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SOME PROBLEMS OF EXPONENTIAL SMOOTHING

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Snmmary. The paper deals with some practical problems connected with the classical exponential smoothing in time series. The fundamental theorem of the exponential smoothing is extended to the case with missing observations and an interpolation procedure in the framework of the exponential smoothing is described. A simple method of the exponential smoothing for multivariate time series is suggested.

Keywords: Time series, exponential smoothing, missing observations, interpolation.

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1. INTRODUCTION

The classical exponential smoothing (see [1]) belongs to popular smoothing and extrapolation methods. Due to its simplicity it is still used frequently in practical time series analysis although more effective, but more complicated methods have been developed.

This paper concentrates on some aspects which are connected with the practical applications of the exponential smoothing. First, the case with missing observations which is usual in practice is investigated. It is shown in Section 2 that the classical exponential smoothing can be extended in a natural way to this case so that the fundamental theorem of the classical exponential smoothing (see [2], [4]) stays valid.

Further, a procedure of interpolation in time series based on the exponential smoothing is suggested in Section 3. This procedure combines the "forward" and "backward" exponential smoothing so that the optimality in the sense of the fundamental theorem is again guaranteed. Finally, Section 4 contains a note on the exponential smoothing for multivariate time series. Although the procedure of the adaptive exponential smoothing in the multivariate case by Enns et al. [5] seems to be effective, its initiation based on maximum likelihood estimation can be complicated for routine applications (this procedure takes advantage of Kalman filtering and is described only for the situation corresponding to the model of the simple exponential smoothing in the non-adaptive case). Therefore a natural multivariate extension of the classical exponential smoothing is suggested which is numerically simple.

2. MISSING OBSERVATIONS

The mathematical method of the classical exponential smoothing of order n for the time series model

$$y_{t+\tau} = \sum_{k=0}^{n} \frac{a_k(t)}{k!} \tau^k + \varepsilon_{t+\tau} \tag{2.1}$$

with integers t and τ ($\mathsf{E}\varepsilon_t = 0$, $\operatorname{var} \varepsilon_t = \sigma^2 > 0$, $\mathsf{E}\varepsilon_s\varepsilon_t = 0$ for $s \neq t$) is based on minimization of the discounted least squares

$$\sum_{j=0}^{\infty} (1 - \alpha)^j \left\{ y_{t-j} - \sum_{k=0}^{n} \frac{a_k(t)}{k!} (-j)^k \right\}^2 , \tag{2.2}$$

where $0 < \alpha < 1$ is a smoothing constant. Let the observations $y_{u+1}, y_{u+2}, \dots$ $\dots, y_{v-2}, y_{v-1}$ $(u < v)$ be missing so that only the observations $\dots, y_{u-1}, y_u, y_v,$ $y_{v+1}, \dots$ can be gradually delivered (for $v = u + 1$ we obviously have the standard case without missing observations). Now the natural modification of the above minimization consists in excluding the summands with missing observations so that the minimized expression is

$$\sum_{\substack{j=0 \\ j \neq t-v+1, \dots, t-u-1}}^{\infty} (1 - \alpha)^j \left\{ y_{t-j} - \sum_{k=0}^{n} \frac{a_k(t)}{k!} (-j)^k \right\}^2 . \tag{2.3}$$

For $u < t < v$ the sum in (2.3) is over $j = t - u, t - u + 1, \dots$ and for $t \leqq u$ the sum is over $j = 0, 1, \dots$. In this section the bare symbol Σ will always denote the sum used in (2.3) in the sense just described.

Let us construct smoothing statistics $S_t^{[p]}$ of order p recursively as

$$\begin{aligned} S_t^{[1]} &= \alpha y_t + (1 - \alpha) S_{t-1}^{[1]} , \quad t = \dots, u - 1, u, v, v + 1, \dots ; \\ &= (1 - \alpha) S_{t-1}^{[1]} , \quad t = u + 1, u + 2, \dots, v - 1 ; \\ S_t^{[p]} &= \alpha S_t^{[p-1]} + (1 - \alpha) S_{t-1}^{[p]} , \\ t &= \dots, u - 1, u, u + 1, \dots, v - 1, v, v + 1, \dots , \quad p = 2, \dots, n + 1 . \end{aligned} \tag{2.4}$$

If one uses the recursive formulas (2.4) for practical computations one must choose suitable initial values for the smoothing statistics (see e.g. [3]). One can see that the only difference from the case without missing observations concerns the values $S_{u+1}^{[1]}, \dots, S_{v-1}^{[1]}$ which are calculated as if the missing observations were replaced by zero values. Let us denote $\mathbf{S}_t = (S_t^{[1]}, S_t^{[2]}, \dots, S_t^{[n+1]})'$ and $\boldsymbol{a}(t) = (a_0(t), a_1(t), \dots$ $\dots, a_n(t))'$.

