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Mohamed N. Goria; Dana Vorlíčková

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ON THE ASYMPTOTIC PROPERTIES OF RANK STATISTICS FOR THE TWO-SAMPLE LOCATION AND SCALE PROBLEM

M. N. GORIA, DANA VORLÍČKOVÁ

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Summary. The equivalence of the symmetry of density of the distribution of observations and the score-generating functions for the location and the scale problem, respectively, is established at first. Then, it is shown that the linear rank statistics with scores generated by these functions are asymptotically independent under the hypothesis of randomness as well as under contiguous alternatives. The linear and quadratic forms of these statistics are considered for testing the two-sample location-scale problem simultaneously in the last part of the paper.

Keywords: linear rank statistics, the hypothesis of randomness, alternatives of difference in location and scale, contiguous alternatives, score generating function, an asymptotic distribution.

1. INTRODUCTION

Let $(X_1, \ldots, X_m)$ and $(X_{m+1}, \ldots, X_N)$, $N = m + n$, be two independent random samples, and suppose that for some unknown value $\theta = (\theta_1, \theta_2) \in \mathbb{R}^2$ the variable X_1 has the same absolutely continuous distribution F as $\theta_1 + X_{m+1}e^{-\theta_2}$, F having continuously differentiable density f. Various authors namely Duran et al. [3], Lepage [9], and Goria [4] have investigated the quadratic form of the linear rank statistics $S_k = \sum_{i=1}^{m} a_{kN}(R_i)$, $k = 1, 2$ with regard to the problem of testing the hypothesis $H : \theta = 0$ against the location-scale alternative $A : \theta \neq 0$, where R_i is the rank of X_i in the combined sample, S_1 is the statistic for testing the difference in location, S_2 is the statistic for the scale problem. The former case corresponds to the alternative $A_1 : \theta_1 \neq 0$, $\theta_2 = 0$, the latter one to $A_2 : \theta_1 = 0$, $\theta_2 \neq 0$. Randles and Hogg [10] showed that the statistics S_1 and S_2 are uncorrelated under the hypothesis provided the scores $a_{kN}(i)$ satisfy the following relation for $i = 1, \ldots, N$:

$$(1.1) \qquad a_{1N}(i) + a_{1N}(N - i + 1) = c \quad \text{and} \quad a_{2N}(i) - a_{2N}(N - i + 1) = 0\,.$$

The purpose of this paper is to show that the score generating functions for (H, A_1) and (H, A_2), respectively, are odd and even if and only if the corresponding pdf is symmetric about the origin. This is done in Section 2, where we also discuss the various consequences resulting from it. In Section 3 the asymptotic independence of the linear rank statistics S_1 and S_2 under H as well as under the local (Pitman type sequence) contiguous alternatives is established. The asymptotic independence of the statistics S_1 and S_2 considerably simplifies the asymptotic power computations of the linear and the quadratic form of the test statistics for the two sample location-scale problem. This is discussed in the last section.

2. SOME RESULTS ON THE SCORE GENERATING FUNCTIONS

Let f, and F^{-1}, respectively, denote the pdf and the inverse of the distribution function F such that $F^{-1}(1/2) = 0$.

Define

$$\phi_1(u, f) = -f'(F^{-1}(u))/f(F^{-1}(u)), \tag{2.1}$$

and

$$\phi_2(u, f) = -1 - F^{-1}(u) \,.\, f'(F^{-1}(u))/f(F^{-1}(u)), \quad 0 < u < 1, \tag{2.2}$$

respectively, the sgf for (H, A_1) and (H, A_2), where f' indicates the derivative of f. Then Lemma 1 characterizes the behaviour of $\phi_1(u, f)$ and $\phi_2(u, f)$.

Lemma 1. *Symmetry of pdf f about the origin is equivalent to each of the following equalities*:

$$\phi_1(u, f) = -\phi_1(1 - u, f), \tag{2.3}$$

$$\phi_2(u, f) = \phi_2(1 - u, f), \quad 0 < u < 1. \tag{2.4}$$

Proof. From the symmetry of the pdf f about the origin, we have

$$F^{-1}(u) = -F^{-1}(1 - u). \tag{2.5}$$

On substituting (2.5) in (2.1) and (2.2), it readily follows that both (2.3) and (2.4) hold.

