Pavel Krejčí

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FORCED PERIODIC VIBRATIONS OF AN ELASTIC SYSTEM WITH ELASTICO-PLASTIC DAMPING

PAVEL KREJČÍ

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Summary, We prove the existence and find necessary and sufficient conditions for the uniqueness of the time-periodic solution to the equation $u_{tt} - \Delta_x u \pm F(u) = g(x, t)$ for an arbitrary (sufficiently smooth) periodic right-hand side g, where Δ_x denotes the Laplace operator with respect to $x \in \Omega \subset R^N$, $N \geqq 1$, and F is the Ishlinskii hysteresis operator. For $N = 2$ this equation describes e.g. the vibrations of an elastic membrane in an elastico-plastic medium.

Keywords: Wave equation, hysteresis, Ishlinskii operator, periodic solutions.

AMS classification: 35B10, 35L70, 73E50.

Hooke's law for elastico-plastic (or non-perfectly elastic) materials in the sense of Ishlinskii is described by a hysteresis scheme which is commonly considered to be sufficiently realistic for „not too large" frequencies of motion ([2]). The basic theory of the Ishlinskii operator was introduced in [2].

In this paper we investigate the existence and uniqueness of weak ω-periodic solutions (with respect to t) to the problem

$$(1)_\pm \qquad u_{tt} - \Delta_x u \pm F(u) = g(x, t), \quad x \in \Omega \subset R^N, \quad t \in R^1,$$

$$(2) \qquad u(x, t) = 0, \quad x \in \partial\Omega,$$

where $N \geqq 1$, $\omega > 0$ are given, $\Omega \subset R^N$ is a bounded open domain with a Lipschitzian boundary, Δ_x is the Laplacian with respect to $x \in \Omega$, g is a given ω-periodic function and F is the Ishlinskii operator. For example, the system $(1)_+$, (2) for $N = 2$ describes the forced vibrations of a membrane in an elastico-plastic medium. Other problems connected with partial differential equations with hysteresis can be found e.g. in [7], [8], [4].

1. FUNCTION SPACES

We introduce the spaces:

L^p_ω, $1 \leqq p \leqq \infty$: the Lebesgue space of all measurable ω-periodic function v: $R^1 \to R^1$ such that

$$|v|_p = \left(\int_0^\omega |v(t)|^p \, dt\right)^{1/p} < \infty \quad \text{for} \quad p < \infty$$

and

$$|v|_\infty = \sup \operatorname{ess} \{|v(t)|, t \in R^1\} \quad \text{for} \quad p = \infty ,$$

with the norm $|\cdot|_p$,

C_ω: the B-space of all continuous real ω-periodic functions with the norm $|\cdot|_\infty$.

In the sequel, $\Omega \subset R^N$ is a bounded open domain with a Lipschitzian boundary. We denote by

$L^p(\Omega; L^q_\omega)$, $1 \leqq p < \infty$, $1 \leqq q \leqq \infty$ the space of all measurable functions u: $\Omega \times \times R^1 \to R^1$ such that $u(x, \cdot) \in L^q_\omega$ for a.e. $x \in \Omega$ and

$$|u|_{p,q} = \left(\int_\Omega |u(x, \cdot)|_q^p \, dx\right)^{1/p} < \infty , \quad \text{with the norm} \quad |\cdot|_{p,q} ;$$

for $p = q$ we write simply $L^p_\omega(\Omega)$;

$L^p(\Omega; C_\omega)$, $1 \leqq p < \infty$: the subspace of all functions $u \in L^p(\Omega; L^\infty_\omega)$ such that $u(x, \cdot) \in C_\omega$ for almost all $x \in \Omega$.

The spaces $L^p(\Omega; L^q_\omega)$ are Banach spaces (cf. [1]) and the same is true for $L^p(\Omega; C_\omega)$ which is a closed subspace of $L^p(\Omega; L^\infty_\omega)$;

$$Z_\omega(\Omega) = \{u \in L^2_\omega(\Omega);\ u_t \in L^2(\Omega; L^3_\omega),\ \nabla_x u \in (L^2_\omega(\Omega))^N\} ,$$

where

$$\nabla_x = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_N}\right),$$

with the norm

$$|u|_z = |u|_{2,2} + |u_t|_{2,3} + \|\nabla_x u\|_{2,2} , \quad \text{where} \quad \|\cdot\|_{2,2}$$

denotes the norm in $(L^2_\omega(\Omega))^N$.

