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Correction to the paper “On the two-sided quality control”

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CORRECTION TO THE PAPER "ON THE TWO-SIDED QUALITY CONTROL"

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Summary. The correction consists of deriving correct explicit formulas for MLE of parameters μ, σ of the normal distribution under the hypothesis $\mu + c\sigma \leqq m + \delta$, $\mu - c\sigma \geqq m - \delta$.

Formulas (4)−(6) for computation of the maximum likelihood estimator T_n under the hypothesis H are in the paper "On the two-sided quality control" (Apl. Mat. 27 (1982), pp. 87−95) wrong, and their correct form is as follows.

(I) If $(\bar{x}, s)' \in H_\Delta$, then

$$M_n(x^{(n)}) = \bar{x} \quad D_n(x^{(n)}) = s \tag{4}$$

(II) Let $(\bar{x}, s)' \notin H_\Delta$. Let us denote

$$\hat{\sigma} = \frac{c_\Delta(\bar{x} - m - \delta)}{2} + [s^2 + (\bar{x} - m - \delta)^2 (1 + c_\Delta^2/4)]^{1/2}$$

and for $\bar{x} \geqq m$ put

$$M_n(x^{(n)}) = m + \delta - c_\Delta D_n(x^{(n)}) \quad D_n(x^{(n)}) = \begin{cases} \min\left\{\dfrac{\delta}{c_\Delta}, \hat{\sigma}\right\}, & \bar{x} + c_\Delta\hat{\sigma} \geqq m + \delta \\ \dfrac{m + \delta - \bar{x}}{c_\Delta}, & \bar{x} + c_\Delta\hat{\sigma} < m + \delta \end{cases} \tag{5}$$

If $\bar{x} < m$, we denote

$$\tilde{\sigma} = \frac{c_\Delta(m - \delta - \bar{x})}{2} + [s^2 + (m - \delta - \bar{x})^2 (1 + c_\Delta^2/4)]^{1/2}$$

and put

$$M_n(x^{(n)}) = m - \delta + c_\Delta D_n(x^{(n)}) \quad D_n(x^{(n)}) = \begin{cases} \min\left\{\dfrac{\delta}{c_\Delta}, \tilde{\sigma}\right\}, & \bar{x} - c_\Delta\tilde{\sigma} \leqq m - \delta \\ \dfrac{\bar{x} - m + \delta}{c_\Delta}, & \bar{x} - c_\Delta\tilde{\sigma} > m - \delta\,. \end{cases} \tag{6}$$

In this notation for $T_n = (M_n, D_n)'$ Theorem 1 of the paper is true. The convergence $T_n \to \theta = (\mu, \sigma)'$ holds whenever $\mu + c_\Delta \sigma \leqq m + \delta$, $\mu - c_\Delta \sigma \geqq m - \delta$ and since proof of (8) and (9) of the paper is connected with (4)–(6) by Theorem 2 only through consistency of the MLE, it is sufficient to prove, that for $s > 0$

(7) $$f_{T_n}^{(n)}(x^{(n)}) = L(x^{(n)}, H_\Delta) .$$

For this purpose we denote

$$\lambda(\mu, \sigma) = \log f_{(\mu,\sigma)'}^{(n)}(x^{(n)})$$

where log stands for logarithm to the base e. Formulas (10) of the paper yield

(11) $\lambda(\cdot, \sigma)$ is increasing on $(-\infty, \bar{x}\rangle$ and decreasing on $\langle \bar{x}, +\infty)$

(12) $\lambda(\bar{x}, \cdot)$ is increasing on $(0, s\rangle$ and decreasing on $\langle s, +\infty)$.

Let $(\bar{x}, s)' \notin H_\Delta$. Assume at first that

$$\bar{x} \geqq m .$$

If $(\mu, \sigma)' \in H_\Delta$, then $\sigma \in (0, \delta/c_\Delta\rangle$ and the inequalities $\mu \leqq m + \delta - c_\Delta \sigma < m + \delta$ imply, that for $\bar{x} \geqq m + \delta$

(13) $$\log L(x^{(n)}, H_\Delta) = \sup \left\{ \lambda(m + \delta - c_\Delta \sigma, \sigma) ; 0 < \sigma \leqq \frac{\delta}{c_\Delta} \right\} .$$

But

$$\lambda(m + \delta - c_\Delta \sigma, \sigma) = -\frac{n}{2} \log 2\pi - n \log \sigma - \frac{n}{2\sigma^2} [s^2 + (\bar{x} - m - \delta + c_\Delta \sigma)^2]$$

$$\frac{\partial \lambda(m + \delta - c_\Delta \sigma, \sigma)}{\partial \sigma} = \frac{n}{\sigma^3} \xi(\sigma) , \quad \xi(\sigma) = -\sigma^2 + \sigma c_\Delta(\bar{x} - m - \delta) + s^2 + $$
$$+ (\bar{x} - m - \delta)^2$$