Now let

$$\phi_1(u, f) = -\phi_1(1 - u, f), \quad 0 < u < 1,$$

that is

$$f'(F^{-1}(u))/f(F^{-1}(u)) = -f'(F^{-1}(1 - u))/f(F^{-1}(1 - u)).$$

This is equivalent to

$$\{f(F^{-1}(u))\}' = \{f(F^{-1}(1 - u)\}'. \tag{2.6}$$

On integrating (2.6) and observing that the constant of integration vanishes at $u = \frac{1}{2}$, we have

$$f(F^{-1}(u)) = f(F^{-1}(1-u))$$

or

$$\{F^{-1}(u)\}' = \{-F^{-1}(1-u)\}' . \tag{2.7}$$

On integrating (2.7), we have (2.5) as the constant of integration is again zero at $u = \frac{1}{2}$. This proves the symmetry of f.

Now suppose

$$\phi_2(u, f) = \phi_2(1-u, f), \quad 0 < u < 1 ,$$

that is

$$\{F^{-1}(u) f(F^{-1}(u))\}' = \{-F^{-1}(1-u) f(F^{-1}(1-u))\}' . \tag{2.8}$$

Then, we have

$$F^{-1}(u) f(F^{-1}(u)) = -F^{-1}(1-u) f(F^{-1}(1-u)) \tag{2.9}$$

or

$$\{\log(F^{-1}(u))\}' = \{\log(-F^{-1}(1-u))\}' ,$$

which yields

$$F^{-1}(u) = -k\, F^{-1}(1-u) . \tag{2.10}$$

By putting (2.10) in (2.9) and then letting $u = \frac{1}{2}$ in the result, we have $k = 1$. This completes the proof of Lemma 1.

Hájek & Šidák (1967, Chap. 1, p. 20) show that the function $\phi_1(u, f)$ is monotone provided the pdf f is strongly unimodal and vice versa.

Below we give several consequences resulting from Lemma 1 and thus we assume throughout this section that the pdf f is symmetric about zero.

Corollary 1. *Let $\phi_k(u, f)$, for $k = 1, 2$ be square integrable. Then* $\operatorname{cov}(\phi_1, (U, f), \phi_2(U, f)) = 0$, *where U is the uniform r.v. on the interval* $(0, 1)$.

Proof. The square integrability of $\phi_k(u, f)$'s guarantees the existence of the $\operatorname{cov}\{\phi_1(U, f), \phi_2(U, f)\}$. The statement of Corollary 1 follows by straightforward computations using (2.3) and (2.4).

Let the alternative A be defined by the density

$$h(x, \theta_1, \theta_2) = \mathrm{e}^{-\theta_2} f\{\mathrm{e}^{-\theta_2}(x - \theta_1)\} ,$$

and let $I(\theta) = (I_{ij}(\theta))$, $i, j = 1, 2$ be the information matrix, where

$$I_{ij}(\theta) = \mathrm{E}_\theta \left(\frac{\partial \log h}{\partial \theta_i} \cdot \frac{\partial \log h}{\partial \theta_j} \right) .$$

Then, we readily find that $I_{12}(\theta) = \mathrm{e}^{-\theta_2} \,.\, \mathrm{E}\{\phi_1(U, f)\, \phi_2(U, f)\} = 0$. Thus the information matrix in the present case is diagonal.

Define the scores and the approximate scores generated by $\phi_k(u, f)$, respectively, as

$$a_{kN}(i, f) = \mathrm{E}\phi_k(U_N^i, f)$$

and

$$a_{kN}(i, f) = \phi_k(\mathrm{E}U_N^i, f),$$

where U_N^i is the ith order statistic in a random sample of size N from the uniform distribution. Obviously,

$$\mathrm{E}U_N^{N-i+1} = 1 - \mathrm{E}U_N^i,$$

and

$$\mathrm{E}\phi_k(U_N^{N-i+1}, f) = \mathrm{E}\phi_k(1 - U_N^i, f).$$

These results together with (2.3) and (2.4) yield:

Corollary 2. *The scores and approximate scores satisfy the condition* (1.1) *with* $c = 0$.

Let the linear rank statistic defined by $\phi_k(u, f)$ be

$$T_k = \sum_{i=1}^{m} a_{kN}(R_i, f), \quad k = 1, 2. \tag{2.11}$$

Then, from Corollary 2, we obtain:

Corollary 3. *Linear rank statistics T_1 and T_2 are uncorrelated under H.*

Note that Corollaries 1, 2 and 3 hold in general for any two square integrable functions satisfying the conditions (2.3) and (2.4).

