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SYMMETRIES OF WOVEN FABRICS

BOHDAN ZELINKA

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The paper [1] develops a mathematical theory of woven fabrics. The usual diagrams of fabrics are used; such a diagram is a regular tiling of the plane by unit squares, each of them being either black or white. The fabric is theoretically considered as infinite in all directions. A vertical (or horizontal) two-way infinite sequence of squares represents a warp (or weft, respectively) strand of the fabric. Each square of the tiling belongs to one warp strand and to one weft strand. It is black (or white), if the weft (or warp) strand passes over the warp (or weft, respectively) strand.

For practical purposes it is convenient to study not all possible tilings with black and white squares, but only periodical ones; they correspond to the so-called periodical fabrics. For a periodical fabric there exists a fundamental $n \times m$ block of squares such that the whole fabric is the union of its non-overlapping copies which are obtained by translating the block in horizontal and vertical directions through multiples of n and m units, respectively.

Let $\mathscr{F}$ be a periodical woven fabric. By $U(\mathscr{F})$ we denote the group of all isometric transformations of the plane which map a strand of $\mathscr{F}$ onto a strand of $\mathscr{F}$. Evidently $U(\mathscr{F})$ is generated by the following five mappings: a horizontal unit translation, a vertical unit translation, an axial symmetry by a horizontal axis, an axial symmetry by a vertical axis, an axial symmetry by an axis forming a 45° angle with the horizontal direction. (All the axes mentioned go through the centre of a chosen square.) Let $U_1(\mathscr{F})$ (or $U_2(\mathscr{F})$) be the subgroup of $U(\mathscr{F})$ generated by the first four (or the first two) generators mentioned above.

Now let $V(\mathscr{F})$ be the group of symmetries of $\mathscr{F}$, i.e. a subgroup of $U(\mathscr{F})$ consisting of such mappings which map either every black square onto a black square and every white square onto a white square, or every black square onto a white square and every white square onto a black square. Let $V_0(\mathscr{F})$ be the subgroup of $V(\mathscr{F})$ consisting of such mappings which preserve the colours of squares. Further, let $V_1(\mathscr{F}) = V_0(\mathscr{F}) \cap$ $\cap U_1(\mathscr{F})$, $V_2(\mathscr{F}) = V_0(\mathscr{F}) \cap U_2(\mathscr{F})$.

As we consider a periodical fabric with a fundamental $n \times m$ block, the subgroup $Z(n, m)$ of $V_2(\mathscr{F})$ generated by a horizontal translation through n units and a vertical translation through m units is a normal subgroup of all groups $U_1(\mathscr{F})$, $U_2(\mathscr{F})$, $V_1(\mathscr{F})$, $V_2(\mathscr{F})$. (The unit length is chosen as the width of a strand.) We denote $W_1(\mathscr{F}) = U_1(\mathscr{F})/Z(n, m)$, $W_2(\mathscr{F}) = U_2(\mathscr{F})/Z(n, m)$, $T_1(\mathscr{F}) = V_1(\mathscr{F})/Z(n, m)$, $T_2(\mathscr{F}) = V_2(\mathscr{F})/Z(n, m)$. If $m = n$, then $Z(n, m)$ is also a normal subgroup of $U(\mathscr{F})$; then we put $W(\mathscr{F}) = U(\mathscr{F})/Z(n, m)$. The group $Z(n, m)$ is also always a normal subgroup of $V(\mathscr{F})$ and $V_0(\mathscr{F})$; we put $T(\mathscr{F}) = V(\mathscr{F})/Z(n, m)$, $T_0(\mathscr{F}) = V_0(\mathscr{F})/Z(n, m)$. We may interpret these groups as follows. Take a fundamental block of $\mathscr{F}$. Then $W_1(\mathscr{F})$ is the group of mappings of this block onto itself generated by the following four mappings: the cyclic permutation φ of the warp strands which maps any strand onto its neighbour from the right (and the last onto the first), the cyclic permutation ψ of the weft strands which maps any strand onto its neighbour from above, the axial symmetry α by the horizontal axis of the block, the axial symmetry β by the vertical axis of the block. The group $W_2(\mathscr{F})$ is generated only by φ and ψ.

If $W(\mathscr{F})$ exists, then it is generated by all these generators and, moreover, by the axial symmetry by the diagonal of the fundamental block.

