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A fast iteration for uniform approximation

Aplikace matematiky, Vol. 33 (1988), No. 4, 269–276

Persistent URL: `http://dml.cz/dmlcz/104308`

A FAST ITERATION FOR UNIFORM APPROXIMATION

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(Received August 20, 1986)

Summary. The paper gives such an iterative method for special Chebyshev approximations that its order of convergence is $\geqq 2$. Somewhat comparable results are found in [1] and [2], based on another idea.

Keywords: best Chebyshev approximation, Chebyshev system of functions, extremal points, Q-order of a convergent iterative method.

AMS Classification: 49D35.

1. THE ITERATION

The fixed symbols are as follows:
$[a, b] \in \boldsymbol{R}$, a closed and bounded interval;
$f(x)$, a continuous function on $[a, b]$;
$g_1(x,)\, g_2(x), \ldots, g_n(x)$, a Chebyshev system of continuous functions on $[a, b]$;
$A_1^*, A_2^*, \ldots, A_n^*$, the coefficients of the best Chebyshev approximation;
$e(x) = A_1^*\, g_1(x) + \ldots + A_n^*\, g_n(x) - f(x)$, the error function of the best Chebyshev approximation;
$x_1^*, x_2^*, \ldots, x_{n+1}^*$, a sequence of extremal points, where $a \leqq x_1^* < x_2^* < \ldots < x_{n+1}^* \leqq \leqq b$ and $e(x_j^*) = -e(x_{j+1}^*)$, $j = 1, 2, \ldots, n$.

Definition 1.1. Assume that $f(x), g_1(x), \ldots, g_n(x)$ are twice continuously differentiable on $[a, b]$. The extremal point x_j^* is *called a simple extremal point*, if

$$e'(x_j^*) \neq 0\,, \quad \text{if} \quad x_j^* = a \quad \text{or} \quad b\,;$$

$$e'(x_j^*) = 0 \quad \text{and} \quad e''(x_j^*) \neq 0\,, \quad \text{if} \quad x_j^* \neq a \quad \text{or} \quad b\,.$$

The number of different determinants in the formula of our iteration is $2n + 2$. These determinants differ from one another in the first row only. For the determinants we shall use the following symbols:

$$\begin{vmatrix} 0 \dots g_k'(x_i) \dots f'(x_i) \\ * \ * \ * \end{vmatrix} = \begin{vmatrix} 0 & g_1'(x_i) & g_2'(x_i) & \dots & g_n'(x_i) & f'(x_i) \\ 1 & g_1(x_1) & g_2(x_1) & \dots & g_n(x_1) & f(x_1) \\ -1 & g_1(x_2) & g_2(x_2) & \dots & g_n(x_2) & f(x_2) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ (-1)^n & g_1(x_{n+1}) & g_2(x_{n+1}) & \dots & g_n(x_{n+1}) & f(x_{n+1}) \end{vmatrix},$$

$$i = 1, 2, \dots, n+1\,;$$

$$\begin{vmatrix} 0 \dots g_k''(x_i) \dots f''(x_i) \\ * \ * \ * \end{vmatrix} = \begin{vmatrix} 0 & g_1''(x_i) & g_2''(x_i) & \dots & g_n''(x_i) & f''(x_i) \\ 1 & g_1(x_1) & g_2(x_1) & \dots & g_n(x_1) & f(x_1) \\ -1 & g_1(x_2) & g_2(x_2) & \dots & g_n(x_2) & f(x_2) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ (-1)^n & g_1(x_{n+1}) & g_2(x_{n+1}) & \dots & g_n(x_{n+1}) & f(x_{n+1}) \end{vmatrix},$$

$$i = 1, 2, \dots, n+1\,.$$

Our iteration starts from $x_1^{(0)}, x_2^{(0)}, \dots, x_{n+1}^{(0)}$ and its aim is to compute the sequence $x_1^*, x_2^*, \dots, x_{n+1}^*$. ($A_1^*, \dots, A_n^*$ are determined from $x_1^*, \dots, x_{n+1}^*$ by solving a linear system of equations.) The connection of the auxiliary variables u, v of the iteration with $x_1 \geqq a$ and $x_{n+1} \leqq b$ is given by the following formulas:

$$\left.\begin{aligned} u &= a + \sqrt{(x_1 - a)} \\ v &= b - \sqrt{(b - x_{n+1})} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} u^{(0)} &= a + \sqrt{(x_1^{(0)} - a)} \\ v^{(0)} &= b - \sqrt{(b - x_{n+1}^{(0)})} \end{aligned}\right\},$$

$$\left.\begin{aligned} x_1^{(j)} &= a + (u^{(j)} - a)^2 \\ x_{n+1}^{(j)} &= b - (v^{(j)} - b)^2 \end{aligned}\right\}, \quad \left.\begin{aligned} u^* &= a + \sqrt{(x_1^* - a)} \\ v^* &= b - \sqrt{(b - x_{n+1}^*)} \end{aligned}\right\}.$$

