s \to 0} \frac{\psi_\varepsilon(s) - \psi_\varepsilon(0)}{|s|_m} > -\infty . \tag{2.13}$$

Problems of the type (1.1) satisfying hypothesis (ii)′ are termed *calm* at $\hat{x}$. It has been shown in [11] that for such a problem there exists a positive constant r for which the function

$$W(x) = f(x) + r f_1(x)$$

attains its local minimum over S at $\hat{x}$. Prop. 2.4.3 in Chap. 2 of [3] implies then the validity of (2.6).

3. APPLICATIONS

Consider an optimization problem of the type (1.1) in which

$$F(x) = \{y \in \mathbf{R}^m \mid y \in -\mu(x) + K\} , \tag{3.1}$$

where $\mu[\mathbf{R}^n \to \mathbf{R}^m]$ is a continuously differentiable operator and $K \subset \mathbf{R}^m$ is a nonempty closed set. Necessary optimality conditions for such a problem can be found in many works under the assumption that K is a convex cone with vertex at the origin. The theory from the previous section enables us to omit this assumption.

Proposition 3.1. *Assume that* $\hat{x}$ *is a local solution of* (1.1) *with* F *given by* (3.1) *and that assumption* (ii) *holds. Then there exists a vector* $k^* \in C_K^*(\mu(\hat{x}))$ *for which one has*

$$0 \in \partial f(\hat{x}) - (\nabla\mu(\hat{x}))^T k^* + N_S(\hat{x}) . \tag{3.2}$$

In the proof we make use of the following lemma.

Lemma 3.1. *Let* $\bar{x} \in R^n$ *be given and let* $\bar{y} = -\mu(\bar{x}) + \bar{v}$, *where* $\bar{v} \in K$. *Then for* F *given by* (3.1) $C_{gfF}(\bar{x}, \bar{y}) = D$, *where*

$$D = \{(h, k) \subset R^n \times R^m \mid k = -\nabla\mu(\bar{x})\, h + \xi, \xi \in C_K(\bar{v})\} . \tag{3.3}$$

Proof. Let $(h, k) \in C_{gfF}(\bar{x}, \bar{y})$. By definition for all $(x_i, y_i) \to (\bar{x}, \bar{y})$, $\lambda_i \downarrow 0$ there exist sequences $h_i \to h$, $k_i \to k$ such that

$$y_i + \lambda_i k_i = -\mu(x_i + \lambda_i h_i) + \tilde{v}_i , \quad \tilde{v}_i \in K$$

for all i. We may certainly express the vectors $\tilde{v}_i$ in the form

$$\tilde{v}_i = v_i + \lambda_i \xi_i ,$$

where $v_i = y_i + \mu(x_i)$ so that $v_i \to \bar{v}$. Hence,

$$k_i = -\frac{\mu(x_i + \lambda_i h_i) - \mu(x_i)}{\lambda_i} + \xi_i . \tag{3.4}$$

As $h_i \to h$ and $k_i \to k$, the sequence ξ_i converges to the vector $\xi = k + \nabla\mu(\bar{x})\, h$. Moreover, we observe that for all sequences $v_i \to \bar{v}$, $\lambda_i \downarrow 0$

$$v_i + \lambda_i \xi_i \in K$$

so that $\xi \in C_K(\bar{v})$ and consequently $(h, k) \in D$.

Conversely, let $(h, k) \in D$, i.e. $k = -\nabla\mu(\bar{x})\, h + \xi$, $\xi \in C_K(\bar{v})$, $\bar{v} = \bar{y} + \mu(\bar{x})$. We take arbitrary sequences $(x_i, y_i) \to (\bar{x}, \bar{y})$, $\lambda_i \downarrow 0$ and denote $v_i = y_i + \mu(x_i)$ so that $v_i \to \bar{v}$. By definition there exists a sequence $\xi_i \to \xi$ such that $v_i + \lambda_i \xi_i \in K$ for all i. We set now $h_i = h$ and assume that k_i are given by (3.4). Clearly, $k_i \to k$ and

$$\begin{aligned} y_i + \lambda_i k_i &= y_i - \mu(x_i + \lambda_i h_i) + \mu(x_i) + \lambda_i \xi_i = \\ &= -\mu(x_i + \lambda_i h_i) + v_i + \lambda_i \xi_i \in -\mu(x_i + \lambda_i h_i) + K . \end{aligned}$$

The assertion has been proved. □

It can be easily shown that

$$C_{gfF}^*(\bar{x}, 0) = \{(h^*, k^*) \in R^n \times R^m \mid h^* = (\nabla\mu(\bar{x}))^T k^*, k^* \in C_K^*(\mu(\bar{x}))\}$$

so that to prove Prop. 3.1 one needs merely to verify assumptions (i) and (iii) of the previous section.