Let X denote one of the symbols V, V_0, V_1, V_2. We say that a fabric $\mathscr{F}$ is X-warp-isonemal (or X-weft-isonemal), if for any two warp (or weft, respectively) strands of $\mathscr{F}$ there exists a mapping from $X(\mathscr{F})$ which maps one of them onto the other. We say that it is X-isonemal, if for any two strands of $\mathscr{F}$ there exists a mapping from $X(\mathscr{F})$ which maps one of them onto the other. Instead of "V-isonemal", "V-warp-isonemal", "V-weft-isonemal" we say shortly "isonemal", "warp-isonemal", "weft-isonemal" [1].

Each strand represents a two-way infinite sequence of black and white squares. If these sequences coincide for any two strands (or any two warp strands, or any two weft strands) of $\mathscr{F}$, we call $\mathscr{F}$ mononemal (or warp-mononemal, or weft-mononemal, respectively).

In [1] it was suggested to study the groups of symmetries of various types of woven fabrics. We shall restrict our consideration to periodical fabrics and study their groups $T(\mathscr{F})$, $T_0(\mathscr{F})$, $T_1(\mathscr{F})$, $T_2(\mathscr{F})$.

Take a fundamental block of a fabric $\mathscr{F}$ which is the least possible one (it cannot be obtained as a union of smaller fundamental blocks). Denote the warp strands from left to right by $u_1, \ldots, u_n$ and the weft strands from below to above by $v_1, \ldots, v_m$.

With help of this notation we shall define a special kind of a woven fabric called satin. A satin is a woven fabric with a square fundamental $n \times n$ block and with the property that there exists an integer $k \geqq 2$ relatively prime to n such that all squares of the block which are intersections of the strands u_i, v_j for $j \equiv ik \pmod{n}$ are black (or white), while all the others are white (or black, respectively). If a satin is a mononemal fabric, it is called a mononemal satin.

Now by $W_1'(\mathscr{F})$ (or $W_1''(\mathscr{F})$) we denote the subgroup of $W_1(\mathscr{F})$ generated by the elements φ, α (or ψ, β, respectively). It is easy to prove that each element of $W_1(\mathscr{F})$ can be uniquely expressed as the product of an element of $W_1'(\mathscr{F})$ and an element of $W_1''(\mathscr{F})$ and that each element of $W_1'(\mathscr{F})$ commutes with each element of $W_1''(\mathscr{F})$. Thus $W_1(\mathscr{F})$ is a direct product of $W_1'(\mathscr{F})$ and $W_1''(\mathscr{F})$. Put $W_2'(\mathscr{F}) = W_2(\mathscr{F}) \cap \cap W_1'(\mathscr{F})$, $W_2''(\mathscr{F}) = W_2(\mathscr{F}) \cap W_1''(\mathscr{F})$. Then again $W_2(\mathscr{F})$ is a direct product of $W_2'(\mathscr{F})$ and $W_2''(\mathscr{F})$.

Now we shall prove some theorems.

Theorem 1. *The group $T_2(\mathscr{F})$ is either a trivial group, or a cyclic group whose order is a common divisor of m and n.*