Theorem 1. *The vector* $\boldsymbol{a}(t)$ *minimizing* (2.3) *is determined by the system of equations*

$$\mathbf{M}\,\boldsymbol{a}(t) = \mathbf{S}_t\,, \tag{2.5}$$

where the elements of the matrix $\mathbf{M}$ *have the form*

$$m_{pk} = \frac{(-1)^k}{k!}\,\alpha^p\,\Sigma(1-\alpha)^j \binom{p-1+j}{j} j^k\,, \tag{2.6}$$

$$p = 1, \ldots, n+1\,, \quad k = 0, \ldots, n\,.$$

Proof. The normal equations corresponding to the minimization of (2.3) have the form

$$\sum_{k=0}^{n} \frac{a_k(t)}{k!}\,\Sigma(1-\alpha)^j j^{p-1}(-j)^k = \Sigma(1-\alpha)^j j^{p-1} y_{t-j}\,, \quad p = 1, \ldots, n+1\,. \tag{2.7}$$

Further we shall show by induction that $S_t^{[p]}$ from (2.4) can be written as

$$S_t^{[p]} = \alpha^p\,\Sigma(1-\alpha)^j \binom{p-1+j}{j} y_{t-j}\,, \quad p = 1, \ldots, n+1\,. \tag{2.8}$$

For $p = 1$, (2.8) is obvious. Let (2.8) hold for p. Then using the induction assumption we can write for $p + 1$

$$S_t^{[p+1]} = \alpha S_t^{[p]} + (1-\alpha)\,S_{t-1}^{[p+1]} = \alpha \sum_{j=0}^{\infty} (1-\alpha)^j\,S_{t-j}^{[p]} =$$

$$= \alpha \sum_{j=0}^{\infty} (1-\alpha)^j\,\alpha^p \sum_{\substack{i=0\\ i\neq t-j-v+1,\ldots,t-j-u-1}}^{\infty} (1-\alpha)^i \binom{p-1+i}{i} y_{t-j-i} =$$

$$= \alpha^{p+1} \sum_{\substack{r=0\\ r\neq t-v+1,\ldots,t-u-1}}^{\infty} (1-\alpha)^r\,y_{t-r} \sum_{s=0}^{r} \binom{p-1+s}{s}\,.$$

Since

$$\sum_{s=0}^{r} \binom{p-1+s}{s} = \binom{p+r}{r}$$

we obtain the required relation

$$S_t^{[p+1]} = \alpha^{p+1}\,\Sigma(1-\alpha)^j \binom{p+j}{j} y_{t-j}\,.$$

The proof is completed by noticing that the system of equations (2.5) is equivalent to the system (2.7).

The formula

$$\boldsymbol{a}(t) = \mathbf{M}^{-1}\mathbf{S}_t \tag{2.9}$$

can be used in the same way as for the exponential smoothing without missing observations. As the calculation of the elements of the matrix $\mathbf{M}$ for a chosen smoothing constant α is concerned the problem can be reduced to the calculation of sums of the type $\Sigma(1 - \alpha)^j j^k$ $(k = 0, 1, \ldots, n)$ with missing summands. One can use formulas for these sums without missing summands given e.g. in [1, p. 135] for $k = 0, 1, \ldots, 6$ and then subtract the values of the missing summands or calculate these sums directly using a computer till a prescribed precision is achieved.