The quadratic equation $\xi(\sigma) = 0$ has the only positive root $\hat{\sigma}$. Since it has also a negative root, ξ is positive on $(0, \hat{\sigma})$, negative on $(\hat{\sigma}, +\infty)$ and

(14) $g(\sigma) = \lambda(m + \delta - c_\Delta \sigma, \sigma)$ is increasing on $(0, \hat{\sigma}\rangle$ and decreasing on $\langle \hat{\sigma}, +\infty)$

for all values of $\bar{x}$. It is obvious from this and (13), that (7) is correct, if $\bar{x} \geqq m + \delta$. Let

$$m \leqq \bar{x} < m + \delta .$$

A straightforward application of (11) and (12) leads to

(15) $$\sup \{\lambda(\mu, \sigma); (\mu, \sigma)' \in H_\Delta, \mu \geqq \bar{x}\} = \sup \{\lambda(\bar{x}, \sigma); (\bar{x}, \sigma)' \in H_\Delta\} =$$
$$= \sup \left\{ \lambda(\bar{x}, \sigma); 0 < \sigma \leqq \frac{m + \delta - \bar{x}}{c_\Delta} \right\} = \lambda\left(\bar{x}, \frac{m + \delta - \bar{x}}{c_\Delta}\right)$$

because $(\bar{x}, s)' \notin H_\Delta$ and

$$s > \frac{m + \delta - \bar{x}}{c_\Delta} .$$

Let $(\mu, \sigma)' \in H_\Delta$ and $\mu \leqq \bar{x}$. If $(\sigma \leqq (m + \delta - \bar{x})/c_\Delta)$, then $\lambda(\mu, \sigma) \leqq \lambda(\bar{x}, \sigma) \leqq \leqq \lambda(\bar{x}, (m + \delta - \bar{x})/c_\Delta)$. If $(m + \delta - \bar{x})/c_\Delta \leqq \sigma \leqq \delta/c_\Delta$, then $\mu \leqq m + \delta - c_\Delta\sigma \leqq \leqq \bar{x}$ and therefore $\lambda(\mu, \sigma) \leqq \lambda(m + \delta - c_\Delta\sigma, \sigma)$. Hence

$$\sup \{\lambda(\mu, \sigma); (\mu, \sigma)' \in H_\Delta, \mu \leqq \bar{x}\} =$$

$$= \sup \left\{\lambda(m + \delta - c_\Delta\sigma, \sigma); \frac{m + \delta - \bar{x}}{c_\Delta} \leqq \sigma \leqq \frac{\delta}{c_\Delta}\right\}.$$

Combining this with (15) and (14) we obtain that (7) holds if $\bar{x} \geqq m$. Since the rest of the proof can be performed similarly, we present here a brief sketch only.

If $\bar{x} \leqq m - \delta$, then

$$\log L(x^{(n)}, H_\Delta) = \sup \left\{\lambda(m - \delta + c_\Delta\,\sigma, \sigma); 0 < \sigma \leqq \frac{\delta}{c_\Delta}\right\}$$

where

$$\frac{\partial\lambda(m - \delta + c_\Delta\sigma, \sigma)}{\partial\sigma} = \frac{n}{\sigma^3}\,\eta(\sigma), \eta(\sigma) = -\sigma^2 + \sigma c_\Delta(m - \delta - \bar{x}) + s^2 + + (m - \delta - \bar{x})^2$$

The equation $\eta(\sigma) = 0$ has the only positive root $\tilde{\sigma}$ and

(16) $h(\sigma) = \lambda(m - \delta + c_\Delta\sigma, \sigma)$ is increasing on $(0, \tilde{\sigma}\rangle$ and decreasing on $\langle\tilde{\sigma}, +\infty)$

for all values of $\bar{x}$. This means, that (7) holds, if $\bar{x} \leqq m - \delta$. If $m - \delta < \bar{x} < m$, then

$$\log L(x^{(n)}, H_\Delta) = \sup \left\{\lambda(m - \delta + c_\Delta\sigma, \sigma); \frac{\bar{x} - m + \delta}{c_\Delta} \leqq \sigma \leqq \frac{\delta}{c_\Delta}\right\}$$

which together with (16) and (6) yields (7).

Súhrn

OPRAVA ČLÁNKU „O DVOJSTRANNEJ KONTROLE KVALITY“

FRANTIŠEK RUBLÍK

V oprave si správne odvodené explicitné výrazy pre odhad maximálnej vierohodnosti parametrov μ, σ normálneho rozdelenia za predpokladu platnosti hypotézy $\mu + c\sigma \leqq m + \delta$, $\mu - c\sigma \geqq m - \delta$.

Резюме

ИСПРАВЛЕНИЕ СТАТЬИ „О ДВУСТОРОННЕМ КОНТРОЛЕ КАЧЕСТВА“

FRANTIŠEK RUBLÍK

В исправлении правильно выведены явные выражения для оценки максимального правдоподобия параметров μ, σ нормального распределения при предположении, что имеет место гипотеза

$$\mu + c\sigma \leqq m + \delta, \mu - c\sigma \geqq m - \delta.$$

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