3. ASYMPTOTIC INDEPENDENCE OF S_1 AND S_2

Here we shall first derive the joint asymptotic distribution of S_1 and S_2 under the hypothesis as well as under the local contiguous alternative, and then show that if the scores defining these statistics satisfy the condition (1.1), then the statistics are asymptotically independent.

Since the asymptotic results deal with a sequence of situations similar to the given testing problem, let (m_ν, n_ν), $\nu = 1, 2, \ldots$ be a sequence of pairs of positive integers such that $N_\nu = m_\nu + n_\nu \to \infty$ as $\nu \to \infty$. For each ν let H_ν be the sequence of null hypotheses such that under H_ν, the joint density of $(X_{\nu 1}, \ldots, X_{\nu N_\nu})$ is given by

$$p_\nu = \prod_{i=1}^{N_\nu} f(x_i).$$

Let $R_{\nu i}$ be the rank of $X_{\nu i}$ among $(X_{\nu 1}, \ldots, X_{\nu N_\nu})$. Now we consider the linear statistics $S_{k\nu} = \sum_{i=1}^{m_\nu} a_{kN_\nu}(R_{\nu i})$, where $a_{kN_\nu}(i)$ are the scores or approximate scores generated by the functions $\phi_k(u)$, such that

$$\text{(3.1)} \qquad \int_0^1 \{\phi_k(u)\}^2 \, \mathrm{d}u < \infty \,, \quad k = 1, 2 \,.$$

Condition (3.1) guarantees the existence of the covariance matrix of the random functions $\phi_1(U)$, and $\phi_2(U)$, which we denote by $D = (d_{ij})$, $i, j = 1, 2$. Then we have the following theorem.

Theorem 1. *Suppose that* (3.1) *hold and* $\min(m_\nu, n_\nu) \to \infty$ *as* $\nu \to \infty$. *Then the random vector* $(S_{1\nu}, S_{2\nu})$, *under* H_ν, *is asymptotically bivariate normal with the expectation* $(m_\nu \bar{a}_{1\nu}, m_\nu \bar{a}_{2\nu})$ *and the covariance matrix* $(m_\nu n_\nu / N_\nu)\, D$, *where*

$$\bar{a}_{k\nu} = N_\nu^{-1}\Big(\sum_{i=1}^{N_\nu} a_{kN_\nu}(i)\Big) \,.$$

Proof. To prove the theorem, we must show that for all values of the real vector (λ_1, λ_2), as $\min(m_\nu, n_\nu) \to \infty$, the statistic

$$S_\nu = \sum_{k=1}^{2} \lambda_k (S_{k\nu} - m_\nu \bar{a}_{k\nu}) \left(\frac{m_\nu n_\nu}{N_\nu}\, \sigma_{k\nu}^2\right)^{-1/2} ,$$

where

$$\sigma_{k\nu}^2 = \sum_{i=1}^{N_\nu} \{a_{kN_\nu}(i) - \bar{a}_{k\nu}\}^2 / (N_\nu - 1) \,,$$

is asymptotically univariate normal with zero expectation and variance $\lambda_1^2 + \lambda_2^2 + 2\lambda_1 \lambda_2 \varrho$, where $\varrho = d_{12}(d_{11} d_{12})^{-1/2}$.

From Hájek & Šidák [5], Chap. 5, we find that it suffices to show that the result holds for the asymptotically equivalent statistic

$$S_\nu^* = \sum_{k=1}^{2} \lambda_k (S_{k\nu} - m_\nu \bar{a}_{k\nu}) \left(\frac{m_\nu n_\nu}{N_\nu}\, d_{kk}\right)^{-1/2} .$$

Now if we define the row vector

$$\begin{aligned} c'_{i\nu} &= \left\{\lambda_1 \left(\frac{m_\nu N_\nu d_{11}}{n_\nu}\right)^{-1/2}, \; \lambda_2 \left(\frac{m_\nu N_\nu d_{22}}{n_\nu}\right)^{-1/2}\right\}, \quad i = 1, 2, \ldots, m_\nu, \\ &= \left\{-\lambda_1 \left(\frac{n_\nu N_\nu d_{11}}{m_\nu}\right)^{-1/2}, \; -\lambda_2 \left(\frac{n_\nu N_\nu d_{22}}{m_\nu}\right)^{-1/2}\right\}, \quad i = m_\nu + 1, \ldots, N_\nu \,. \end{aligned}$$

and the row vector

$$a'_{N\nu}(R_{\nu i}) = \{a_{1N_\nu}(R_{\nu i}) \,, \; a_{2N_\nu}(R_{\nu i})\} \,, \quad i = 1, \ldots, N_\nu \,,$$

