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DYNAMIC VON KÁRMÁN EQUATIONS INVOLVING NONLINEAR DAMPING: TIME-PERIODIC SOLUTIONS

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Summary. In the present paper, time-periodic solutions to dynamic von Kármán equations are investigated. Assuming that there is a damping term in the equations we are able to show the existence of at least one solution to the problem. The Faedo-Galerkin method is used together with some basic ideas concerning monotone operators on Orlicz spaces.

Keywords: Dynamic von Kármán equations, time-periodic solution, nonlinear damping.
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When dealing with damped transversal vibrations of a thin plate occupying a bounded domain $\Omega \subset \mathbb{R}^2$ with a regular clamped boundary $\partial\Omega$, we are led to the problem

$$(E_1) \qquad u'' + \beta(u') + a_1 \Delta^2 u - [u, \Phi] = f \quad \text{on} \quad \Omega \times \mathbb{R}^1 ,$$

$$(E_2) \qquad a_2 \Delta^2 \Phi + [u, u] = 0 \quad \text{on} \quad \Omega \times \mathbb{R}^1 ,$$

$$(B) \qquad u = \frac{\partial u}{\partial \nu} = \Phi = \frac{\partial \Phi}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega \times \mathbb{R}^1$$

where $a_1, a_2 > 0$, $u' = \partial u/\partial t$, ν is the outward normal to $\partial\Omega$, and

$$[u, v] \stackrel{\text{def}}{=} \frac{\partial^2 u}{\partial x^2} \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} \frac{\partial^2 v}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} \frac{\partial^2 v}{\partial x \partial y} .$$

Here the unknown functions u, Φ of $(x, y, t) \in \Omega \times \mathbb{R}^1$ are interpreted as the transversal displacement and the Airy-stress function, respectively.

Our aim is to establish the existence of at least one time-periodic solution to the above problem with the period $\omega > 0$, i.e.

$$(P) \qquad u(x, y, t + \omega) = u(x, y, t) , \quad (x, y, t) \in \Omega \times \mathbb{R}^1$$

provided that (of course) the right-hand side f satisfies (P) as well (see Theorem 1 in Section 1). To this end, the function β is supposed to satisfy some "reasonable" conditions among which monotonicity plays an essential part.

First of all, let us remark shortly on some works related to the subject. If β is linear in u' (or $\beta \equiv 0$), we refer to Lions [4], Morozov [6], von Wahl [13], Stahel [11] for the corresponding initial-boundary value problem and to Morozov [6], Vejvoda et al. [12] (see also the literature listed in this book) if the time-periodic solutions are concerned.

Involving the nonlinear damping term $\beta(u')$, the problem resembles an analogous task for the telegraph equation (see e.g. Prodi [8], Prouse [9], Haraux [2], Nakao [7]). Unfortunately, there seem to be obstacles to exploiting these methods in a direct fashion.

In this paper, we make use of the classical method of Faedo-Galerkin in order to obtain a sequence of approximate solutions (Section 2). In Section 4 we will carry out the corresponding limit passage taking advantage of a priori estimates derived in Section 3 and some basic ideas connected with monotonicity. Finally, to remove the inherent difficulty of this method – the lack of continuity of the solution in a corresponding energy space, the regularization technique is employed (see Section 5) due to Lions-Magenes [5].

Seeing that no essential differences occur in comparison with the general case, we confine our attention to the situation $a_1 = a_2 = 1$, $\omega = 2\pi$.

From now on all strictly positive constants will be denoted as c_i or $c_i(L)$, where the latter symbol should emphasize the dependence of c_i on the quantity L only.

1. FORMULATION OF THE MAIN RESULT

To begin with, let us mention the function spaces which will be useful throughout the later discussion. Denote by Γ the unit sphere in $\mathbb{R}^2$ identified with $[0, 2\pi]/\{0, 2\pi\}$ in a standard way. Further set $Q = \Omega \times \Gamma$.