Let $\{e_k, k = 1, 2, \ldots\}$ be the complete system of eigenfunctions of $-\Delta_x$ in Ω with zero Dirichlet boundary condition on $\partial\Omega$, i.e.

$$(1.1) \qquad -\Delta_x e_k = \nu_k e_k , \quad e_k(x) = 0 \quad \text{for} \quad x \in \partial\Omega , \quad 0 < \nu_1 < \nu_2 \leqq \ldots .$$

We define

$$(1.2) \qquad w_{jk}(x, t) = \left\langle \begin{array}{lll} \sin(2\pi j/\omega)\, t\, e_k(x) , & k \geqq 1 , & j \geqq 1 \\ \cos(2\pi j/\omega)\, t\, e_k(x) , & k \geqq 1 , & j \leqq 0 \end{array} \right. .$$

We denote by $Z^0_\omega(\Omega)$ the closure of the linear hull of $\{w_{jk}, k \text{ natural}, j \text{ integer}\}$ in $Z_\omega(\Omega)$.

2. ISHLINSKII OPERATOR

We recall here the properties of the Ishlinskiĭ operator (cf. [3], [4]). Throughout the paper c, c_k denote any independent positive constants.

(2.1) F is an odd continuous operator $C_\omega \to C_\omega$,

(2.2) φ: $(0, \infty) \to (0, \infty)$ is a given twice continuously differentiable function such that

(i) φ is increasing, $\varphi(0+) = 0$, $0 < \varphi'(0+) < \infty$,

(ii) $\varphi(h) \leqq c_1 h^\alpha$ for some $\alpha \in (0, 1)$ and every $h > 0$,

(iii) $\gamma(r) \geqq c_2 r^{\beta-2}$ for some $\beta \in (0, \alpha]$, $r_0 > 0$ and every $r > r_0$, where $\gamma(r) = \inf\{-\varphi''(h), 0 < h \leqq r\}$,

$$(2.3) \qquad |F(u) - F(v)|_\infty \leqq 2\varphi(|u - v|_\infty) \quad \text{for every} \quad u, v \in C_\omega\,,$$

$$(2.4) \qquad \int_\omega^{2\omega} F(v)\, v'''\, dt \leqq -\tfrac{1}{4}\gamma(|v|_\infty) \int_0^\omega |v'|^3\, dt$$

for each $v \in C_\omega$ such that v'' is absolutely continuous. These properties are proved in [4]. From (2.19) and (2.24) of [4] we immediately derive

(2.5) Let $v \in C_\omega$ be given. Then for an arbitrary real constant z the difference $F(v + z) - F(v)$ is independent of t for $t \geqq \omega$ and

$$\psi(v, z) \equiv F(v + z)(t) - F(v)(t) =$$
$$= \operatorname{sign}(\mu + z)\,[\varphi(\lambda + |\mu + z|) - \varphi(\lambda)] - \operatorname{sign}(\mu)\,[\varphi(\lambda + |\mu|) - \varphi(\lambda)]\,,$$

where

$$\mu = \tfrac{1}{2}(\max v + \min v)\,, \quad \lambda = \tfrac{1}{2}(\max v - \min v)\,.$$

The function $\psi(v, \cdot)$ is continuously differentiable and for every $v \in C_\omega$ and $z, z_1, z_2 \in R^1$ we have

(i) $|\psi(v, z_1) - \psi(v, z_2)| \leqq 2\varphi(\frac{1}{2}|z_1 - z_2|)$,

(ii) $(\partial/\partial z)\, \psi(v, z) \geqq \varphi'(|v|_\infty + |z|)$,

(iii) $\psi(v, 0) = 0$.