The formula of our iteration is as follows:

$$u = u - \frac{(u - a)\begin{vmatrix} 0 \dots g_k'(x_1) \dots f''(x_1) \\ * \ * \ * \end{vmatrix}}{\begin{vmatrix} 0 \dots g_k'(x_1) \dots f'(x_1) \\ * \ * \ * \end{vmatrix} + 2(u - a)^2 \begin{vmatrix} 0 \dots g_k''(x_1) \dots f''(x_1) \\ * \ * \ * \end{vmatrix}}$$

$$x_i = x_i - \frac{\begin{vmatrix} 0 \dots g_k'(x_i) \dots f'(x_i) \\ * \ * \ * \end{vmatrix}}{\begin{vmatrix} 0 \dots g_k''(x_i) \dots f''(x_i) \\ * \ * \ * \end{vmatrix}}, \quad i = 2, 3, \dots, n\,;$$

$$v = v - \frac{(v - b)\begin{vmatrix} 0 \dots g_k'(x_{n+1}) \dots f'(x_{n+1}) \\ * \ * \ * \end{vmatrix}}{\begin{vmatrix} 0 \dots g_k'(x_{n+1}) \dots f'(x_{n+1}) \\ * \ * \ * \end{vmatrix} - 2(v - b)^2 \begin{vmatrix} 0 \dots g_k''(x_{n+1}) \dots f''(x_{n+1}) \\ * \ * \ * \end{vmatrix}}$$

In the end of this part we remark that throughout the paper the order of convergence is used like the Q-order in [3].

Theorem 2.1. *Assume that* $f(x), g_1(x), \ldots, g_n(x)$ *are four times continuously differentiable on* $[a, b]$ *and* $x_1^*, x_2^*, \ldots, x_{n+1}^*$ *are simple extremal points. If* δ *is a sufficiently small positive number and* $|x_j^* - x_j^{(0)}| < \delta$, $x_1^{(0)} \geqq a$, $x_{n+1}^{(0)} \leqq b$, $j = 1, 2, \ldots, n+1$, *then the iteration is convergent and the order of the convergence is* $\geqq 2$.

Proof. In the first place we shall prove that $\boldsymbol{x}^* = \{x_1^*, \ldots, x_{n+1}^*\}$ satisfies the equations of our iteration. To this aim we show:

(a) If $x_i^* \neq a$ or b, then $\boldsymbol{x}^*$ satisfies the equation

$$\begin{vmatrix} 0 \ldots g_k'(x_i) \ldots f'(x_i) \\ * \ * \ * \end{vmatrix} = 0\,,$$

but $\boldsymbol{x}^*$ does not satisfy the equation

$$\begin{vmatrix} 0 \ldots g_k''(x_i) \ldots f''(x_i) \\ * \ * \ * \end{vmatrix} = 0\,.$$

We can see that $\boldsymbol{x}^*$ satisfies the system of equations

$$\begin{cases} A_1^* g_1'(x_i^*) + \ldots + A_n^* g_n'(x_i^*) - s f'(x_i^*) + 0\,. \quad d^* = 0\,, \\ A_1^* g_1(x_1^*) + \ldots + A_n^* g_n(x_1^*) - s f(x_1^*) + \quad d^* = 0\,, \\ \vdots \\ A_1^* g_1(x_{n+1}^*) + \ldots + A_n^* g_n(x_{n+1}^*) - s f(x_{n+1}^*) + (-1)^n d^* = 0\,, \end{cases}$$

since $e'(x_i^*) = 0$ and $e(x_j^*) - (-1)^j d^* = 0$, $j = 1, 2, \ldots, n+1$. Our system of equations is linear in the variables $A_1^*, \ldots, A_n^*, s, d^*$ and has a nontrivial solution. Therefore the determinant of the homogeneous linear system of equations equals 0, thus the first property of $\boldsymbol{x}^*$ is proved. Moreover, $\boldsymbol{x}^*$ satisfies the system of equations

