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RELIABILITY OF SYSTEM WITH DEPENDENT UNITS

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Availability of a system with dependent units is obtained in the case that the system fails when one of the essential units fails. The Markov model is assumed. The system considered consists of n dependent units of which $r \leqq n$ are essential. A unit is said to be essential if its failure causes the system to fail. The mean and variance of time to a system failure are given. Unit importance is also discussed.

Key Words: Essential unit, System reliability, Unit importance, Unit reliability, System (stationary) availability.

1. INTRODUCTION

In this paper we consider a complex system in which the assumption of independent unit is not realistic. Most systems consist of dependent unit where a failure of one unit is responsible for a premature failure of another unit. In this paper, it is assumed that the system consists of n dependent units with $r \leqq n$ essential units. The state-space of the system is $K = \{0, 1, 2, \ldots, n\}$. Let X_T denote the state of the system at a time T, $T = 0, t, 2t, 3t, \ldots$. The system is inspected at times $T = 0, t, 2t, \ldots$. It is assumed that the infinite sequence $\{X_T \mid T = 0, t, 2t, \ldots\}$ is a finite state Markov chain. The system is said to be in state 0 at time T if all the unit are functioning regardless of the ages of the units. The system is said to be in state $i \in K - \{0\}$ at time T if the unit i have failed during $(T - t; T)$, i.e. if the unit i is observed to be in the failed state at the inspection time T. Any failed unit is replaced immediately at the next inspection time.

We assume that changes of states of the system form a time-homogeneous Markov chain with the state-space K and with the transition probabilities p_{ij} defined by

$p_{ij} = \mathsf{P}$ (that after a failure of the unit i during an inter-inspection period, the unit j will fail during the next inter-inspection period), for all i corresponding to non-essential units and $j \in K - \{0\}$;

p_{i0} = P (that after a failure of the unit i, no failure of any unit occurs during the next inter-inspection period).

We also assume that:

(i) The probability of two or more units failing in a single inter-inspection period is negligible since t is assumed to be very small.

(ii) The reliability behaviour of the essential units depends only on the state of the system at the last inspection and not on the number of failures of non-essential units or on the age of units.

2. STATIONARY AVAILABILITY

2.1. The Model

Consider a system with r essential units and $n - r$ non-essential units. That is, the system fails only when one of the essential units fails. Any failed unit is replaced at the next inspection time. We assume, without loss of generality, that the essential units are labelled $1, 2, 3, \ldots, r$.

The transition probability matrix for the above model is $\mathsf{P}_1 = (p_{ij})_{i,j \in K}$.

The stationary probabilities are given by

$$\pi = \pi \mathsf{P}_1\,, \quad \pi \mathbf{1} = \mathbf{1}$$

where

$$\pi = \{\pi_0, \pi_1, \pi_2, \ldots, \pi_n\}\,.$$

The stationary availability of the system is given by

$$\pi_0 + \sum_{i=r+1}^{n} \pi_i\,.$$

3. EXPECTATION AND VARIANCE OF TIME TO SYSTEM FAILURE

By making the states $1, 2, \ldots, r$ absorbing states, we are in a position to find the expected value of T and the variance of T where T is the time to system failure. Thus, we have

$$\mathsf{P}_2 = (p_{ij})_{i,j \in K}\,.$$

In the canonical form, we have

$$\mathsf{P}_3 = (p_{ij})_{ij \in K} = \left[\begin{array}{c|c} \boldsymbol{I} & \mathbf{0} \\ \hline \mathbf{R} & \mathbf{Q} \end{array}\right]$$