For $n = 0$ (the simple exponential smoothing) the following explicit formulas can be derived (let us denote $c = t - v + 1$, $d = t - u$ for simplicity)

$$\begin{aligned} \hat{a}_0(t) &= \{1 - (1 - \alpha)^c + (1 - \alpha)^d\}^{-1} S_t^{[1]}, & t &\geqq v; \\ &= (1 - \alpha)^{-d} S_t^{[1]}, & u &< t < v; \\ &= S_t^{[1]}, & t &\leqq u. \end{aligned}$$

For $n = 1$ (the double exponential smoothing) the matrix $\boldsymbol{M}$ in (2.9) has the explicit form

$$\boldsymbol{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

where

$$A = 1 - (1 - \alpha)^c + (1 - \alpha)^d,$$

$$\begin{aligned} B = {} & -(1/\alpha)\left[1 - \alpha + \{(1 - \alpha)(c - 1) - c\}(1 - \alpha)^c - \right. \\ & \left. - \{(1 - \alpha)(d - 1) - d\}(1 - \alpha)^d\right], \end{aligned}$$

$$C = 1 + \{(1 - \alpha)c - (c + 1)\}(1 - \alpha)^c - \{(1 - \alpha)d - (d + 1)\}(1 - \alpha)^d,$$

$$\begin{aligned} D = {} & \{-(1 - \alpha)/\alpha\}\left[2 + \{-(c - 1)c(1 - \alpha)^2 + \right. \\ & + 2(c - 1)(c + 1)(1 - \alpha) - c(c + 1)\}(1 - \alpha)^{c-1} - \\ & - \{-(d - 1)d(1 - \alpha)^2 + 2(d - 1)(d + 1)(1 - \alpha) - \\ & \left. - d(d + 1)\}(1 - \alpha)^{d-1}\right], \quad t \geqq v; \end{aligned}$$

$$A = (1 - \alpha)^d, \quad B = (1/\alpha)\{(1 - \alpha)(d - 1) - d\}(1 - \alpha)^d,$$

$$C = -\{(1 - \alpha)d - (d + 1)\}(1 - \alpha)^d,$$

$$\begin{aligned} D = {} & (1/\alpha)\{-(d - 1)d(1 - \alpha)^2 + 2(d - 1)(d + 1)(1 - \alpha) - \\ & - d(d + 1)\}(1 - \alpha)^d, \quad u < t < v; \end{aligned}$$

$$A = C = 1, \quad B = -(1 - \alpha)/\alpha, \quad D = -2(1 - \alpha)/\alpha, \quad t \leqq u.$$

Remark 1. If the values of $c = t - v + 1$ and $d = t - u$ are large one can go back to the formulas of the exponential smoothing without missing observations. Then e.g. in the simple exponential smoothing with $\alpha = 0{\cdot}2$ and $c = 9$, $d = 10$ one replaces the formula $\hat{a}_0(t) = 1{\cdot}0276\, S_t^{[1]}$ by $\hat{a}_0(t) = S_t^{[1]}$.

Remark 2. The above procedure can be generalized to the case when more groups of observations are missing. For example if observations $y_{u_1+1}, y_{u_1+2}, \ldots, y_{v_1-1}, y_{u_2+1}, y_{u_2+2}, \ldots, y_{v_2-1}$ $(u_1 < v_1 \leqq u_2 < v_2)$ are missing then one must use

$$S_t^{[1]} = (1 - \alpha)\, S_{t-1}^{[1]}\,,$$

$$t = u_1 + 1, u_1 + 2, \ldots, v_1 - 1, u_2 + 1, u_2 + 2, \ldots, v_2 - 1$$

in (2.4) and omit all summands with missing observations in (2.6).

3. INTERPOLATION

The interpolation procedure described in this section is very similar to the procedure of exponential smoothing for time series with missing observations from Section 2. The particular observations used for the construction of the interpolated value are discounted according to their time distances from this value.

Assume a value y_s in a group of missing observations $y_{u+1}, y_{u+2}, \ldots, y_{v-1}$ $(u + 1 \leqq s \leqq v - 1)$ is to be interpolated using the known observations $\ldots, y_{u-1}, y_u, y_v, y_{v+1}, \ldots$. Let us suppose that the number of known observations is sufficiently large in both directions (forward and backward) from y_s. Then it is natural for the time series model (2.1) to construct the interpolation $\hat{y}_s$ as the first component a_0 of the vector $\boldsymbol{a} = (a_0, a_1, \ldots, a_n)'$ minimizing the expression

$$\sum_{\substack{j=-\infty \\ j \neq -v+s+1,\ldots,s-u-1}}^{\infty} (1 - \alpha)^{|j|} \left\{ y_{s-j} - \sum_{k=0}^{n} \frac{a_k(s)}{k!} (-j)^k \right\}^2 . \tag{3.1}$$