then S_ν^* can be rewritten as

$$S_\nu^* = \sum_{i=1}^{N_\nu} c'_{i\nu} a_{N\nu}(R_{\nu i}) .$$

To complete the proof, we merely need to verify the following conditions of Theorem 2.2 of Beran [1]:

$$\sum_{i=1}^{N_\nu} \|c_i\|^2 < d^2 < \infty , \tag{3.2}$$

$$\max_{1 \leq i \leq N_\nu} \|c_{i\nu}\| \to 0 , \quad \nu \to \infty . \tag{3.3}$$

Condition (3.2) is satisfied with $d^2 = \lambda_1^2/d_{11} + \lambda_2^2/d_{22}$, whereas (3.3) is equivalent to the assumption $\min(m_\nu, n_\nu) \to \infty$ as $\nu \to \infty$.

From Corollary 3 and Theorem 1, we have

Corollary 4. *Suppose $a_{k,N_\nu}(i)$'s are generated by the square integrable functions $\phi_k(u)$, $k = 1, 2$, satisfying (2.3) and (2.4). Then, as $\min(m_\nu, n_\nu) \to \infty$, the statistics $S_{1\nu}$ and $S_{2\nu}$, under H_ν, are asympotically independent normal with expectation $m_\nu \bar{a}_{k\nu}$ and variance $(m_\nu n_\nu/N_\nu)\, d_{kk}$, $k = 1, 2$.*

Corollary 5. *The lmp rank statistics $T_{1\nu}$ and $T_{2\nu}$, defined by (2.11), corresponding to the symmetric pdf f with finite Fisher information matrix $I(0)$, under H_ν, as $\min(m_\nu, n_\nu) \to \infty$, are asymptotically independent normal with zero expectation and variance $(m_\nu n_\nu/N_\nu)\, I_{kk}(0)$, $k = 1, 2$.*

Let the location-scale alternative A be defined by the density

$$h(x, \theta) = e^{-\theta(2)} \cdot f\{e^{-\theta(2)}(x - \theta(1))\} , \tag{3.4}$$

where the vector $\theta' = (\theta(1), \theta(2))$.

We assume that the sgf $\phi_k(u, f)$, $k = 1, 2$ exist and that

$$\int_0^1 \{\phi_k(u, f)\}^2 \, du < \infty , \quad k = 1, 2 , \tag{3.5}$$

holds. Let A_ν be a sequence of alternatives such that under A_ν, the random variables $(X_{\nu 1}, \ldots, X_{\nu N_\nu})$ are jointly distributed with the density

$$q_\nu = \prod_{i=1}^{m_\nu} h(x_i, \theta_{i\nu}) \prod_{i=m_\nu+1}^{N_\nu} h(x_i, 0) , \tag{3.6}$$

where $\theta'_{i\nu} = \{\theta_\nu(1), \theta_\nu(2)\} = \theta'_\nu$, $i = 1, \ldots, m_\nu$.

Let

$$\|\theta_\nu\| \to 0 , \tag{3.7}$$

$$m_\nu \|\theta_\nu\|^2 < b^2 < \infty , \tag{3.8}$$

$$m_\nu \theta'_\nu I(0)\, \theta_\nu \to \beta^2 < \infty \tag{3.9}$$

hold. Then the densities q_ν are contiguous to p_ν (see Beran [1]). Proceeding as in the proof of Theorem 1 and using Theorem 3.2 of Beran [1], we obtain the following assertion.

Theorem 2. *Let* (3.1), (3.5), (3.7)–(3.9) *hold. Then, for* $\min(m_\nu, n_\nu) \to \infty$, *the random vector* $(S_{1\nu}, S_{2\nu})$ *is, under* A_ν *given by* (3.6), *asymptotically jointly normal with the expectation*

$$m_\nu \bar{a}_{k\nu} + \frac{m_\nu n_\nu}{N_\nu}\left\{\int_0^1 \phi_k(u)\,(\theta_\nu(1)\,\phi_1(u,f) + \theta_\nu(2)\,\phi_2(u,f))\,\mathrm{d}u\right\}, \quad k = 1, 2,$$

and the covariance matrix $(m_\nu n_\nu / N_\nu)\, D$.