Further we have (cf. [3], [5])

(2.6) Let u, $v \in C_\omega$ be absolutely continuous. Then

$$\int_\omega^{2\omega} (F(u) - F(v))\,(u' - v')\, dt \geqq 0\,.$$

If moreover

$$\int_\omega^{2\omega} (F(u) - F(v)\,(u' - v')\, dt = 0\,, \quad \text{then} \quad u' = v' \quad \text{a.e.}$$

The next property is obvious (cf. also [7] for another type of hysteresis operators):

(2.7) For $u \in L^p(\Omega; C_\omega)$ we define $F(u)(x, t) = F(u(x, \cdot))(t)$ for a.e. $x \in \Omega$ and every $t \in R^1$. We have

$$F(u) \in L^{p/\alpha}(\Omega; C_\omega) \quad \text{and} \quad |F(u) - F(v)|_{p/\alpha,\infty} \leqq C|u - v|^{\alpha}_{p,\infty}$$

for every $u, v \in L^p(\Omega; C_\omega)$.

3. EXISTENCE THEOREM

Theorem. *Let $\varepsilon = \pm 1$ and $N \geqq 1$ be given. Let β* (cf. (2.2) (iii)) *be arbitrary for $N \leqq 2$ and greater than $(7N - 16)/(7N - 8)$ for $N \geqq 3$. Then for each $g \in L^q(\Omega; L^1_\omega)$, $q = (4 - 2\beta)/(3 - 2\beta)$, such that $g_{tt} \in L^2(\Omega; L^{3/2}_\omega)$ there exists at least one $u \in Z^o_\omega(\Omega)$ such that for every $w \in Z^o_\omega(\Omega)$ we have*

$$\int_\Omega \int_\omega^{2\omega} (-u_t w_t + \langle \nabla_x u, \nabla_x w \rangle + \varepsilon F(u) w - g w) \, dt \, dx = 0 \,, \tag{3.1}$$

where $\langle \cdot, \cdot \rangle$ denotes the scalar product in R^N.

Remarks. (i) The term $F(u)$ is meaningful, since the space $Z_\omega(\Omega)$ is (compactly) embedded into $L^p(\Omega; C_\omega)$ for $p = 4 - 2\beta$ (see Appendix).

(ii) In fact, for $v \in C_\omega$, $F(v)$ is ω-periodic for $t \geqq \omega$. For this reason we integrate from ω to 2ω. In particular, the equation is sastisfied for $t \geqq \omega$.

Proof. We make use of the classical Galerkin method. Put

$$_m u(x, t) = \sum_{k=1}^{m} \sum_{j=-m}^{m} u_{jk} \, w_{jk}(x, t) \,,$$

where w_{jk} are given by (1.2). The real numbers u_{jk} have to satisfy the system

$$\int_\Omega \int_\omega^{2\omega} ({_m u_{tt}} - \Delta_x \, {_m u} + \varepsilon F({_m u}) - g) \, w_{jk} \, dx \, dt = 0 \,, \tag{3.2}$$

$$j = -m, \ldots, m \,, \quad k = 1, \ldots, m \,.$$

We first derive apriori estimates (cf. [4]): we multiply (3.2) by $(2\pi j/\omega)^3 \, u_{-jk}$ and sum over j and k. We get (using (2.2) (iii))

$$\int_\Omega \gamma(|{_m u}(x, \cdot)|_\infty) \int_0^\omega |{_m u_t}(x, t)|^3 \, dt \, dx \leqq 4 \int_\Omega \int_0^\omega |g_{tt}(x, t)| \, |{_m u_t}(x, t)| \, dt \, dx \,. \tag{3.3}$$

From the Hölder inequality

$$\int_\Omega \left(\int_0^\omega |{_m u_t}(x, t)|^3 \, dt \right)^{2/3} dx \leqq \left(\int_\Omega (\gamma(|{_m u}(x, \cdot)|_\infty))^{-2} \, dx \right)^{1/3} \cdot$$

$$\cdot \left(\int_\Omega \gamma({_m u}(x, \cdot)|_\infty) \int_0^\omega |{_m u_t}(x, t)|^3 \, dt \, dx \right)^{2/3}$$

and (3.3) we have

$$(3.4)\qquad |{}_m u_t|_{2,3} \leqq c(1 + |{}_m u|_{4-2\beta,\infty}^{1-\beta/2}) .$$