The symbols $L_q(K)$, $1 \leqq q \leqq +\infty$ are reserved for the ordinary Lebesgue spaces of integrable functions on the sets $K = \Omega, Q$ with the norms $| \ |_q$, $[\![\ ]\!]_q$, respectively.

More generally, the Orlicz spaces $L_G(K)$ are considered where G is a convex, coercive function. When we set

$$G^*(v^*) \stackrel{\text{def}}{=} \sup \{v^* v - G(v) \mid v \in \mathbb{R}^1\}$$

– the Legendre-Fenchel transform of G, the space $L_G(K)$ coincides with the dual space to $E_{G^*}(K)$, the Banach space $E_{G^*}(K)$ being determined as the closure of all bounded functions in $L_{G^*}(K)$. We will refer systematically to [3] concerning this subject.

Next we make use of the Sobolev space $H_0^2(\Omega)$ obtained as the completion of the set of all functions being both smooth on $\bar{\Omega}$ and satisfying (B), with respect to the norm $\|v\| = |\Delta v|_2$. Now we can (and will) identify $H_0^2(\Omega)$ with a subspace of its dual $H^{-2}(\Omega)$ via the relation

$$H_0^2(\Omega) \subsetneq L_2(\Omega) \subsetneq H^{-2}(\Omega)$$

where the duality pairing as well as the inner product on $L_2(\Omega)$ are denoted as $(\ , \ )$.

As remarked earlier we look for time-periodic solutions and so the spaces $L_q(\Gamma, B)$, $C(\Gamma, B)$ of periodic functions ranging in a Banach space B are of interest. The former is provided with the norm

$$|v|_{L_q(\Gamma,B)} = (\int_\Gamma |v(s)|_B^q \, ds)^{1/q},$$

the latter being equipped with

$$|v|_{C(\Gamma,B)} = \sup \{|v(s)|_B \mid s \in \Gamma\}.$$

We refer to [12] for precise definitions and basic properties of these spaces.

At this stage, we proceed to the definition of the weak solution of the problem given by (E_1), (E_2), (B), (P). A pair of functions u, Φ is called a *weak solution* if $u, \Phi \in C(\Gamma, H_0^2(\Omega))$, $u' \in C(\Gamma, L_2(\Omega))$, $\beta(u') \in L_1(Q)$, and

$$\begin{aligned}(1.1) \qquad & \int_\Gamma - (u'(s), \varphi'(s)) + (\beta(u'(s)), \varphi(s)) + (\Delta u(s), \Delta\varphi(s)) - \\ & - ([u(s), \Phi(s)], \varphi(s)) \, ds = \int_\Gamma (f(s), \varphi(s)) \, ds,\end{aligned}$$

$$(1.2) \qquad (\Delta\Phi(t), \Delta\varphi(t)) + ([u(t), u(t)], \varphi(t)) = 0 \quad t \in \Gamma$$

for all $\varphi \in L_2(\Gamma, H_0^2(\Omega))$, $\varphi' \in L_2(\Gamma, L_2(\Omega))$.

We are about to formulate the existence theorem whose proof forms the bulk of the paper.

Theorem 1. *Let $\beta \in C(\mathbb{R}^1)$ be an odd increasing function satisfying*

$$(1.3) \qquad \frac{1}{p} v\,\beta(v) - B(v) > 0 \quad \text{for all} \quad v, |v| \geqq v_0$$

where $p > 2$ is a fixed number and

$$B(v) \overset{\text{def}}{=} \int_0^v \beta(s) \, ds.$$

Then there exists at least one weak solution to (E_1), (E_2), (B), (P) *whenever* $f \in L_2(\Gamma, L_2(\Omega))$.

Remark. The result claimed above is by no means the best possible. For instance, β is supposed to be odd so that the Orlicz space theory might be used directly without any modification. Actually, the oddness is not really necessary.