$$\begin{cases} A_1^* g_1''(x_i^*) + \ldots + A_n^* g_n''(x_i^*) - s f''(x_i^*) + 0\,. \quad d^* = c\,, \\ A_1^* g_1(x_1^*) + \ldots + A_n^* g_n(x_1^*) - s f(x_1^*) + \quad d^* = 0\,, \\ \vdots \\ A_1^* g_1(x_{n+1}^*) + \ldots + A_n^* g_n(x_{n+1}^*) - s f(x_{n+1}^*) + (-1)^n d^* = 0\,, \end{cases}$$

since $e''(x_i^*) = c \neq 0$ and $e(x_j^*) - (-1)^j d^* = 0$, $j = 1, 2, \ldots, n+1$. The inhomogeneous linear system of equations is solvable for the variables $A_1^*, \ldots, A_n^*, s, d^*$. Besides,

$$\begin{vmatrix} g_1''(x_i^*) & \ldots & g_n''(x_i^*) & 0 & c \\ g_1(x_1^*) & \ldots & g_n(x_1^*) & 1 & 0 \\ \vdots & & \vdots & \vdots & \vdots \\ g_1(x_{n+1}^*) & \ldots & g_n(x_{n+1}^*) & (-1)^n & 0 \end{vmatrix} \neq 0\,,$$

since by a well-known property of the Chebyshev system the sign of the determinant

$$\begin{vmatrix} g_1(z_1) & \dots & g_n(z_1) \\ \vdots & & \vdots \\ g_1(z_n) & \dots & g_n(z_n) \end{vmatrix}$$

is permanent if $a \leqq z_1 < z_2 < \dots < z_n \leqq b$.

Hence

$$\begin{vmatrix} g_1''(x_i^*) & \dots & g_n''(x_i^*) & f''(x_i^*) & 0 \\ g_1(x_1^*) & \dots & g_n(x_1^*) & f(x_1^*) & 1 \\ \vdots & & \vdots & \vdots & \vdots \\ g_1(x_{n+1}^*) & \dots & g_n(x_{n+1}^*) & f(x_{n+1}^*) & (-1)^n \end{vmatrix} \neq 0,$$

and the second property of $\boldsymbol{x}^*$ is proved.

(b) If $x_1^* = a$, then $\boldsymbol{x}^*$ does not satisfy the equation

$$\begin{vmatrix} 0 \dots g_k'(x_1) \dots f'(x_1) \\ * \; * \; * \end{vmatrix} = 0.$$

The proof is similar to the former case (starting from the equations $e'(x_1^*) = c \neq 0$ and $e(x_j^*) - (-1)^j d^* = 0, j = 1, 2, \dots, n + 1$).

(c) If $x_{n+1}^* = b$, then $\boldsymbol{x}^*$ does not satisfy the equation

$$\begin{vmatrix} 0 \dots g_k'(x_{n+1}) \dots f'(x_{n+1}) \\ * \; * \; * \end{vmatrix} = 0.$$

We can get this result from the equations $e'(x_{n+1}^*) = c \neq 0$ and $e(x_j^*) - (-1)^j d^* = 0, j = 1, 2, \dots, n + 1$.

Now, if we analyse the cases

$$x_1^* \neq a (\Rightarrow u^* \neq a), \quad x_{n+1}^* \neq b (\Rightarrow v^* \neq b);$$

$$x_1^* = a (\Rightarrow u^* = a), \quad x_{n+1}^* \neq b (\Rightarrow v^* \neq b);$$

$$x_1^* \neq a (\Rightarrow u^* \neq a), \quad x_{n+1}^* = b (\Rightarrow v^* = b);$$

$$x_1^* = a (\Rightarrow u^* = a), \quad x_{n+1}^* = b (\Rightarrow v^* = b),$$

then we can see that $\boldsymbol{x}^*$ indeed satisfies the equations of our iteration.