Similarly, multiplying (3.2) by u_{jk} and summing over j and k we obtain

$$\|\nabla_x\, {}_m u\|_{2,2}^2 = |{}_m u_t|_{2,2}^2 + \int_\Omega\int_\omega^{2\omega} |F({}_m u)|\, |{}_m u|\, dt\, dx + \int_\Omega\int_\omega^{2\omega} |g|\, |{}_m u|\, dt\, dx ,$$

hence by (3.4), (2.1) (ii),

$$(3.5)\qquad \|\nabla_x\, {}_m u\|_{2,2} \leqq c(1 + |{}_m u|_{4-2\beta,\infty}^{1-\beta/2} + |{}_m u|_{4-2\beta,\infty}^{(1+\alpha)/2} + |{}_m u|_{4-2\beta,\infty}^{1/2}) .$$

Notice that for $p = 4 - 2\beta$ we have $0 < \frac{1}{2} - (1/p) < (4/7N)$. On the other hand, $|{}_m u|_z \leqq c(|{}_m u_t|_{2,3} + \|\nabla_x\, {}_m u\|_{2,2})$. Therefore (3.4), (3.5) and the embedding theorem (A.1) (see Appendix) imply $|{}_m u|_{4-2\beta,\infty} \leqq c$, where c is independent of m. Consequently $|{}_m u|_z \leqq c$. This estimate implies the solvability of (3.2) for arbitrary m (cf. [4]). Moreover, since the corresponding embedding is compact, there exists a subsequence $\{{}_n u\} \subset \{{}_m u\}$ and $u \in Z_\omega^o(\Omega)$ such that ${}_n u \to u$ in $Z_\omega^o(\Omega)$ weak and ${}_n u \to u$ in $L^p(\Omega; C_\omega)$ strong. We now pass to the limit in (3.2) for $n \to \infty$ and conclude that u satisfies (3.1). Theorem is proved.

4. UNIQUENESS THEOREM

Theorem. *Let the assumptions of Existence Theorem be fulfilled. Then the solution $u \in Z_\omega^o(\Omega)$ of (3.1) for an arbitrary righ-hand side g is unique if and only if $\varepsilon = +1$ or $\varphi'(0+) \leqq \nu_1$ (cf. (1.1), (2.2) (i)).*

Proof. Let $u, v \in Z_\omega^o(\Omega)$ be two solutions of (3.1). We put $z(x, t) = v(x, t) - u(x, t)$. For arbitrary $w \in Z_\omega^o(\Omega)$ we have

$$(4.1)\qquad \int_\Omega\int_\omega^{2\omega} (-z_t w_t + \langle\nabla_x z, \nabla_x w\rangle + \varepsilon(F(v) - F(u))\, w)\, dt\, dx = 0 .$$

Let $\varrho \in C_\infty(-\infty, \infty)$ be an even nonnegative function, $\operatorname{supp} \varrho \subset (-\omega/2, \omega/2)$, $\int_{-\infty}^{\infty} \varrho(s)\, ds = 1$, and put

$${}_n w(x, t) = n\int_{-\infty}^{\infty} \varrho(n(t - s))\, z(x, s)\, ds = \int_{-\infty}^{\infty} \varrho(s)\, z(x, t - s/n)\, ds ,$$

$n = 1, 2, \ldots$. Relation (4.1) holds in particular for $w = {}_n w_t$. Notice that ϱ' is odd, hence for arbitrary $f \in L_\omega^1$ we have

$$\int_0^\omega\int_{-\infty}^{\infty} \varrho'(n(t - s))\, f(t)\, f(s)\, ds\, dt = 0 ,$$

consequently

$$\int_\Omega\int_\omega^{2\omega} (F(v) - F(u))\, {}_n w_t\, dt\, dx = 0 .$$

This yields, for $n \to \infty$,

$$(4.2) \qquad \int_\Omega \int_\omega^{2\omega} (F(v) - F(u))\,(v_t - u_t)\,dt\,dx = 0\,.$$