From Corollary 3 and Theorem 2 we have

Corollary 5. *Under the assumption of Theorem* 2, *if the functions* $\phi_k(u)$, $k = 1, 2$, *satisfy* (2.3) *and* (2.4), *the statistics* $S_{1\nu}, S_{2\nu}$ *are, under* A_ν, *asymptotically independent.*

As an example we may choose

$$\theta_\nu(1) = \left(\frac{n_\nu N_\nu}{m_\nu}\right)^{-1/2} \Delta_1\,, \quad \theta_\nu(2) = \left(\frac{m_\nu N_\nu}{n_\nu}\right)^{-1/2} \Delta_2\,. \tag{3.10}$$

It can be easily verified that, for $\min(m_\nu, n_\nu) \to \infty$ and m_ν/n_ν tending to a finite limit, the conditions (3.7)–(3.9) are satisfied.

Corollary 6. *Under* (3.1) *and* (3.5), *for* $\min(m_\nu, n_\nu) \to \infty$ *and* m_ν/n_ν *tending to a finite limit, the random vector* $(S_{1\nu}, S_{2\nu})$ *is asymptotically jointly normal under* A_ν *defined by* (3.6) *and* (3.10) *with the expectation*

$$m_\nu \bar{a}_{k\nu} + (m_\nu n_\nu/N_\nu)^{1/2}\left\{\int_0^1 \phi_k(u)\left(\frac{m_\nu}{N_\nu}\,\Delta_1 \phi_1(u,f) + \frac{n_\nu}{N_\nu}\,\Delta_2 \phi_2(u,f)\right)\mathrm{d}u\right\}, \quad k = 1, 2,$$

and the covariance matrix $(m_\nu n_\nu/N_\nu)\, D$. *If, in addition,* $\phi_k(u)$, $k = 1, 2$, *satisfy* (2.3) *and* (2.4) *the statistics* $S_{1\nu}, S_{2\nu}$ *are under* A_ν *asymptotically indepedent.*

Corollary 7. *Suppose that, in addition to the conditions of Theorem* 2, *the pdf* h *in* (3.4) *with* $\theta(1) = 0$ *is symmetric about the origin. Then, the lmp rank statistics* $T_{1\nu}$ *and* $T_{2\nu}$ *defined by* (2.11) *are, under* A_ν *given by* (3.6), *asymptotically independent and normal with expectations* $(m_\nu n_\nu/N_\nu)^{1/2}\, \theta_\nu(1)\, I_{11}(0), (m_\nu n_\nu/N_\nu)^{1/2}\, \theta_\nu(1)\,.$ $.\, I_{22}(0)$, *and variances* $m_\nu n_\nu\, I_{kk}(0)/N_\nu$, $k = 1, 2$, *where* $I_{kk}(0)$ *is the element of the Fisher's information matrix.*

4. RANK TESTS FOR THE TWO SAMPLE LOCATION-SCALE PROBLEM (H, A)

In the absence of lmp rank test for the problem (H, A), analogically to the parametric approach, the linear and quadratic form of the lmp rank test statistics T_1

and T_2 have been suggested in the literature; see Goria [4] for details. Clearly, the asymptotic power computations of these statistics are considerably simplified if the underlying pdf is symmetric about the origin. Below we discuss each of these combinations.

4.1. Linear combination of T_1 and T_2

The statistic $T = k_1 T_1 + k_2 T_2$ arises if one considers the problem of testing H against the alternative $A(\theta)$ defined by the density

$$h(x, \theta) = b(\theta) f\{b(\theta)(x - a(\theta))\},$$

such that $a(0) = 0$ and $b(0) = 1$.

It can be easily found that the sgf in this case is

$$\phi(u, f) = k_1 \phi_1(u, f) + k_2 \phi_2(u, f),$$

and the corresponding linear rank statistic T can be shown to be lmp for $(H, A(\theta))$ under the conditions of Theorem 4.8 of Hájek & Šidák (Chap. 2, p. 70–71), and its asymptotic normality under H_ν and A_ν follows from Theorems 1 and 2.

Lepage [7] considers the statistic $T^* = k_1 T_1 + T_2$ and shows that T^*, under contiguous alternatives A_ν^*, is asymptotically normal where A_ν^* is defined by the density q_ν in (3.6) such that

$$\theta'_{i\nu} = (\Delta_1 (m_\nu n_\nu / N_\nu)^{-1/2}, \quad \Delta_2 (m_\nu n_\nu / N_\nu)^{-1/2}), \quad i = 1, 2, \ldots, m_\nu,$$

and $k_1 = \Delta_1 / \Delta_2$.