Proof of Prop. 3.1. Consider the "distance" problem (2.3) which attains for a fixed $x \in \mathbf{R}^n$ the form

$$(3.5) \qquad \begin{gathered} |y|_m \to \inf \\ \text{subject to} \\ y \in -\mu(x) + K . \end{gathered}$$

One immediately sees that in this case

$$f_1(x) = \operatorname{dist}_K(\mu(x)) .$$

As the distance function is known to be Lipschitz (cf. [3]) assumption (i) is fulfilled. Concerning assumption (iii), observe that

$$(3.6) \qquad A(x) = -\mu(x) + \operatorname{Proj}_K(\mu(x))$$

and the map A is indeed set-valued due to the nonconvexity of K. To show the existence of a selection α of A, continuous at $\hat{x}$, observe that for x from a neighbourhood O of $\hat{x}$ and $y \in A(x)$ one has

$$|y - A(\hat{x})|_m = |-\mu(x) + \operatorname{Proj}_K(\mu(x))|_m \leqq |-\mu(x) + \mu(\hat{x})|_m \leqq L|x - \hat{x}|_n ,$$

where L is the Lipschitz constant of μ on O. Thus A is upper Lipschitz on a neighbourhood of $\hat{x}$ which implies the validity of assumption (iii). □

In optimal control problems μ is often given implicitly, by means of a system equation

$$(3.7) \qquad G(x, y) = 0 ,$$

where $G[\mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^m]$ is a continuously differentiable operator and

$$(3.8) \qquad G(x, \mu(x)) = 0 \quad \text{for all} \quad x \in \mathbf{R}^n .$$

Suppose that μ is continuously differentiable on a neighbourhood of $\hat{x}$. Then on the basis of Prop 3.1 we may derive the optimality conditions for the finite-dimensional abstract control problem

$$(3.9) \qquad \begin{gathered} f(x) \to \inf \\ \text{subject to} \\ G(x, y) = 0 \\ x \in S \\ y \in K . \end{gathered}$$

Proposition 3.2. *Assume that the pair* $(\hat{x}, \hat{y})$ *is a local solution of problem* (3.9) *which satisfies all requirements of Prop.* 3.1 *with* μ *being the implicit function of* (3.7). *Then there exists a vector* $\hat{p} \in \mathbf{R}^m$ *such that the Lagrangian*

$$\mathscr{L}(x, y, p) = f(x) - \langle p, G(x, y) \rangle$$

satisfies the relations

$$(3.10) \qquad \begin{gathered} 0 \in \partial_x \mathscr{L}(\hat{x}, \hat{y}, \hat{p}) + N_s(\hat{x}) \\ \nabla_y \mathscr{L}(\hat{x}, \hat{y}, \hat{p}) \in C_K^*(\hat{y}) . \end{gathered}$$

Proof. Clearly, relation (3.2) holds with some $k^* \in C_K^*(\hat{y})$ and we have to express $\nabla\mu(\hat{x})$ in terms of G. Assume that $\hat{p}$ is a solution of an "adjoint" equation

$$(3.11) \qquad (\nabla_y G(\hat{x}, \hat{y}))^T \hat{p} + k^* = 0 .$$

Then for a vector $h \in \mathbf{R}^n$ one has

$$- \langle(\nabla\mu(\hat{x}))^T k^*, h\rangle = \langle(\nabla\mu(\hat{x}))^T (\nabla_y G(\hat{x}, \hat{y}))^T \hat{p}, h\rangle =$$
$$= \langle\hat{p}, \nabla_y G(\hat{x}, \hat{y}) \nabla\mu(\hat{x}) h\rangle = -\langle\hat{p}, \nabla_x G(\hat{x}, \hat{y}) h\rangle$$

because $\nabla_x G(\hat{x}, \hat{y}) + \nabla_y G(\hat{x}, \hat{y}) \nabla\mu(\hat{x}) = 0$. Hence,

$$(\nabla\mu(\hat{x}))^T k^* = (\nabla_x G(\hat{x}, \hat{y}))^T \hat{p}$$

so that (3.2) directly implies the first relation from (3.10). The second is merely a transcription of (3.11). □

Remark. In a dynamic context this second relation of (3.10) generates the appropriate adjoint inclusion.

CONCLUSION

It is true that in the most important applications of model (1.1) (like in the optimum design problems with variational inequalities) the map F is convex-valued. Then, of course, the optimality conditions (support principle), derived e.g. in [11] under less stringent requirements than those of Sect. 2, suffice. On the other hand it seems reasonable to investigate possible extensions of this principle for general maps F, and Sect 3 shows that we can find interesting applications also in this case, Moreover, the technique used in the proof of Prop. 2.1 may well be applied also to the numerical solution of (1.1) independently of whether F is convex- or nonconvex-valued. The idea relies on the combination of the exact penalization with some modern numerical method for nonsmooth optimization. In particular, such a method could be applied to the "augmented "problem

$$W(x) = f(x) + r f_1(x) \to \inf$$

subject to

$$x \in S .$$

This possibility will be investigated elsewhere.

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Souhrn

PODMÍNKY OPTIMALITY V JEDNÉ TŘÍDĚ OPTIMALIZAČNÍCH ÚLOH

JIŘÍ V. OUTRATA

V článku jsou odvozeny nutné podmínky optimality pro minimalizaci lokálně lipschitzovské kriteriální funkce na množině bodů vyhovujících omezením $x \in S$, $0 \in F(x)$, kde S je uzavřená množina a F je mnohoznačné zobrazení, u něhož se nepředpokládá konvexnost v žádném smyslu. Získané podmínky jsou aplikovány na zobecněnou úlohu matematického programování a abstraktní úlohu optimálního řízení konečné dimenze.

Резюме

НЕОБХОДИМЫЕ УСЛОВИЯ ОПТИМАЛЬНОСТИ ДЛЯ ОДНОГО КЛАССА ЭКСТРЕМАЛЬНЫХ ЗАДАЧ

JIŘÍ V. OUTRATA

Изучаются необходимые условия экстремума в задачах минимизации локально липшицевой целевой функции при наличии ограничений $x \in S$, $0 \in F(x)$, где S — замкнутое множество и F — многозначное отображение. Не предполагаются ни выпуклость ни выпуклозначность F. Условия используются в обобщенной задаче математического программирования и в абстрактной задаче оптимального управления конечной размерности.

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