Proof. Suppose that $T_2(\mathscr{F})$ is non-trivial, i.e. that it contains at least one non-unit element. Such an element can be expressed as $\varphi^p\psi^q$, where p, q are some non-negative integers. Let the order of φ^p in $W_2'(\mathscr{F})$ be r and let the order of ψ^q in $W_2''(\mathscr{F})$ be s. If $r < s$, take the element $(\varphi^p\psi^q)^r = \psi^{qr}$; this element is not the unit element of $W_2(\mathscr{F})$. All strands $u_1, \ldots, u_n$ are fixed in this mapping and $\psi^{qr}(v_i) = v_{i+qr}$ for $i = 1, \ldots, m$ (the sum being taken modulo m). Thus a translation in the vertical direction through t units, where t is the greatest common divisor of qr and m, is a symmetry of $\mathscr{F}$ and thus there exists a fundamental block of $\mathscr{F}$ with t weft strands and $t < m$, which contradicts the assumption that the fundamental $n \times m$ block of $\mathscr{F}$ is the least possible one. Analogously for $r > s$. Thus we must have $r = s$. Now suppose that there exist integers p, q, r such that both $\varphi^p\psi^q$ and $\varphi^p\psi^r$ are in $T_2(\mathscr{F})$. Then $(\varphi^p\psi^q)^{-1} \cdot (\varphi^p\psi^r) = \psi^{r-q} \in T_2(\mathscr{F})$. This implies that either $r - q = 0$, or there exists a fundamental block of $\mathscr{F}$ with $r - q$ weft strands; then $r - q$ is divisible by m and thus $r \equiv q \pmod{m}$ and $\psi^q = \psi^r$. Hence for each φ^p there exists at most one ψ^q such that $\varphi^p\psi^q \in T_2(\mathscr{F})$ and $T_2(\mathscr{F})$ is isomorphic to the group formed by all elements φ^p for which such a ψ^q exists. This group, being a subgroup of the cyclic group $W_2'(\mathscr{F})$, is cyclic and its order is a divisor of n; the same must hold for $T_2(\mathscr{F})$. Analogously we can prove that to each ψ^q there exists at most one φ^p such that $\varphi^p\psi^q \in T_2(\mathscr{F})$ and $T_2(\mathscr{F})$ is isomorphic to the group formed by all elements ψ^q for which such a φ^p exists; this implies that the order of $T_2(\mathscr{F})$ is a divisor of m. Hence the order of $T_2(\mathscr{F})$ is a common divisor of m and n.

Theorem 2. *Let $\mathscr{F}$ be a V_2-warp-isonemal woven fabric with a fundamental $n \times m$ block. Then the order of $T_2(\mathscr{F})$ is n.*

Proof. As $\mathscr{F}$ is V_2-warp-isonemal, for any warp strand there exist n pairwise distinct mappings from $T_2(\mathscr{F})$ which map it onto all the other strands. Thus the order of $T_2(\mathscr{F})$ is at least n. As $T_2(\mathscr{F})$ is isomorphic to a subgroup of the cyclic group $W_2'(\mathscr{F})$ of the order n, its order is at most n. Hence the order of $T_2(\mathscr{F})$ is equal to n.

Theorem 2'. *Let $\mathscr{F}$ be a V_2-weft-isonemal woven fabric with a fundamental $n \times m$ block. Then the order of $T_2(\mathscr{F})$ is m.*

Proof is dual to the proof of Theorem 2.

Corollary 1. *Let $\mathscr{F}$ be a V_2-warp-isonemal woven fabric with a fundamental $n \times m$ block. Then n divides m.*

Corollary 1′. *Let $\mathscr{F}$ be a V_2-weft-isonemal woven fabric with a fundamental $n \times m$ block. Then m divides n.*

Corollary 2. *Let $\mathscr{F}$ be a woven fabric with a fundamental $n \times m$ block which is simultaneously V_2-warp-isonemal and V_2-weft-isonemal. Then $m = n$.*

An example of a fabric which is simultaneously V_2-warp-isonemal and V_2-weft-isonemal is a twill in Fig. 1. In Fig. 2 we see a fabric which is V_2-warp-isonemal, but not V_2-weft-isonemal.

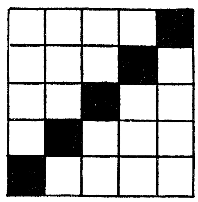

Fig. 1.

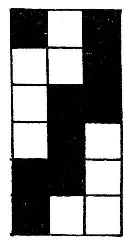

Fig. 2.

In the proof of Theorem 1 we have used the subgroup of $W_2'(\mathscr{F})$ formed by all elements φ^p for which there exists ψ^q such that $\varphi^p\psi^q \in T_2(\mathscr{F})$. Denote this group by $T_2'(\mathscr{F})$. Analogously, let $T_2''(\mathscr{F})$ be the subgroup of $W_2''(\mathscr{F})$ formed by all elements ψ^q for which there exists an element φ^p such that $\varphi^p\psi^q \in T_2(\mathscr{F})$.

We have seen that $T_2(\mathscr{F})$ is isomorphic to both $T_2'(\mathscr{F})$ and $T_2''(\mathscr{F})$. Further, if $\mathscr{F}$ is V_2-warp-isonemal (or V_2-weft-isonemal), then $T_2'(\mathscr{F}) = W_2'(\mathscr{F})$ (or $T_2''(\mathscr{F}) = W_2''(\mathscr{F})$, respectively).