In this section let the bare symbol Σ denote the sum used in (3.1). In addition to the smoothing statistics $S_t^{[p]}$ (see (2.4)) we shall also use "backward" smoothing statistics $T_t^{[p]}$ defined as

(3.2)

$$\begin{aligned} T_t^{[1]} &= \alpha y_t + (1 - \alpha)\, T_{t+1}^{[1]}\,, && t = \ldots, v + 1, v, u, u - 1, \ldots\,; \\ &= (1 - \alpha)\, T_{t+1}^{[1]}\,, && t = v - 1, v - 2, \ldots, u + 1\,; \\ T_t^{[p]} &= \alpha T_t^{[p-1]} + (1 - \alpha)\, T_{t+1}^{[p]}\,, && t = \ldots, v + 1, v, v - 1, \ldots, u + 1, u, u - 1, \ldots, \\ & && p = 2, \ldots, n + 1\,. \end{aligned}$$

The statistics $T_t^{[p]}$ are constructed recursively in the opposite time direction than $S_t^{[p]}$ and their initial values can be constructed analogously as for $S_t^{[p]}$ (e.g. using regression estimates based on several last observations of the time series). Let us denote $\boldsymbol{T}_t = (T_t^{[1]}, T_t^{[2]}, \ldots, T_t^{[n+1]})'$.

For given n and α let $\boldsymbol{J}$ be the matrix of the type $(n + 1) \times (n + 1)$ fulfilling

$$(3.3)\qquad \begin{Vmatrix} \alpha \\ \frac{\alpha^2}{1!}(1 - j) \\ \frac{\alpha^3}{2!}(2 - j)(1 - j) \\ \vdots \\ \frac{\alpha^{n+1}}{n!}(n - j)\dots(1 - j) \end{Vmatrix} = \boldsymbol{J} \begin{Vmatrix} \alpha \\ \frac{\alpha^2}{1!}(1 + j) \\ \frac{\alpha^3}{2!}(2 + j)(1 + j) \\ \vdots \\ \frac{\alpha^{n+1}}{n!}(n + j)\dots(1 + j) \end{Vmatrix}$$

for all $j = \dots, -1, 0, 1, \dots$. The matrix $\boldsymbol{J}$ is uniquely determined by (3.3). For example, for $n = 3$ it has the form

$$(3.4)\qquad \boldsymbol{J} = \begin{pmatrix} 1, & 0, & 0, & 0 \\ 2\alpha, & -1, & 0, & 0 \\ 3\alpha^2, & -3\alpha, & 1, & 0 \\ 4\alpha^3, & -6\alpha^2, & 4\alpha, & -1 \end{pmatrix}.$$

Theorem 2. *The vector* $\boldsymbol{a}$ *minimizing* (3.1) *is determined by the system of equations*

$$(3.5)\qquad \boldsymbol{M}\boldsymbol{a} = \boldsymbol{S}_s + \boldsymbol{J}\boldsymbol{T}_s\,,$$

where the elements of the matrix $\boldsymbol{M}$ *have the form*

$$(3.6)\qquad m_{pk} = \frac{(-1)^k}{k!}\,\frac{\alpha^p}{(p - 1)!}\,\Sigma(1 - \alpha)^{|j|}\,(p - 1 + j)\dots(1 + j)\,j^k\,,$$
$$p = 1, \dots, n + 1\,, \quad k = 0, \dots, n\,.$$

Proof is similar to that of Theorem 1 since the normal equations corresponding to the minimization of (3.1) have the form

$$(3.7)\qquad \sum_{k=0}^{n} \frac{a_k(s)}{k!}\,\Sigma(1 - \alpha)^{|j|}\,j^{p-1}\,(-j)^k = \Sigma(1 - \alpha)^{|j|}\,j^{p-1}\,y_{s-j}\,,$$
$$p = 1, \dots, n + 1\,,$$

and it is possible to show by induction that the p-th component of the vector $\boldsymbol{S}_s + \boldsymbol{J}\boldsymbol{T}_s$ can be expressed as

$$(3.8)\qquad \frac{\alpha^p}{(p - 1)!}\,\Sigma(1 - \alpha)^{|j|}\,(p - 1 + j)\dots(1 + j)\,y_{s-j}\,.$$

For $n = 0$ the following explicit formula holds (let us denote for simplicity $f = s - u$, $g = v - s$):

$$\hat{y}_s = \hat{a}_0 = \{(1-\alpha)^f + (1-\alpha)^g\}^{-1} (S_s^{[1]} + T_s^{[1]}) .$$