Let us pause to list some observations related to the function β. The condition (1.3) can be traced back to the paper [10] of Rabinowitz, who also showed that

$$(1.4) \qquad B(v) \geqq c_1 |v|^p - c_2, \quad v \in \mathbb{R}^1$$

provided (1.3) holds.

Passing to the conjugate function B^*, we get

$$B^*(\beta(v)) - \left(1 - \frac{1}{p}\right) \beta^{-1}(\beta(v))\,\beta(v) = \frac{1}{p} v\,\beta(v) - B(v).$$

Setting $v^* = \beta(v)$, the inequality (1.3) takes the "conjugate" form

$$B^*(v^*) \geqq \left(1 - \frac{1}{p}\right) \beta^{-1}(v^*)\, v^* \quad \text{for all} \quad v^*, |v^*| \geqq v_0^* .$$

Accordingly, in view of [3], the function B^* satisfies the Δ_2-condition, i.e.

$$B^*(2v^*) \leqq c_3\, B^*(v^*) + c_4\,, \quad v^* \in \mathbb{R}^1 . \tag{1.5}$$

Consequently, we are allowed to identify (see [3])

$$E_{B^*}(K) = L_{B^*}(K) . \tag{1.6}$$

2. THE FAEDO-GALERKIN APPROXIMATION

In this section, the original problem will be replaced by a system of ordinary differential equations. For this purpose consider a sequence $\{e_j\}_{j=1}^{\infty}$ of linearly independent smooth functions from $H_0^2(\Omega)$, span $\{e_j\}_{j=1}^{\infty}$ being dense in $H_0^2(\Omega)$.

For a fixed integer n, we look for a vector function $u_n = u_n(t) \in \operatorname{span}\{e_1, \ldots, e_n\}$ satisfying

(2.1)

$$(u_n''(t) + \beta(u_n'(t)), v) + (\Delta u_n(t), \Delta v) - ([u_n(t), \Phi_n(t)], v) + \frac{1}{n}(\beta(u_n(t)), v) = (f(t), v)$$

for all $v \in \operatorname{span}\{e_1, \ldots, e_n\}$, $t \in [0, 2\pi]$,

$$\begin{aligned} u_n(0) &= u_n^0 \in \operatorname{span}\{e_1, \ldots, e_n\} \\ u_n'(0) &= u_n^1 \in \operatorname{span}\{e_1, \ldots, e_n\} \end{aligned} \tag{2.2}$$

where $\Phi_n(t) = -\Delta^{-2}[u_n(t), u_n(t)]$ (Δ^{-2} is the inverse operator to Δ^2 in Ω with the boundary conditions (B)).

In fact, (2.1) together with (2.2) is nothing else but a Cauchy problem for a system of ordinary differential equations which is known to possess a unique local solution on some interval $[0, t_n]$.

First of all, we are going to show, by virtue of a priori estimates, that $t_n = 2\pi$. Then the existence of a 2π-periodic solution will be established via the Poincaré method, i.e. by finding a fixed point of the translation operator

$$T\colon (u_n^0, u_n^1) \mapsto (u_n(2\pi), u_n'(2\pi)) .$$

Note that we have added the term $(1/n)\,\beta(u_n)$ in (2.1) in order to obtain decay estimates of the function u_n in a certain energy space.