We shall be interested also in the number of orbits of $T_2'(\mathscr{F})$ and of $T_2''(\mathscr{F})$. Evidently the number of orbits of $T_2'(\mathscr{F})$ (or of $T_2''(\mathscr{F})$) is equal to one if and only if $\mathscr{F}$ is V_2-warp-isonemal (or V_2-weft-isonemal, respectively).

Two warp strands in a fundamental block are called neighbouring, if they lie immediately beside each other or one is the first and the other is the last in the block. Analogously for weft strands.

Theorem 3. *Let $\mathscr{F}$ be a woven fabric which is V_1-warp-isonemal, but not V_2-warp-isonemal. Then the group $T_2'(\mathscr{F})$ has two or four orbits, all of the same cardinality. The order of $T_2(\mathscr{F})$ is $n/2$ or $n/4$.*

Proof. As $\mathscr{F}$ is not V_2-warp-isonemal, the group $T_2'(\mathscr{F})$ has at least two orbits. Let A be the orbit of $T_2'(\mathscr{F})$ which contains u_1. As $T_2'(\mathscr{F})$ is cyclic, there exists a generator of $T_2'(\mathscr{F})$. Suppose that k is the least positive integer with the property that φ^k is a generator of $T_2'(\mathscr{F})$; then there exists ψ^l such that $\varphi^k\psi^l \in T_2(\mathscr{F})$. Evidently k

divides n and $k \geqq 2$. The orbit A consists of the elements u_j such that $j \equiv 1 \pmod{k}$. As $\mathcal{F}$ is V_1-warp-isonemal, any warp strand of $\mathcal{F}$ which is not in A must be an image of u_1 in a mapping $\lambda \in T_1(\mathcal{F}) - T_2(\mathcal{F})$. As all the elements α, β, $\alpha\beta$ are of the order 2, the element λ has the form $\alpha\mu$, $\beta\mu$ or $\alpha\beta\mu$, where $\mu \in T_2(\mathcal{F})$. Let there exist two warp strands x_1, x_2 such that $x_1 = \alpha\,\mu_1(u_1)$, $x_2 = \alpha\,\mu_2(u_1)$, where μ_1, μ_2 are elements of $T_2(\mathcal{F})$. Then x_2 is an image of x_1 in the mapping $\alpha\mu_2(\alpha\mu_1)^{-1} = \alpha\mu_2\mu_1^{-1}\alpha$. As the order of α is 2, the group $T_2(\mathcal{F})$ is a normal subgroup of the subgroup of $T_1(\mathcal{F})$ generated by α and the elements of $T_2(\mathcal{F})$ and thus $\alpha\mu_2\mu_1^{-1}\alpha \in T_2(\mathcal{F})$; the elements x_1, x_2 belong to the same orbit of $T_2(\mathcal{F})$. Analogously we prove this assertion for any two elements x_1, x_2 such that $x_1 = \beta\,\mu_1(u_1)$, $x_2 = \beta\,\mu_2(u_1)$ or $x_1 = \alpha\beta\,\mu_1(u_1)$; $x_2 = \alpha\beta\,\mu_2(u_1)$, where μ_1, μ_2 are elements of $T_2(\mathcal{F})$. This implies that $T_2(\mathcal{F})$ has at most four orbits. It remains to prove that $T_2(\mathcal{F})$ cannot have three orbits. Suppose the contrary. Then u_2, u_3 are not in A and lie in different orbits of $T_2(\mathcal{F})$. If $u_2 = \alpha\,\mu_1(u_1)$, $u_3 = \beta\,\mu_2(u_1)$, then consider the element $\alpha\beta\,\mu_1(u_1)$. If it is in A, then $\alpha\beta\,\mu_1(u_1) = \mu_3(u_1)$, where μ_3 is an element of $T_2(\mathcal{F})$. This implies $u_1 = \alpha\beta\mu_1\,\mu_3^{-1}(u_1)$. From the above mentioned normality of $T_2(\mathcal{F})$ it follows that $u_2 = \mu_4\,\alpha(u_1)$ for some $\mu_4 \in T_2(\mathcal{F})$; then $u_2 = \mu_4\,\alpha(u_1) = \mu_4\beta\mu_1\mu_2^{-1}(u_1) = \beta\,\mu_5(u_1)$ for some $\mu_5 \in T_2(\mathcal{F})$ (this follows from the analogous normality of $T_2(\mathcal{F})$ in the subgroup of $T_1(\mathcal{F})$ generated by β and the elements of $T_2(\mathcal{F})$). Then u_2, u_3 are in the same orbit of $T_2(\mathcal{F})$, which is a contradiction. Analogously we obtain a contradiction in the other cases (e.g. $u_2 = \alpha\,\mu_1(u_1)$, $u_3 = \alpha\beta\,\mu_2(u_1)$). Hence the number of orbits of $T_2(\mathcal{F})$ is either 2 or 4. If it is 2 (or 4), then evidently the order of $T_2(\mathcal{F})$ is $n/2$ (or $n/4$, respectively).