Moreover, we shall prove that all partial derivatives of our iterative functions equal 0 at $\hat{\boldsymbol{x}}^* = \{u^*, x_2^*, \dots, x_n^*, v^*\}$. (In the formulas, x_1 and x_{n+1} are used for the sake of convenience only.) To this end we employ the following properties ($\hat{\boldsymbol{x}} = \{u, x_2, \dots, x_n, v\}$):

(1) Since $x_1 = a + (u - a)^2$, therefore

$$D_{11}(\hat{\boldsymbol{x}}) = \frac{\partial}{\partial u} \begin{vmatrix} 0 \dots g_k'(x_1) \dots f'(x_1) \\ * \; * \; * \end{vmatrix} = 2(u - a) \begin{vmatrix} 0 \dots g_k''(x_1) \dots f''(x)_1 \\ * \; * \; * \end{vmatrix};$$

$$D_{i1}(\hat{x}) = \frac{\partial}{\partial u}\begin{vmatrix} 0 \dots g_k'(x_i) \dots f'(x_i) \\ * \;\; * \;\; * \end{vmatrix} = 2(u-a)\begin{vmatrix} 0 & \dots g_k'(x_1) & \dots f'(x_i) \\ 0 & g_k'(x_1) \dots & f'(x_1) \\ -1 & g_k(x_2) \dots & f(x_2) \\ \vdots & \vdots & \vdots \\ (-1)^n & \dots g_k(x_{n+1}) & \dots f(x_{n+1}) \end{vmatrix},$$

if $i = 2, 3, \dots, n+1$.

(2) For $j = 2, 3, \dots, n$,

$$D_{ij}(\hat{x}) = \frac{\partial}{\partial x_j}\begin{vmatrix} 0 \dots g_k'(x_i) \dots f'(x_i) \\ * \;\; * \;\; * \end{vmatrix} = \begin{vmatrix} 0 \dots g_k''(x_i) \dots f''(x_i) \\ * \;\; * \;\; * \end{vmatrix},$$

if $i = 2, 3, \dots, n$ and $i = j$;

$$D_{ij}(\hat{x}) = \frac{\partial}{\partial x_j}\begin{vmatrix} 0 \dots g_k'(x_i) \dots f'(x_i) \\ * \;\; * \;\; * \end{vmatrix} = \begin{vmatrix} 0 & \dots g_k'(x_i) & \dots f'(x_i) \\ 1 & \dots g_k(x_1) & \dots f(x_1) \\ \vdots & \vdots & \vdots \\ (-1)^{j-2} & \dots g_k(x_{j-1}) & \dots f(x_{j-1}) \\ 0 & \dots g_k'(x_j) & \dots f'(x_j) \\ (-1)^j & \dots g_k(x_{j+1}) & \dots f(x_{j+1}) \\ \vdots & \vdots & \vdots \\ (-1)^n & \dots g_k(x_{n+1}) & \dots f(x_{n+1}) \end{vmatrix},$$

if $i = 1, 2, \dots, n+1$ and $i \neq j$.

(3) Since $x_{n+1} = b - (v-b)^2$, therefore

$$D_{n+1n+1}(\hat{x}) =$$

$$= \frac{\partial}{\partial v}\begin{vmatrix} 0 \dots g_k'(x_{n+1}) \dots f'(x_{n+1}) \\ * \;\; * \;\; * \end{vmatrix} = -2(v-b)\begin{vmatrix} 0 \dots g_k''(x_{n+1}) \dots f''(x_{n+1}) \\ * \;\; * \;\; * \end{vmatrix};$$

$$D_{in+1}(\hat{x}) = \frac{\partial}{\partial v}\begin{vmatrix} 0 \dots g_k'(x_i) \dots f'(x_i) \\ * \;\; * \;\; * \end{vmatrix} =$$

$$= -2(v-b)\begin{vmatrix} 0 & \dots g_k'(x_i) & \dots f'(x_i) \\ 1 & \dots g_k(x_1) & \dots f(x_1) \\ \vdots & \vdots & \vdots \\ (-1)^{n-1} & \dots g_k(x_n) & \dots f(x_n) \\ 0 & \dots g_k'(x_{n+1}) & \dots f'(x_{n+1}) \end{vmatrix},$$

if $i = 1, 2, \dots, n$.

(4) Among the functions $D_{ij}(\hat{x})$ of (1)–(3) the values of $n(n+1) - 2$ functions equal 0 at $\hat{x}^*$.

Namely,

$D_{i1}(\hat{x}^*) = 0$, if $i = 2, 3, \dots, n$;
$D_{ij}(\hat{x}^*) = 0$, if $i = 1, 2, \dots, n+1$; $j = 2, 3, \dots, n$ and $i \neq j$;
$D_{in+1}(\hat{x}^*) = 0$, if $i = 2, 3, \dots, n$.