We start with inserting $v = u_n'(t) + \delta\, u_n(t)$ in (2.1). With the relations

$$-([u_n(t), \Phi_n(t)], u_n'(t)) = \frac{1}{4}\frac{\mathrm{d}}{\mathrm{d}t}\|\Phi_n(t)\|^2\,, \tag{2.3}$$

$$(2.4) \qquad -([u_n(t), \Phi_n(t)], u_n(t)) = \|\Phi_n(t)\|^2$$

in mind (see [4]) it is a matter of computation to get

$$(2.5) \qquad \frac{\mathrm{d}}{\mathrm{d}t} F_n(u_n(t), u_n'(t)) + 2\delta F_n(u_n(t), u_n'(t)) = -\delta\|\Phi_n(t)\|^2 + \sum_{j=1}^{5} r_j(t)$$

where (cf. [12])

$$F_n(u, v) = |v|_2^2 + \|u\|^2 + \frac{1}{2}\|\Delta^{-2}[u, u]\|^2 + \frac{2}{n}|B(u)|_1 + 2\delta(u, v),$$

$$r_1(t) = 4\delta|u_n'(t)|_2^2 - \tfrac{1}{2}(\beta(u_n'(t)), u_n'(t)),$$

$$r_2(t) = 4\delta^2(u_n'(t), u_n(t)),$$

$$r_3(t) = \frac{4\delta}{n}\left(|B(u_n(t))|_1 - \frac{1}{p}(\beta(u_n(t)), u_n(t))\right),$$

$$r_4(t) = 2(f(t), u_n'(t) + \delta\, u_n(t)) - \tfrac{1}{2}(\beta(u_n'(t)), u_n'(t)),$$

$$r_5(t) = -\frac{4\delta}{n}\left(\frac{1}{2} - \frac{1}{p}\right)(\beta(u_n(t)), u_n(t)) - (\beta(u_n'(t)), u_n'(t)) - 2\delta(\beta(u_n'(t)), u_n(t)).$$

It follows from (1.3), (1.4) and the inequality $ab \leqq a^2/2 + b^2/2$ that

$$(2.6) \qquad \sum_{j=1}^{3} r_j(t) \leqq \delta\, F_n(u_n(t), u_n'(t)) + c_5$$

$\delta > 0$ being chosen small enough.

Next, the term r_4 can be treated as

$$r_4(t) \leqq \frac{1}{\varepsilon^2}|f(t)|_2^2 + \varepsilon^2(|u_n'(t)|_2^2 + \delta|u_n(t)|_2^2) - c_6|u_n'(t)|_p^p + c_7, \quad \varepsilon > 0.$$

Taking $\varepsilon > 0$ sufficiently small, we therefore see that

$$(2.7) \qquad r_4(t) \leqq c_8|f(t)|_2^2 + \delta\, F_n(u_n(t), u_n'(t)) + c_9.$$

Finally, taking advantage of the Fenchel inequality, we get

$$r_5(t) \leqq 2\delta(c_{10}(\varepsilon)\,|B^*(\beta(u_n'(t)))|_1 + \varepsilon|B(u_n(t))|_1 + c_{11}) -$$
$$- (\beta(u_n'(t)), u_n'(t)) - \frac{4\delta}{n}\left(\frac{1}{2} - \frac{1}{p}\right)(\beta(u_n(t)), u_n(t)),$$

(1.5) being taken into account.

Setting consecutively $0 < \varepsilon(n) \leqq 2(\frac{1}{2} - 1/p)/n$, $0 < \delta(n) \leqq 1/(2c_{10}(\varepsilon))$ we obtain

$$(2.8) \qquad r_5(t) \leqq c_{12}.$$

Thus combining (2.6)–(2.8) together with (2.5), we conclude

$$(2.9) \qquad e^{2\delta t} F_n(u_n(t), u_n'(t)) \leqq$$
$$\leqq F_n(u_n^0, u_n^1) + \int_0^t e^{2\delta s}(c_8|f(s)|_2^2 + \delta F_n(u_n(s), u_n'(s)) + c_{13})\, ds .$$

By virtue of the Gronwall lemma, we arrive at the desirable relation

$$(2.10) \qquad F_n(u_n(t), u_n'(t)) \leqq e^{-\delta t} F_n(u_n^0, u_n^1) + c_{14}([\![f]\!]_2) .$$

As a consequence we have $t_n = 2\pi$. Moreover, there is $\varrho > 0$ such that the mapping T transforms the set

$$N_\varrho = \{(u, v) \mid u, v \in \operatorname{span}\{e_1, \ldots, e_n\}, F_n(u, v) \leqq \varrho\}$$

into itself. Since N_ϱ is homeomorphic to the unit ball in $\mathbb{R}^{2n}$ (cf. [12]), the existence of at least one 2π-periodic solution to the equation (2.1) is ensured via the famous theorem of Brouwer.