By virtue of (2.6), (2.5) z is independent of t and $F(v)(x, t) - F(u)(x, t) = \psi(u(x, \cdot), z(x))$. We have $z \in W_o^{1,2}(\Omega)$ and from (4.1) we obtain for each $w \in W_o^{1,2}(\Omega)$

$$(4.3) \qquad \int_\Omega (\langle \nabla_x z, \nabla_x w \rangle + \varepsilon\, \psi(u(x, \cdot), z(x))\, w(x))\,dx = 0\,.$$

We distinguish three cases:

(4.4) (i) $\varepsilon = +1$. We put $w = z$ in (4.3) and from (2.5) (ii), (iii) we immediately obtain $z = 0$.

(ii) $\varepsilon = -1$, $\varphi'(0+) \leqq \nu_1$. We put again $w = z$ in (4.3).
We have

$$\int_\Omega \langle \nabla_x z, \nabla_x z \rangle\,dx \geqq \nu_1 \int_\Omega z^2(x)\,dx$$

and

$$\int_\Omega \psi(u(x, \cdot), z(x))\, z(x)\,dx \leqq \int_\Omega 2|z(x)|\, \psi(\tfrac{1}{2}|z(x)|)\,dx$$

(cf. (2.5) (i)). On the other hand, $\varphi(h) < \varphi'(0+)\, h$ for every $h > 0$, hence $z = 0$.

(iii) $\varepsilon = -1$, $\varphi'(0+) > \nu_1$. We put $g = 0$ in (3.1). Then $u = 0$ is a solution of (3.1) and (4.2), (4.3) imply that $v \neq u$ is a solution of (3.1) if and only if $v(x, t) = z(x)$, where $z \in W_o^{1,2}(\Omega)$ and

$$(4.5) \qquad \int_\Omega (\langle \nabla_x z, \nabla_x w \rangle - \psi(0, z(x))\, w(x))\,dx = 0$$

for every $w \in W_o^{1,2}(\Omega)$.

Let us define $G(z) = \frac{1}{2} \int_\Omega \langle \nabla_x z, \nabla_x z \rangle\,dx - \int_\Omega \int_0^{z(x)} \psi(0, \zeta)\,d\zeta\,dx$ for $z \in W_o^{1,2}(\Omega)$. We find $\delta > 0$ such that $\varphi'(\delta) > \nu_1$, and $\eta > 0$ such that $\eta \max \{|e_1(x)|, x \in \Omega\} \leqq \delta$. From (2.5) (ii), (iii) we obtain $G(\eta e_1) = (\frac{1}{2}\eta^2 \nu_1 - \frac{1}{2}\eta^2\, \varphi'(\delta)) \int_\Omega e_1^2(x)\,dx < 0$. On the other hand, (2.5) (i) and (2.2) (ii) yield $|\int_\Omega \int_0^{z(x)} \varphi(0, \xi)\,d\zeta\,dx| \leqq c \int_\Omega |z(x)|^{1+\alpha}\,dx$ for arbitrary $z \in W_o^{1,2}(\Omega)$. This inequality implies that there exists $R > 0$ such that $G(z) > 0$ for $\|z\| > R$, where $\|\cdot\|$ denotes the norm in $W_o^{1,2}(\Omega)$. The functional G is weakly lower semicontinuous in $B_R = \{z \in W_o^{1,2}(\Omega), \|z\| \leqq R\}$. Consequently, there exists $z_o \in B_R$ such that $G(z_o) = \inf\{G(z), z \in B_R\} < 0$. In particular, z_o (and also $-z_o$), is a nontrivial solution of (4.5).

APPENDIX. AN EMBEDDING THEOREM

The following theorem is not explicitly proved in [1], but the proof we sketch here is based on the same method.