For $n = 1$ the interpolation $\hat{y}_s = \hat{a}_0$ can be calculated as

$$\begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \begin{pmatrix} S_s^{[1]} + T_s^{[1]} \\ S_s^{[2]} + 2\alpha T_s^{[1]} - T_s^{[2]} \end{pmatrix},$$

where

$$A = (1-\alpha)^f + (1-\alpha)^g ,$$

$$B = (1/\alpha) [(1-\alpha)^g \{g - (1-\alpha)(g-1)\} - (1-\alpha)^f \{f - (1-\alpha)(f-1\}] ,$$

$$C = (1-\alpha)^f \{(f+1) - (1-\alpha) f\} - (1-\alpha)^g \{g - (1-\alpha)(g-1) - \alpha\} ,$$

$$D = (1/\alpha) [\{-(f-1) f(1-\alpha)^2 + 2(f-1)(f+1)(1-\alpha) - \\ - f(f+1)\} (1-\alpha)^f + \{-(g-2)(g-1)(1-\alpha)^2 + \\ + 2(g-2) g(1-\alpha) - (g-1) g\} (1-\alpha)^g] .$$

Remark 3. If the time distance of the last known observation y_t from the interpolated value y_s is not large one can proceed in the following way improving the interpolation gradually with each new observation y_t in the time series $\ldots, y_{u-1}, y_u, y_v, y_{v+1}, \ldots, y_t$ $(u < s < v \leqq t)$. Let us construct the statistics $U_t^{[p]}$ recursively as

$$U_v^{[p]} = \frac{\alpha^p}{(p-1)!} (1-\alpha)^g (p-1-g) \ldots (1-g) \, y_v , \tag{3.9}$$

$$U_{t+1}^{[p]} = U_t^{[p]} + \frac{\alpha^p}{(p-1)!} (1-\alpha)^{t+1-s} (p-2+s-t) \ldots (s-t) \, y_{t+1} ,$$

$$t = v, v+1, \ldots, p = 1, \ldots, n+1 ,$$

where $f = s - u$, $g = v - s$. Then the interpolation $\hat{y}_s(t)$ based on the known observations $\ldots, y_{u-1}, y_u, y_v, y_{v+1}, \ldots, y_t$ can be constructed as the first component $\hat{a}_0(t)$ of the vector $\boldsymbol{a}^\wedge(t) = (\hat{a}_0(t), \hat{a}_1(t), \ldots, \hat{a}_n(t))'$ given by the formula

$$\boldsymbol{a}^\wedge(t) = \boldsymbol{M}(t)^{-1} (\boldsymbol{S}_s + \boldsymbol{U}_t) , \tag{3.10}$$

where $\boldsymbol{U}_t = (U_t^{[1]}, U_t^{[2]}, \ldots, U_t^{[n+1]})'$ and the elements of the matrix $\boldsymbol{M}(t)$ have the form

$$m_{pk}(t) = \frac{\alpha^p}{k!\,(p-1)!} \{ \sum_{j=f}^{\infty} (1-\alpha)^j (p-1+j) \ldots (1+j)(-j)^k + \\ + \sum_{j=g}^{t-s} (1-\alpha)^j (p-1-j) \ldots (1-j) j^k \} , \quad p = 1, \ldots, n+1 , \quad k = 0, \ldots, n . \tag{3.11}$$

For example, for $n = 0$ we obtain

$$\hat{y}_s(t) = \hat{a}_0(t) = \{(1-\alpha)^f + (1-\alpha)^g - (1-\alpha)^{t-s+1}\}^{-1} (\boldsymbol{S}_s^{[1]} + \boldsymbol{U}_t^{[1]}) .$$

Let $\mathbf{y}_t = (y_{1t}, \ldots, y_{dt})'$ be a d-dimensional time series with the model

$$\mathbf{y}_t = \mathbf{f}(t) + \boldsymbol{\varepsilon}_t\,, \tag{4.1}$$

lwhere the i-th component $f_i(t)$ of the vector $\mathbf{f}(t) = (f_1(t), \ldots, f_d(t))'$ is a polynomia of order n_i and $\mathsf{E}\boldsymbol{\varepsilon}_t = 0$, $\operatorname{var} \boldsymbol{\varepsilon}_t = \boldsymbol{\Sigma} > 0$, $\mathsf{E}\boldsymbol{\varepsilon}_s\boldsymbol{\varepsilon}_t' = 0$ for $s \neq t$.