3. A PRIORI ESTIMATES

Up to now, only one estimate has been obtained concerning the periodic solution u_n, namely (2.10). Unfortunately, its dependence on the number n prevents us from using it in a limit passage. To remove this difficulty, a priori estimates of u_n will be deduced independent of the value n.

First we integrate (2.1) inserting $v = u_n'(t)$. By the help of (2.3), we obtain

$$\int_\Gamma (\beta(u_n'(s)), u_n'(s))\, ds = \int_\Gamma (f(s), u_n'(s))\, ds .$$

Consequently, by virtue of (1.4),

$$(3.1) \qquad \int_\Gamma (\beta(u_n'(s)), u_n'(s))\, ds \leqq c_{15} .$$

Using the Fenchel inequality, we derive

$$(3.2) \qquad [\![B(u_n')]\!]_1 + [\![B^*(\beta(u_n'))]\!]_1 \leqq c_{16} ,$$

$$(3.3) \qquad [\![u_n']\!]_p \leqq c_{17} .$$

Next, setting $v = u_n'(t)$ in (2.1) again, we integrate from s to t, $0 \leqq s < t \leqq 2\pi$. Denoting

$$E_n(t) = |u_n'(t)|_2^2 + \|u_n(t)\|^2 + \tfrac{1}{2}\|\Phi_n(t)\|^2 + \frac{2}{n}|B(u_n(t))|_1$$

we get

$$E_n(t) - E_n(s) = 2(\int_s^t (f(z), u_n'(z)) - (\beta(u_n'(z)), u_n'(z))\, dz)$$

and consequently, due to (3.1),

$$(3.4) \qquad |E_n(t) - E_n(s)| \leqq c_{18} , \quad s, t \in \Gamma .$$

Finally, taking (2.4) into account, we can substitute $v = u_n(t)$ in (2.1):

$$\int_\Gamma \|u_n(s)\|^2 + \|\Phi_n(s)\|^2 + \frac{1}{n}(\beta(u_n(s)), u_n(s))\, ds = \tag{3.5}$$

$$= \int_\Gamma (f(s), u_n(s)) - (\beta(u_n'(s)), u_n(s))\, ds + [\![u_n']\!]_2^2 .$$

The hardest term to estimate seems to be

$$\left|\int_\Gamma (\beta(u_n'(s)), u_n(s))\, ds\right| \leqq [\![\beta(u_n')]\!]_1 \, [\![u_n]\!]_\infty . \tag{3.6}$$

Since $H_0^2(\Omega) \subsetneq C(\bar{\Omega})$ (see [12]), it follows easily

$$[\![u_n]\!]_\infty^2 \leqq c_{19} \sup \{E_n(s) \mid s \in \Gamma\}$$

(by the mean value theorem)

$$\leqq c_{19} \sup \{|E_n(s) - E_n(\xi)| \mid s \in \Gamma\} + \frac{1}{2\pi} \int_\Gamma E_n(s)\, ds$$

(according to (3.4))

$$\leqq c_{20} \int_\Gamma E_n(s)\, ds + c_{21} . \tag{3.7}$$

Combining (3.2), (3.3)–(3.6) with (3.7) leads to

$$\int_\Gamma E_n(s)\, ds \leqq c_{22} \tag{3.8}$$

which, together with (3.4), yields

$$\sup \{E_n(s) \mid s \in \Gamma\} \leqq c_{23} . \tag{3.9}$$

The estimates (3.1), (3.9) are crucial in the limit process. One observes easily that the corresponding constants do not depend on the number n.