Theorem 3'. *Let $\mathcal{F}$ be a woven fabric which is V_1-weft-isonemal, but not V_2-weft-isonemal. Then the group $T_2''(\mathcal{F})$ has two or four orbits, all of the same cardinality. The order of $T_2(\mathcal{F})$ is $m/2$ or $m/4$.*

Corollary 3. *Let $\mathcal{F}$ be a V_1-warp-isonemal woven fabric. Then m is a multiple of $n/4$.*

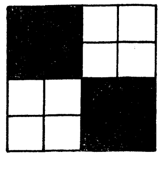

Fig. 3.

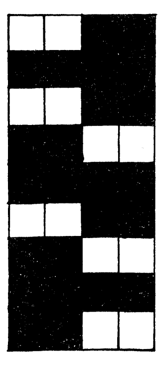

Fig. 4.

Corollary 3′. *Let $\mathcal{F}$ be a V_1-weft-isonemal woven fabric. Then n is a multiple of $m/4$.*

In Figs. 3 and 4 we see woven fabrics which are V_1-warp-isonemal, but not V_2-warp-isonemal. For the fabric in Fig. 3 the group $T_2(\mathcal{F})$ has two orbits, for the fabric in Fig. 4 it has four orbits.

Corollary 4. *Let $\mathcal{F}$ be a V_1-warp-isonemal and V_1-weft-isonemal woven fabric. Then $m/n \in \{\frac{1}{4}, \frac{1}{2}, 1, 2, 4\}$.*

The notion of V_0-warp-isonemality (or V_0-weft-isonemality) coincides with that of V_1-warp-isonemality (or V_1-weft-isonemality).

Now we turn to the V-warp-isonemality and the V-weft-isonemality.

Theorem 4. *Let $\mathcal{F}$ be a woven fabric which is V-warp-isonemal, but not V_1-warp-isonemal. Then the group $T_1(\mathcal{F})$ has exactly two orbits and the group $T_2(\mathcal{F})$ has two, four or eight orbits.*

Proof. By γ denote the interchanging of the colours black and white and let $T^*(\mathcal{F})$ be the subgroup of $T(\mathcal{F})$ generated by γ and the elements of $T_1(\mathcal{F})$. As $\mathcal{F}$ is V-warp-isonemal, for any two warp strands there exists a mapping from $T^*(\mathcal{F})$ which maps one of them onto the other. The group $T_1(\mathcal{F})$ is a normal subgroup of $T^*(\mathcal{F})$ of the index 2. Thus the proof is analogous to the proof of Theorem 3.

Theorem 4′. *Let $\mathcal{F}$ be a woven fabric which is V-weft-isonemal, but not V_1-weft-isonemal. Then the group $T_1''(\mathcal{F})$ has exactly two orbits and the group $T_2''(\mathcal{F})$ has two, four or eight orbits.*

Corollary 5. *Let $\mathcal{F}$ be a warp-isonemal fabric with a fundamental $n \times m$ block, where n is odd. Then $\mathcal{F}$ is T_2-warp-isonemal.*

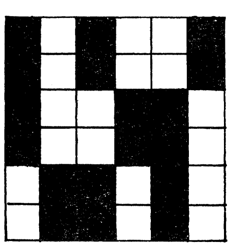

Fig. 5.

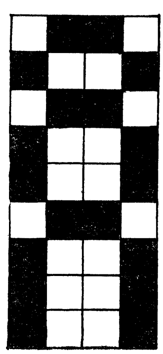

Fig. 6.