Let us suppose that the matrix $\boldsymbol{I} - \mathbf{Q}$ is regular; the expected time to system failure is given by

$$(\boldsymbol{I} - \mathbf{Q})^{-1}\,\mathbf{1}t$$

and the variance of time to system failure is given by

$$[\boldsymbol{N}(2\boldsymbol{N}_{dg} - \boldsymbol{I}) - \boldsymbol{N}_{sq}]\,\mathbf{1}t^2\,,$$

where

$$\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} \quad (\text{see } [1])\,;$$

$$\boldsymbol{N}_{dg} = \{n_{ij}\} \quad \text{where} \quad n_{ij} = \begin{cases} n_{ij} & \text{for } j = i \\ 0 & \text{for } j \neq i \end{cases}$$

$$\boldsymbol{N}_{sq} = \{n_{ij}^2\} \quad \text{for all} \quad i, j\,.$$

4. UNIT IMPORTANCE

The Unit importance (for an essential unit) is defined as the probability that the cause of a failure of the system is not due to its failure. In the above model, let

$$B = (b_{ij}) \quad i \in K - \{1; \ldots; r\}$$
$$j \in \{1; 2; \ldots; r\}\,,$$

where

$$b_{ij} = \mathsf{P}\,(\text{that the system starting in the state } i \text{ ends up in the state } j)\,,$$

for each $i \in K - \{1; 2; \ldots; r\}$ and $j \in \{1; 2; \ldots; r\}$.

We know from [1] that $B = NR$, where N and R are defined in Section 3.

5. ILLUSTRATIVE EXAMPLE

Consider a system with 5 units having 3 essential units. Suppose the transition matrix is given by

$$\boldsymbol{P}_1 = \begin{bmatrix} \cdot8 & 0 & 0 & 0 & \cdot1 & \cdot1 \\ \cdot5 & 0 & \cdot1 & 0 & \cdot2 & \cdot2 \\ \cdot6 & \cdot1 & 0 & \cdot1 & \cdot1 & \cdot1 \\ \cdot6 & \cdot1 & \cdot1 & 0 & \cdot1 & \cdot1 \\ \cdot6 & \cdot1 & \cdot1 & \cdot1 & 0 & \cdot1 \\ \cdot5 & \cdot1 & \cdot1 & \cdot1 & \cdot2 & 0 \end{bmatrix} \qquad \boldsymbol{P}_2 = \begin{bmatrix} \cdot8 & 0 & 0 & 0 & \cdot1 & \cdot1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \cdot6 & \cdot1 & \cdot1 & \cdot1 & 0 & \cdot1 \\ \cdot5 & \cdot1 & \cdot1 & \cdot1 & \cdot2 & 0 \end{bmatrix}$$

The stationary probabilities are

$$[0{\cdot}735,\ 0{\cdot}024,\ 0{\cdot}024,\ 0{\cdot}022,\ 0{\cdot}102,\ 0{\cdot}093]$$

The stationary availability of the system = 0·93.

The expected time to system failure is given by

$$\mathbf{N}\mathbf{1}t = \begin{bmatrix} 17\cdot53 \\ 12\cdot75 \\ 12\cdot32 \end{bmatrix} t$$

and the variance of time to system failure is given by

$$[\mathbf{N}(2\mathbf{N}_{dg} - \mathbf{I}) - \mathbf{N}_{sq}]\,\mathbf{1}t^2 = \begin{bmatrix} 192\cdot63 \\ 174\cdot19 \\ 170\cdot30 \end{bmatrix} t^2$$

The units importance is given by

$$\mathbf{B} = \mathbf{N}\mathbf{R} = \begin{bmatrix} 0\cdot333 & 0\cdot333 & 0\cdot333 \\ 0\cdot333 & 0\cdot333 & 0\cdot333 \\ 0\cdot333 & 0\cdot333 & 0\cdot333 \end{bmatrix}$$

which shows that the components 1, 2, and 3 have equal importance of 0·667.

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Reference

[1] *John G. Kemeny, J. Laurie Snell:* Finite Markov Chain, D. Van Nostrand Company Inc., New York (1960).

Souhrn

SPOLEHLIVOST SYSTÉMU SE ZÁVISLÝMI PRVKY

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Zkoumaný model se skládá z n závislých prvků, z nichž $r \leqq n$ je podstatných, tj. poruchy systému nastávají jako důsledek poruch oněch podstatných prvků. Je odvozena stacionární pohotovost systému a střední hodnota a rozptyl doby do poruchy systému. Vyšetřována je rovněž spolehlivostní důležitost prvků.

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