4. PASSING TO THE LIMIT

4. A Compactness. For convenience of notation we still denote any subsequence of $\{u_n\}_{n=1}^\infty$ by the same symbol, $\{u_n\}_{n=1}^\infty$ being the sequence of time-periodic solutions obtained in Section 2.

The estimates (3.1)–(3.3) and (3.9) allow us to suppose

$$u_n \to u \text{ weakly-star in } L_\infty(\Gamma, H_0^2(\Omega)) , \tag{4.1}$$

$$\Phi_n \to \Phi \text{ weakly-star in } L_\infty(\Gamma, H_0^2(\Omega)) , \tag{4.2}$$

$$u_n' \to u' \; E_{B^*}(Q)\text{-weakly in } L_B(Q) , \tag{4.3}$$

$$\beta(u_n') \to h \; E_B(Q)\text{-weakly in } E_{B^*}(Q) \tag{4.4}$$

and so, according to [5],

$$u \in C(\Gamma, L_2(\Omega)) . \tag{4.5}$$

Making use of the symmetry of $[\ ,\ ]$, we are able to pass to the limit in (2.1), the underlying idea stemming from [4]. The sequence $\{u_n\}_{n=1}^{\infty}$ being bounded in $L_\infty(Q)$, the term $1/n(\beta(u_n(t)), v)$ disappears in the final expression. Thus, we finally take (2.1) to be

(4.6)
$$\int_\Gamma -(u'(s), \varphi'(s)) + (h(s), \varphi(s)) + (\Delta u(s), \Delta\varphi(s)) - \\ - ([u(s), \Phi(s)], \varphi(s))\, \mathrm{d}s = \int_\Gamma (f(s), \varphi(s))\, \mathrm{d}s$$

where φ is as in (1.1). Further, we easily verify that (1.2) holds.

Since $h \in L_2(\Gamma, L_1(\Omega))$, we deduce

(4.7)
$$u' \in C(\Gamma, H^{-2}(\Omega))$$

as a consequence of (4.3), (4.6).

At this stage, to prove that u, Φ is a weak solution to the problem in question, we have but to prove

(4.8)
$$\beta(u') = h\,.$$

5. B Monotonicity. To show (4.8), we need

(4.9)
$$\lim_{n\to\infty} \int_\Gamma (\beta(u_n'(s)), u_n'(s))\, \mathrm{d}s = \int_\Gamma (h(s), u'(s))\, \mathrm{d}s\,.$$

Note that the right-hand side has sense due to (4.3), (4.4). To undertake the final step from (4.9) to (4.8), the standard Minty's trick can be used.

To demonstrate (4.9), we set $v = u_n'(t)$ in (2.1). After integrating we easily see

(4.10)
$$\lim_{n\to\infty} \int_\Gamma (\beta(u_n'(s)), u_n'(s))\, \mathrm{d}s = \int_\Gamma (f(s), u'(s))\, \mathrm{d}s\,.$$

As a rule, the second step – inserting (formally) $\varphi = u'$ in (4.6) would represent a rather technical matter.

Pursuing [5], consider a function $\varrho_k\colon \Gamma \to \mathbb{R}^+$ such that

$$\varrho_k(s) = \varrho_k(2\pi - s)\,,$$
$$\int_\Gamma \varrho_k(s)\, \mathrm{d}s = 1\,,$$
$$\operatorname{supp} \varrho_k \subset \left[0, \frac{1}{k}\right] \cup \left[2\pi - \frac{1}{k}, 2\pi\right], \quad k = 1, 2, \ldots$$

Now, we set $\varphi = \psi_k = u' * \varrho_k * \varrho_k$ in (4.6) where $*$ denotes the convolution on Γ. Following [4], we have

$$\int_\Gamma -(u'(s), \psi_k'(s)) + (\Delta u(s), \Delta\psi_k(s))\, \mathrm{d}s = 0\,.$$