For example, $D_{21}(\hat{x}^*) = 0$ because $u \neq a \Rightarrow e'(x_1^*) = 0$ and by equations $e'(x_2^*) = 0$, $e'(x_1^*) = 0$, $e(x_j^*) - (-1)^j d^* = 0$, $j = 2, 3, \ldots, n + 1$ we obtain

$$\begin{vmatrix} 0 & \cdots g_k'(x_2) & \ldots f'(x_2) \\ 0 & \cdots g_k'(x_1) & \ldots f'(x_1) \\ -1 & \cdots g_k(x_2) & \ldots f(x_2) \\ \vdots & & \\ (-1)^n & \cdots g_k(x_{n+1}) & \ldots f(x_{n+1}) \end{vmatrix} = 0$$

$\Rightarrow D_{21}(\hat{x}^*) = 0$, or $u = a \Rightarrow D_{21}(\hat{x}^*) = 0$. In the other $n(n + 1) - 3$ cases the proof is quite similar to the proof if $D_{21}(\hat{x}^*) = 0$.

$$(5) \qquad (v - b)\, D_{n+1\,1}(\hat{x}^*) = 0 \quad \text{and} \quad (u - a)\, D_{1\,n+1}(\hat{x}^*) = 0\,,$$

since $v \neq b \Rightarrow e'(x_{n+1}^*) = 0$ and $u \neq a \Rightarrow e'(x_1^*) = 0$.

If we now determine the partial derivatives of our iterative functions (as derivatives of fractional functions), then we can see (using (1)–(5) and (a)–(c)) that they equal 0 at $\hat{x}^*$. By using Theorem 10.1.7 of [3] the proof can be completed.

Theorem 2.2. *Let* $g_1(x) = 1$, $g_2(x) = x, \ldots, g_n(x) = x^{n-1}$, *where* $n \geqq 2$. *Assume that* $f(x)$ *is four continuously differentiable on* $[a, b]$, *and* $f^{(n)}(x)$ *exists on* $[a, b]$ *and* $f^{(n)}(x) \neq 0$, $\forall x \in (a, b)$. *If* δ *is a sufficiently small positive number and* $|x_j^* - x_j^{(0)}| < \delta$, $j = 2, 3, \ldots, n$ *and* $x_1^{(0)} = a(= u^{(0)} = u^* = x_1^*)$, $x_{n+1}^{(0)} = b(= v^{(0)} = v^* = x_{n+1}^*)$, *then the iteration is convergent (the limit is the unique solution* $x_1^*, x_2^*, \ldots, x_{n+1}^*$ *of our problem) and the order of the convergence is* $\geqq 2$.

Proof. If we prove that sequence $x_1^*, x_2^*, \ldots, x_{n+1}^*$ is unique and $x_1^* = a$, $x_{n+1}^* = b$ and the points x_j^*, $j = 1, 2, \ldots, n + 1$ are simple extremal points, then by Theorem 2.1. the proof is complete. The above mentioned properties of $\{x_1^*, x_2^*, \ldots, x_{n+1}^*\}$ are proved in two parts.

(1) If $e'(x) = 0$ has at least n roots on $[a, b]$, then $e''(x) = 0$ has at least $n - 1$ roots on $(a, b) \Rightarrow \ldots \Rightarrow e^{(n)}(x) \equiv f^{(n)}(x) = 0$ has at least one root on (a, b). Hence $x_1^* = a$, $x_{n+1}^* = b$ and the sequence $x_2^*, x_3^*, \ldots, x_n^*$ is unique and x_1^*, x_{n+1}^* are simple extremal points.

(2) If $e''(x_j^*) = 0$, $j \in \{2, 3, \ldots, n\}$, then (since $e'(x_j^*) = 0$, $j = 2, 3, \ldots, n$) $e''(x) = 0$ has at least $n - 1$ roots on $(a, b) \Rightarrow \ldots \Rightarrow e^{(n)}(x) \equiv f^{(n)}(x) = 0$ has at least one root on (a, b). Hence x_j^*, $j = 2, 3, \ldots, n$ are simple extremal points.