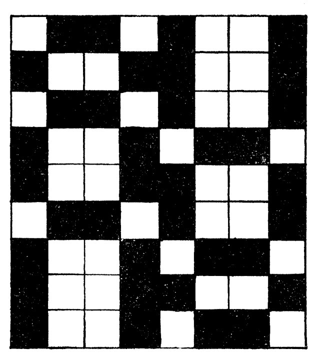

Fig. 7.

Corollary 5′. *Let $\mathscr{F}$ be a weft-isonemal fabric with a fundametal $n \times m$ block, where m is odd. Then $\mathscr{F}$ is T_2-weft-isonemal.*

In Fig. 3 we see a V_1-warp-isonemal fabric with two orbits of $T_2(\mathscr{F})$, in Fig. 4 one with four orbits. In Fig. 5 there is a V-warp-isonemal fabric with two orbits of $T_1(\mathscr{F})$ and two orbits of $T_2(\mathscr{F})$. In Fig. 6 we see a V-warp-isonemal fabric with two orbits of $T_1(\mathscr{F})$ and four orbits of $T_2(\mathscr{F})$ and in Fig. 7 with two orbits of $T_1(\mathscr{F})$ and eight orbits of $T_2(\mathscr{F})$.

Now we turn to the isonemality. If a fabric $\mathscr{F}$ is isonemal, then $m = n$ and a fundamental block can be chosen so that it is axially symmetric by one of its diagonals, i.e. there exists a mapping δ which maps $\{u_1, \ldots, u_n\}$ onto $\{v_1, \ldots, v_n\}$ and $\{v_1, \ldots, v_n\}$ onto $\{u_1, \ldots, u_n\}$ and either preserves or mutually interchanges the colours of squares and has the property that either $\delta(u_i) = v_i$, $\delta(v_i) = u_i$, or $\delta(u_i) = v_{n+1-i}$, $\delta(v_i) = u_{n+1-i}$ for $i = 1, \ldots, n$. In the sequel we shall refer to a fundamental block with these properties.

If $\delta(u_i) = v_i$, then $\psi = \delta\varphi\delta$; if $\delta(u_i) = v_{n+1-i}$, then $\psi = \delta\varphi^{-1}\delta$. (The reader may verify this himself.)

Theorem 5. *Let $\mathscr{F}$ be an isonemal and V_2-warp-isonemal woven fabric with a fundamental $n \times n$ block. Then $\mathscr{F}$ is also V_2-weft-isonemal and $T_2(\mathscr{F})$ is a cyclic group generated by the element $\varphi\delta\varphi^k\delta$, where k is such an integer that k is relatively prime to n and $k^2 \equiv 1 \pmod{n}$ or $k^2 \equiv -1 \pmod{n}$.*

Proof. The V_2-weft-isonemality is evident. There exists a mapping from $T_2(\mathscr{F})$ which maps u_1 onto u_2; this maping has (see Theorem 1) the form $\varphi\psi^k$, where k is such an integer that ψ^k has the same order as φ, namely n, hence k must be relatively prime to n. If $\delta(u_i) = v_i$ for $i = 1, \ldots, n$, then all properties of $\mathscr{F}$ are preserved, if the warp and the weft are interchanged. Thus $T_2(\mathscr{F})$ is generated also by $\varphi^k\psi$ and there exists an integer l such that $(\varphi\psi^k)^l = \varphi^k\psi$. As φ, ψ commute with each other, we have $(\varphi\psi^k)^l = \varphi^l\psi^{kl}$. As each element of $T_2(\mathscr{F})$ is uniquely determined as a product of a power of φ and a power of ψ, we have $\varphi^k = \varphi^l$ and $\psi = \psi^{kl}$. This implies $k \equiv l \pmod{n}$, $kl \equiv 1 \pmod{n}$ and hence $k^2 \equiv 1 \pmod{n}$. If $\delta(u_i) = v_{n+1-i}$ for $i = 1, \ldots, n$, then all properties of $\mathscr{F}$ are preserved, if the warp and the weft are interchanged and the ordering of the weft strands is reversed. Thus $T_2(\mathscr{F})$ is generated also by $\varphi^k\psi^{-1}$. By analogous considerations we obtain $k^2 \equiv -1 \pmod{n}$.