Consequently, to achieve the desirable result

(4.11)
$$\int_\Gamma (h(s), u'(s))\, \mathrm{d}s = \int_\Gamma (f(s), u'(s))\, \mathrm{d}s$$

we are to prove both

(4.12) $$\lim_{k\to\infty} \int_\Gamma ([u(s), \Phi(s)], \psi_k(s))\, ds = 0$$

and

(4.13) $$\lim_{k\to\infty} \int_\Gamma (h(s), \psi_k(s))\, ds = \int_\Gamma (h(s), u'(s))\, ds .$$

Arguing as in [4], we get successively

$$[u, u] \in L_\infty(\Gamma, L_1(\Omega)) \hookrightarrow L_\infty(\Gamma, H^{-1-\varepsilon}(\Omega)), \quad \varepsilon > 0 .$$

Consequently, $\Phi \in L_\infty(\Gamma, H_0^{3-\varepsilon}(\Omega))$, $D_x^2\Phi \in L_\infty(\Gamma, H^{1-\varepsilon}(\Omega))$. Due to $H^{1-\varepsilon}(\Omega) \hookrightarrow L_{2/\varepsilon}(\Omega)$ and (3.3) we have

(4.14) $$\lim_{k\to\infty} \int_\Gamma [u(s), \Phi(s)], \psi_k(s))\, ds = \int_\Gamma ([u(s), \Phi(s)], u'(s))\, ds$$

seeing that the right-hand side is well defined.

On the other hand, repeating the same regularization procedure we can show that the right-hand side of (4.14) equals zero, which proves (4.12).

As to the relation (4.13), the following auxiliary result will be of interest.

Lemma 4.1. *Consider a function $v \in L_B(Q)$.*
Then $v(\cdot, \cdot, t + s) \to v(\cdot, \cdot, t)$ $E_{B^*}(Q)$ — weakly whenever $s \to 0$.

Proof. First of all, note that $|v(\cdot, \cdot, t + s)|_{L_B(Q)} = |v|_{L_B(Q)}$. Since $v(\cdot, \cdot, t + s) \to v$ strongly in $L_1(Q)$ (see [1]), the lemma follows since $L_\infty(Q)$ is dense in $E_{B^*}(Q)$. Q.E.D.

Setting $\sigma_k = \varrho_k * \varrho_k$, we get

$$\left|\int_\Gamma (h(s), u'(s) - (u' * \sigma_k)(s))\, ds\right| =$$
$$= \left| \int_\Gamma \int_\Gamma (h(s), (u'(z + s) - u'(s))\, \sigma_k(z))\, ds\, dz \right. .$$

By virtue of Lemma 4.1 the right-hand side tends to zero since $|\sigma_k|_{L_1(\Gamma)}$ is bounded. Consequently, (4.13) and thus (4.8) follow.

5. A REGULARITY RESULT

The underlying idea of this section is taken from [5, Chapter 3]. Combining (4.1), (4.3) together with (4.5), (4.6), we obtain

(5.1) $$u \in C_w(\Gamma, H_0^2(\Omega)) ,$$

(5.2) $$u' \in C_w(\Gamma, L_2(\Omega))$$

where the subscript w indicates continuity with respect to the weak-topology on the corresponding spaces (see [5]). To complete the proof of Theorem 1, we desire to remove the letter w from the above relations.