Since the exponential smoothing if performed separately for particular components of $\mathbf{y}_t$ does not take into account the correlations which may exist among these components, a simple method respecting this fact is looked for. The feature which is substantial for the following procedure and which can unpleasantly contradict a real situation consists in the assumption that the correlation structure given by $\boldsymbol{\Sigma}$ does not change in time.

Let $\mathbf{y}_1, \ldots, \mathbf{y}_N$ be observations which can be used for the initiation of the procedure (such initial observations are used also in the method of adaptive exponential smoothing by Enns et al. [5] for the construction of initial ML estimates). Then one can estimate the elements σ_{ij} of the matrix $\boldsymbol{\Sigma}$ consistently using the theory of the seemingly unrelated regressions (see e.g. [6, p. 160]) as

$$s_{ij} = \mathbf{e}_i^{\wedge\prime}\mathbf{e}_j^{\wedge}/N\,, \quad i, j = 1, \ldots, d\,, \tag{4.2}$$

where $\mathbf{e}_i^{\wedge} = (\hat{e}_{i1}, \ldots, \hat{e}_{iN})'$ is the vector of the OLS residuals from the regression of $(y_{i1}, \ldots, y_{iN})'$ on $(f_i(1), \ldots, f_i(N))'$. The initial estimates of the parameters of $\mathbf{f}(t)$ can be obtained in the second stage by applying the OLS method to the particular components of the transformed observations

$$\mathbf{z}_t = \mathbf{S}^{-1/2}\mathbf{y}_t \tag{4.3}$$

for $t = 1, \ldots, N$ (the elements s_{ij} of $\mathbf{S}$ are given in (4.2)).

The components of $\mathbf{z}_t$ can be taken as uncorrelated and the previous estimates of the parameters of $\mathbf{f}(t)$ are fully efficient (see e.g. [6, Theorem 8.4.2]). This fact motivates the suggestion to use the classical univariate exponential procedure (including the choice of the order of the smoothing and the choice of the smoothing constant) for particular components of the transformed time series $\mathbf{z}_t$ constructed according to (4.3) for all t. Moreover, it is possible to take advantage of the previous initial estimate of $\mathbf{f}(t)$ for starting the procedure. Finally, one must revert from the smoothed values $\mathbf{z}_t^{\wedge}$ of the transformed time series to the smoothed values $\mathbf{y}_t^{\wedge}$ of the original time series according to the formula

$$\mathbf{y}_t^{\wedge} = \mathbf{S}^{1/2}\mathbf{z}_t^{\wedge}\,. \tag{4.4}$$

In particular, if for univariate time series $\{z_{1t}\}, \ldots, \{z_{dt}\}$ it is possible to use the simple exponential smoothing with smoothing constants $\alpha_1, \ldots, \alpha_d$, respectively, then the following direct formula

$$y_t^{\wedge} = A^* y_t + (I - A^*) y_{t-1}^{\wedge} \tag{4.5}$$

holds, where

$$A^* = S^{1/2} A S^{-1/2} \tag{4.6}$$

and A is the diagonal matrix with $\alpha_1, \ldots, \alpha_d$ on the main diagonal.

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Souhrn

NĚKTERÉ PROBLÉMY EXPONENCIÁLNÍHO VYROVNÁVÁNÍ

TOMÁŠ CIPRA

Článek se zabývá některými praktickými problémy spojenými s klasickým exponenciálním vyrovnáváním v časových řadách. Základní věta exponenciálního vyrovnávání je rozšířena na případ s chybějícími pozorováními a v rámci exponenciálního vyrovnávání je popsána interpolační procedura. Je navržena jednoduchá metoda exponenciálního vyrovnávání pro mnohorozměrné časové řady.

Резюме

НЕКОТОРЫЕ ПРОБЛЕМЫ ЭКСПОНЕНЦИАЛЬНОГО СГЛАЖИВАНИЯ

TOMÁŠ CIPRA

Работа занимается некоторыми практическими проблемами, касающимися классического экспоненциального сглаживания во временных рядах. Основная теорема экспоненциального сглаживания обобщена на случай с отсутствующими наблюдениями и описана также интерполяционная процедура в рамках экспоненциального сглаживания. Предложен простой метод экспоненциального сглаживания для многомерных временных рядов.

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