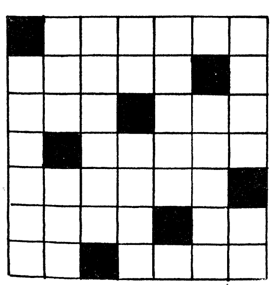

Fig. 8.

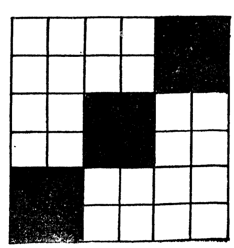

Fig. 9.

In Fig. 8 we see a V_2-isonemal fabric with $n = 8$, $k = 3$; this is a satin. In Fig. 9 there is a V_1-isonemal, but not V_2-isonemal fabric.

At the end of the paper we shall prove an assertion on 2-isonemal fabrics. A woven fabric is called 2-isonemal, if the group $V(\mathscr{F})$ has exactly two orbits. In [1] the authors propose a problem, whether there are any interesting 2-isonemal fabrics apart from the mononemal satins and those that can be obtained by "doubling" any isonemal fabric.

Theorem 6. *Let $\mathscr{F}$ be a 2-isonemal woven fabric, let A, B be the orbits of $V(\mathscr{F})$. Then one of the following four cases occurs:*

(i) *$\mathscr{F}$ is warp-isonemal and weft-isonemal, but not isonemal (Fig. 10).*

(ii) *The warp strands of $\mathscr{F}$ are ordered according to the scheme ... ABABAB ... and so are the weft strands (Fig. 11).*

(iii) *The warp strands of $\mathscr{F}$ are ordered according to the scheme ... AABBAABB .. and so are the weft strands (Fig. 12).*

(iv) *The warp strands of $\mathscr{F}$ are ordered according to the scheme ... AABAAB ... and so are the weft strands (Fig. 13).*

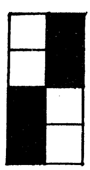

Fig. 10.

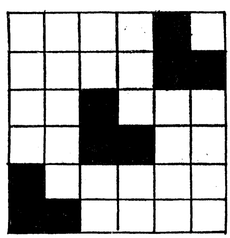

Fig. 11.

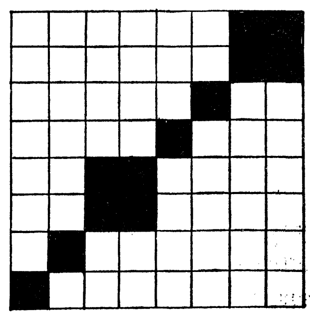

Fig. 12.

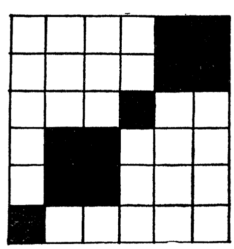

Fig. 13.

Proof. If $\mathscr{F}$ is warp-isonemal and weft-isonemal, but not isonemal, then $V(\mathscr{F})$ has two orbits, one formed by all warp strands and the other by all weft strands. In any other case, the strands of both the orbits A, B must occur among warp strands.

Thus there exists a strand of A neighbouring to a strand of B. As A and B are orbits of $V(\mathscr{F})$ and any strand can be mapped by a mapping from $V(\mathscr{F})$ onto any other strand fiom the same orbit, this implies that each strand of A is neighbouring to a strand of B and similarly each strand of B is neighbouring to a strand of A. If any warp strand is neighbouring to two strands of the other orbit, the warp strands are ordered according to the scheme ... $ABABAB$ ... If any warp strand is neighbouring to exactly one strand of the other orbit, we get the scheme ... $AABBAABB$ ... Finally, if the first assertion holds for the strands of B while the second holds for the strands of A, we have ... $AABAAB$ ... This exhausts all possible cases (the scheme ... $ABBABB$ ... is not essentially different from ... $AABAAB$ ...). As the weft strands belong also to the orbits A and B, they must be ordered according to the same scheme as the warp strands. As Figs. 10–13 show, all described cases are realizable.

Reference

[1] *B. Grünbaum, G. C. Shephard:* Satins and Twills: An Introduction to the Geometry of Fabrics. Math. Magazine 53 (1980), 139–161.

Souhrn

SYMETRIE TKANIN

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Diagramy tkanin představují rozklad roviny na jednotkové čtverce bez společných vnitřních bodů, z nichž každý je černý nebo bílý. Zkoumají se symetrie těchto diagramů.

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