Consider a function Θ_δ such that

$$\Theta_\delta = 0 \quad \text{on} \quad [t, 2\pi]\,,$$
$$1 \quad \text{on} \quad [\delta, t - \delta]\,.$$

Θ_δ is linear and continuous on $[0, \delta] \cup [t - \delta, t]$. Setting $\varphi = \psi_{\delta k} = \Theta_\delta(\Theta_\delta u^1 * * \varrho_k * \varrho_k)$ in (1.1) and integrating over Γ, we get (cf. [5])

$$|u'(t)|_2^2 + |\Delta u(t)|_2^2 = |u'(0)|_2^2 + |\Delta u(0)|_2^2 +$$
$$+ \lim_{k\to\infty} \lim_{\delta\to 0} \int_\Gamma (\beta(u'(s)) + [u(s), \Phi(s)] + f(s), \psi_{\delta k}(s))\, ds\,.$$

We are going to examine the term

$$\int_\Gamma (\beta(u'(s)), \psi_{\delta k}(s))\, ds = \int_\Gamma (\Theta_\delta(s)\, \beta(u'(s)), (\Theta_\delta u' * \varrho_k * \varrho_k)(s))\, ds\,.$$

One has $\Theta_\delta\, \beta(u') \to \Theta_0\, \beta(u')$ in $L_1(\Gamma, L_{B^*}(\Omega))$, $\Theta_\delta u' \to \Theta_0 u'$ in $L_1(\Gamma, L_B(\Omega))$ if $\delta \to 0$. Consequently $\Theta_\delta\, \beta(u') * \varrho_k \to \Theta_0\, \beta(u') * \varrho_k$ in $L_\infty(\Gamma, L_{B^*}(\Omega))$. Thus

$$\lim_{k\to\infty} \lim_{\delta\to 0} \int_\Gamma(\beta(u'(s)), \psi_{\delta k}(s))\, ds =$$
$$= \lim_{k\to\infty} \int_\Gamma (\Theta_0(s)\, \beta(u'(s)), (\Theta_0 u' * \varrho_k * \varrho_k)(s))\, ds\,.$$

Using the same arguments as in Section 4, we conclude that

$$\lim_{k\to\infty} \lim_{\delta\to 0} \int_\Gamma (\beta(u'(s)), \psi_{\delta k}(s))\, ds = \int_0^t (\beta(u'(s)), u'(s))\, ds\,.$$

In a similar way we are able to show

(5.3)
$$|u'(t)|_2^2 + |\Delta u(t)|_2^2 = |u'(0)|_2^2 + |\Delta u(0)|_2^2 +$$
$$+ \int_0^t (-\beta(u'(s)) + [u(s), \Phi(s)] + f(s), u'(s))\, ds\,.$$

As a consequence of (5.3) we deduce

(5.4)
$$|u'(\cdot)|_2^2 + \|u(\cdot)\|^2 \in C(\Gamma, \mathbb{R}^1)$$

and thus, using (5.1), (5.2),

(5.5)
$$u \in C(\Gamma, H_0^2(\Omega))\,, \quad u' \in C(\Gamma, L_2(\Omega))\,.$$

Theorem 1 has been proved.

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Souhrn

DYNAMICKÉ VON KÁRMÁNOVY ROVNICE OBSAHUJÍCÍ NELINEÁRNÍ TLUMENÍ: ČASOVĚ PERIODICKÁ ŘEŠENÍ

EDUARD FEIREISL

V článku jsou studována časově periodická řešení dynamických von Kármánových rovnic. Za předpokladu, že rovnice obsahují člen odpovídající tlumení, je dokázána existence slabého řešení úlohy. Je užito Faedo-Galerkinovy metody a základních poznatků teorie monotonních operátorů na Orliczových prostorech.

Резюме

ДИНАМИЧЕСКИЕ УРАВНЕНИЯ КАРМАНА С НЕЛИНЕЙНЫМ ЗАТУХАНИЕМ: ПЕРИОДИЧЕСКИЕ ВО ВРЕМЕНИ РЕШЕНИЯ

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Автор изучает периодические во времени решения динамических уравнений Кармана. Предполагая существование нелинейного затухания, он получает по крайней мере одно решение проблемы. Основные методы доказательства — метод Фаэдо — Галеркина и теория монотонных операторов в пространствах Орлича.

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