Notice that Lepage's assumptions for proving the contiguity of q_ν to p_ν are not comparable to those of Beran as the densities q_ν fail to meet the requirements of Beran. The asymptotic normality of T under contiguous alternative can be established by an appropriate modification of the results of Lepage. In [8], he studied further the asymptotic efficiency of the statistics of the type T^*.

The linear statistic T has the defect that it may have negligible power in the direction far from the one chosen to maximize its power.

4.2. Quadratic form of T_1 and T_2

Let T_k^* be the standardized T_k under H and let

$$B = \begin{pmatrix} 1 & \varrho \\ \varrho & 1 \end{pmatrix}, \quad \text{where} \quad \varrho = I_{12}(0) \,.\, (I_{11}(0)\, I_{22}(0))^{-1/2},$$

then the quadratic form

$$Q = (T_1^*, T_2^*)\, B^{-1} \begin{pmatrix} T_1^* \\ T_2^* \end{pmatrix},$$

appears to be a natural choice of a test statistic for the problem (H, A). This statistic is particularly suitable if it seems likely that there may be relatively small departures from H in several respects simultaneously. Clearly it follows from Theorems 1 and 2 that Q under H_ν and A_ν is asymptotically χ_2^2 and $\chi_2^2(\delta)$, where the non-centrality parameter δ can be obtained from Q by replacing T_k^* by its expectation under q_ν in Theorem 2.

Further, the statistic Q reduces to

$$Q_1 = (T_1^*)^2 + (T_2^*)^2 \,,$$

if the pdf f is symmetric about the origin. It is the statistic Q_1, which has been the subject of research by several authors.

Lepage [9] derives its distribution under the alternative A_ν^*, and shows further that Q_1 is asymptotically most powerful maximin test for testing H_ν against the alternative $A_\nu^*(\delta)$, where $A_\nu^*(\delta)$ is defined by the densities q_ν such that the pdf f is symmetric about the origin and $\Delta_1^2 I_{11}(0) + \Delta_2^2 I_{22}(0) = \delta$.

Duran et al. [3] derive the expression for the asymptotic power efficiency of the statistics of the type Q_1 in the setting of Chernoff & Savage, while Goria [4] uses this expression and shows that it is better to use the statistic Q_1 among all statistics of this type, obtained through an arbitrary mixture of the rank statistics satisfying conditions (1.1).

Another statistic having a similar advantage as Q_1 is $Q_2 = |T_1^*| + |T_2^*|$, which surprisingly has not been investigated in the literature even though its asymptotic power can be easily evaluated under continuous alternatives A_ν. Both Q_1 and Q_2 have the disadvantage that if there is clear evidence of discrepancy from H, they by themselves give no indication of the nature of the departures. Further inspection of the data is always necessary to interpret what has been already found.

Cox & Hinkley ([2], pp. 122–123) advocate that it is more useful to use T_1^* and T_2^* separately, and, in case of significant departures from H, the statistic $\operatorname{Max}(|T_1^*|, |T_2^*|)$ can be employed using bivariate normal tables or the univariate normal in the symmetric case, to detect the most significant departure from H.

The relative performance of these tests from the asymptotic power efficiency point of view and their asymptotic power behaviour in the non-symmetric case is referred to a later publication.

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Souhrn

O ASYMPTOTICKÝCH VLASTNOSTECH POŘADOVÝCH STATISTIK PRO PROBLÉM DVOU VÝBĚRŮ LIŠÍCÍCH SE POLOHOU A MĚŘÍTKEM

M. N. GORIA, DANA VORLÍČKOVÁ

V článku je nejprve dokázána ekvivalence symetrie hustoty rozdělení a lichosti a sudosti funkce generující skóry statistik pro test rozdílu v poloze resp. měřítku. Potom je ukázáno, že lineární pořadové statistiky se skóry odpovídajícími těmto funkcím jsou asymptoticky nezávislé jak při hypotéze náhodnosti, tak při kontiguitních alternativách. V závěru jsou uvažovány kvadratické formy těchto statistik, na nichž lze založit test hypotézy náhodnosti proti alternativě rozdílu v poloze i měřítku současně.

Authors' addresses: Prof. *M. N. Goria*, Istituto di Statistica e Ricerca Operativa, Universita degli Studi di Trento, Via Verdi 26, 38100 Trento, Italy, RNDr. *Dana Vorlíčková*, CSc., katedra pravděpodobnosti a matematické statistiky MFF UK, Sokolovská 83, 186 00 Praha 8.