(A.1) **Theorem.** *Let* $0 < \frac{1}{2} - 1/p < 4/7N$, $N \geqq 1$, *and let* $\Omega \subset R^N$ *be a bounded open domain with a Lipschitzian boundary. Then the space* $Z_\omega(\Omega)$ *is compactly embedded into* $L^p(\Omega; C_\omega)$.

Proof. Let $P: W^{1/2}(\Omega) \to W^{1,2}(R^N)$ be the linear continuous prolongation operator (cf. [6], p. 75), and put $Q\,u(x, t) = P\,u(\cdot, t)\,(x)$ for $u \in Z_\omega(\Omega)$. Repeating the proof of Theorem 3.9 of [6], Chapter 2 we see that Q is a linear continuous prologation operator $Z_\omega(\Omega) \to Z_\omega(R^N)$. Further, for $\sigma \in (0, 1)$, $(x, t) \in \Omega \times R^1$, set

$$\text{(A.2)} \qquad u^\sigma(x, t) = \sigma^{-\lambda - N} \int_{R^{N+1}} \varphi\left(\frac{y - x}{\sigma}, \frac{s - t}{\sigma}\right) \bar{u}(y, s)\, dy\, ds\,,$$

where $\lambda > 0$, $\bar{u} = Qu$, and φ is a C^∞-function such that $\operatorname{supp} \varphi \subset (-1, 1)^N \times \times (-\omega/2, \omega/2)$,

$$\int_{R^{N+1}} \varphi(\xi, \tau)\, d\xi\, d\tau = 1\,,$$

We have

$$\frac{\partial}{\partial \sigma}\, u^\sigma(x, t) = \sigma^{-N-1} \int_{R^{N+1}} \lambda \frac{s - t}{\sigma^\lambda}\, \varphi\left(\frac{y - x}{\sigma}, \frac{s - t}{\sigma^\lambda}\right) \bar{u}_t(y, s)\, dy\, ds +$$

$$+ \sigma^{-N-\lambda} \int_{R^{N+1}} \left\langle \frac{y - x}{\sigma}\, \varphi\left(\frac{y - x}{\sigma}, \frac{s - t}{\sigma^\lambda}\right), \nabla_y\, \bar{u}(y, s) \right\rangle dy\, ds\,.$$

For $0 < \alpha < \beta < 1$ we obtain

$$\text{(A.3)} \qquad |u^\beta(x, \cdot) - u^\alpha(x, \cdot)|_\infty \leqq \lambda \int_\alpha^\beta \sigma^{-N-1+2\lambda/3}\,.$$

$$\cdot \int_{R^N} \left(\int_{R^1} \left| s\varphi\left(\frac{y - x}{\sigma}, s\right)\right|^{3/2} ds \right)^{2/3} \left(\int_0^\omega |\bar{u}_t(y, s)|^3\, ds \right)^{1/3} dy\, d\sigma +$$

$$+ \int_\alpha^\beta \sigma^{-N-\lambda/2} \int_{R^N} \left(\int_{R^1} \left| \frac{y - x}{\sigma}\, \varphi\left(\frac{y - x}{\sigma}, s\right)\right|^2 ds \right)^{1/2} \left(\int_0^\omega \left|\nabla_y\, \bar{u}(y, s)\right|^2 ds \right)^{1/2} dy\, d\sigma\,.$$

Put $\lambda = 6/7$, $\varkappa = 4/7 - N(\frac{1}{2} - 1/p) > 0$. We use the Young inequality ([1]):

Let $v \in L^q(R^N)$, $w \in L^r(R^N)$, $1/q + 1/r \geqq 1$. Then the function z given by the formula $z(x) = \int_{R^N} v(y - x)\, w(y)\, dy$ belongs to $L^p(R^N)$, where $1/p = 1/q + 1/r - 1$, and $\|z\|_p \leqq \|v\|_q \cdot \|w\|_r$, where $\|\cdot\|_p$ denotes the norm in $L^p(R^N)$.