3. REMARKS FOR NUMERICAL APPLICATIONS

First we comment on the number of operations of one iterative step. The determinants require $3\,(n + 1)^2$ new values of the functions $g_i(x), f(x), g_i'(x), f'(x), g_i''(x), f''(x)$ in one iterative step. In addition, we must execute approximately $2(n + 1)^3/3 + 2(n + 1)(n + 2)^2 < 3(n + 2)^3$ arithmetic operations, if we use the simple Gauss-elimination for the determination of values of the $2(n + 1)$determinants.

(The value of an nth order determinant can be computed by $\sim 2n^3/3$ arithmetic operations in the case of simple Gauss-elimination.) The number of operations of our method is essentially fixed by the above two facts.

The use of our iteration is recommended in two versions.

(1) If the conditions of Theorem 2.2 are fulfilled, then we can make a trial with the initial values

$$x_1^{(0)} = a\,; \quad x_j^{(0)} = \frac{a+b}{2} + \frac{b-a}{2}\cos\left(\pi\,\frac{n-j+1}{n}\right), \quad j = 2, 3, \ldots, n\,;$$

$$x_{n+1}^{(0)} = b\,.$$

We have used the iteration in the following 12 examples:

$$f(x) = e^x\,, \quad [a, b] = [0, 1]\,, \qquad n = 2, 3, 4, 5, ;$$

$$f(x) = \ln x, \quad [a, b] = [1, \mathrm{e}]\,, \qquad n = 2, 3, 4, 5\,;$$

$$f(x) = \sin x, [a, b] = [0, \pi/4]\,, \quad n = 2, 3, 4, 5\,.$$

(The iteration is convergent in each of these cases.)

(2) If we have a problem and have already computed "sufficiently much" by a certain convergent (and slow) method, then we can compute a very accurate approximation of $\{x_1^*, x_2^*, \ldots, x_{n+1}^*\}$ in a few steps. Now we show a simple example for this version. Let $[a, b] = [-1{\cdot}2, 1{\cdot}3]$, $f(x) = x^4 - 2x^2 + x$, $g_1(x) = 1$, $g_2(x) = x$. (The exact solution is $x_1^* = -1$, $x_2^* = 0$, $x_3^* = 1$.) Here we must compute the values of the determinants

$$\begin{vmatrix} 0 & 0 & 1 & f'(x_i) \\ 1 & 1 & x_1 & f(x_1) \\ -1 & 1 & x_2 & f(x_2) \\ 1 & 1 & x_3 & f(x_3) \end{vmatrix} \quad \text{abd} \quad \begin{vmatrix} 0 & 0 & 0 & f''(x_i) \\ 1 & 1 & x_1 & f(x_1) \\ -1 & 1 & x_2 & f(x_2) \\ 1 & 1 & x_3 & f(x_3) \end{vmatrix} \qquad (i = 1, 2, 3)$$

in each of the iterative steps. Hence, first we compute 9 new values of the functions $f(x)$, $f'(x)$, $f''(x)$ in every iterative step. (The values $g_j(x_i)$, $g_j'(x_i)$, $g_j''(x_i)$, $j = 1, 2$; $i = 1, 2, 3$ are given without computation.) Then the determination of values of the six determinants needs 109 arithmetic operations, if we do not use the special features of this example. ($3(n+2)^3 = 192$.) If we start with the values

$$x_1^{(0)} = -0{\cdot}9\,, \quad x_2^{(0)} = 0{\cdot}1\,, \quad x_3^{(0)} = 0{\cdot}9\,,$$

then we get

$$x_1^{(1)} = -0{\cdot}9914, x_2^{(1)} = 0{\cdot}0021, x_3^{(1)} = 0{\cdot}9973.$$

and

$$x_1^{(2)} = -0{\cdot}99986\,, \quad x_2^{(2)} = 0{\cdot}00003\,. \quad x_3^{(2)} = 0{\cdot}99998\,.$$

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Souhrn

RYCHLÉ ITERACE PRO STEJNOMĚRNOU APROXIMACI

FERENC KÁLOVICS

V článku je podána iterační metoda pro speciální Čebyševovy aproximace, jejíž řád konvergence je $\geqq 2$.

Резюме

БЫСТРАЯ ИТЕРАЦИЯ ДЛЯ РАВНОМЕРНОЙ АППРОКСИМАЦИИ

FERENC KÁLOVICS

В статье изложен итерационный метод для специальной аппроксимации Чебышева, порядок сходимости которого $\geqq 2$.

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