We put $q = 2$, $1/r = 1/p + \frac{1}{2}$, p being given. From (A.3), the Young inequality and the continuity of the prolongation operator we conclude

$$|u^\beta - u^\alpha|_{p,\infty} \leqq c|u|_z\,.$$

$$\cdot \left(\left[\int_{R^N} \left(\int_\alpha^\beta \sigma^{-N-3/7} \left(\int_{R^1} \left| s\varphi\left(\frac{y}{\sigma}, s\right)\right|^{3/2} ds \right)^{2/3} d\sigma \right)^r dy \right]^{1/r} + \right.$$

$$\left. + \left[\int_{R^N} \left(\int_\alpha^\beta \sigma^{-N-3/7} \left(\int_{R^1} \left| \frac{y}{\sigma}\, \varphi\left(\frac{y}{\sigma}, s\right)\right|^2 ds \right)^{1/2} d\sigma \right)^r dy \right]^{1/r} \right).$$

Let us write $N + 3/7 = (N + 3/7)\varrho + (N + 3/7)(1 - \varrho)$, where $\varrho = (1 - \varkappa)(r - 1)/((1 - \varkappa) r + N)$ (notice that $(N + 3/7)(r/(r - 1)) = 1 - \varkappa$, $(N + 3/7)(1 - \varrho) r = N + 1 - \varkappa$). The Hölder inequality yields

$$|u^\beta - u^\alpha|_{p,\infty} \leqq c|u|_z \cdot \int_\alpha^\beta \sigma^{\varkappa - 1} \, d\sigma \leqq c(\beta^\varkappa - \alpha^\varkappa)|u|_z .$$

We see that $\{u^\sigma\}$ is a fundamental sequence in $L^p(\Omega; C_\omega)$ as $\sigma \to 0$, therefore there exists $w \in L^p(\Omega; C_\omega)$ such that $|u^\sigma - w|_{p,\infty} \to 0$. On the other hand, $u^\sigma \to u$ in $L^2_\omega(\Omega)$, hence $u = w$. Moreover,

$$|u|_{p,\infty} \leqq c\,\sigma^\varkappa |u|_z + |u^\sigma|_{p,\infty} \leqq c(\sigma^\varkappa |u|_z + \sigma^{\varkappa - 1}|u|_{2,2}) \quad \text{for all} \quad \sigma \in (0, 1),$$

where c is independent of σ. The proof now follows immediately from the compact embedding of $Z_\omega(\Omega)$ into $L^2_\omega(\Omega)$.

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Souhrn

VYNUCENÉ PERIODICKÉ KMITY PRUŽNÉHO SYSTÉMU S PRUŽNĚ PLASTICKÝM TLUMENÍM

Pavel Krejčí

V článku je dokázána existence a odvozena nutná a postačující podmínka pro jednoznačnost časově periodického řešení rovnice $u_{tt} - \Delta_x u \pm F(u) = g(x, t)$ pro libovolnou (dostatečně hladkou) periodickou pravou stranu g, přičemž Δ_x je Laplaceův operátor vzhledem k $x \in \Omega \subset R^N$, $N \geqq 1$, a F je Išlinského hysterezní operátor. Pro $N = 2$ rovnice popisuje např. kmity pružné membrány v pružně plastickém prostředí.

Резюме

ВЫНУЖДЕННЫЕ ПЕРИОДИЧЕСКИЕ КОЛЕБАНИЯ УПРУГОЙ СИСТЕМЫ С УПРУГО-ПЛАСТИЧЕСКИМ ДЕМПФИРОВАНИЕМ

PAVEL KREJČÍ

В работе доказывается существование и находятся необходимые и достаточные условия для однозначности периодического по времени решения уравнения $u_{tt} - \Delta_x u \pm F(u) = g(x, t)$ для произвольной (достаточно гладкой) периодической правой части g, причем Δ_x обозначает оператор Лапласа относительно $x \in \Omega \subset R^N$, $N \geqq 1$, и F-гистерезисный оператор Ишлинского. Для $N = 2$ это уравнение описывает напр. колебания упругой мембраны в упруго пластической среде.

Author's address: RNDr. *Pavel Krejčí*, CSc., Matematický ústav ČSAV Žitná 25, 115 67 Praha 1.