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REALISATION OF RENDEZVOUS BY THE TRANSFER ORBIT WHICH IS TANGENTIAL TO THE ORIGINAL AND TERMINAL ORBITS

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The rendezvous realised by the cotangential transfer between two prescribed flightpathes is studied. General formulas based on the two bodies problem are in the conclusion applied to a numerical example calculated for the earth gravitational field.

Contents Page

§ 0. LIST OF NOTATION

Roman numerals in brackets indicate the corresponding columns of the tables in § 8.

a	semi-major axis of the ellipse (4, 1)
$a_{\mathcal{O},\mathcal{P},\mathcal{T}}$	semi-major axis of the orbit (XI), (6, 7), the parking flightpath and the trajectory
A	space ship transferred from $\mathcal{P}$ to $\mathcal{T}$ (§ 2)
b	semi-minor axis of the ellipse (4, 5)
$b_{\mathcal{O}}$	semi-minor axis of the orbit (6, 7)
B	the object passing along the final trajectory $\mathcal{T}$ (§ 2)

e	half the distance of foci of the ellipse (4, 4)
$e_{\mathcal{O},\mathcal{P},\mathcal{T}}$	half the distance of foci of the orbit (XIII), (6, 7), the parking-flightpath and the trajectory
E	eccentric anomaly of Kepler's ellipse (4, 10)
$f(\Phi_M)$	functional dependence characterising the type of the orbit in the dependence on the launch point (IX), (7, 1)
M	tangent point of the transfer orbit and the parking flightpath (§ 6)
N	tangent point of the transfer orbit and the final trajectory (§ 6)
$\mathcal{O}$	transfer orbit tangential to both the parking flightpath $\mathcal{P}$ and the final trajectory $\mathcal{T}$ (§ 2)
$\mathcal{O}_j$	transfer orbits distinguished with respect to the launch points (§ 3)
p	parameter of the ellipse (4, 1), semi-latus rectum (half the focal chord perpendicular to the axis of the curve)
$p_{\mathcal{O},\mathcal{P},\mathcal{T}}$	parameter of the orbit (X), the parking flightpath (2, 1) and the trajectory (2, 1)
$\mathcal{P}$	parking flightpath (§ 2)
r	radius vector of a point of the conical section (XVIII, XXVIII), (2, 1)
r_M	radius vector of the point M (III), (6, 2)
t	time (§ 4)
$t_{\mathcal{O},\mathcal{P},\mathcal{T}}$	(or t_i, (5, 1)) time of the periapsis passage of the orbit (§ 1), the parking flightpath (§ 2) and the trajectory (ibid.)
$t_{\mathrm{I},\mathrm{II}}$	common tangents of the conic sections $\mathcal{P}$, $\mathcal{T}$ (§ 7)
$t(\varphi)$	or $t(\varphi_a + \varphi_i)$, dependence of time on the vectorial angle (II, XXVI or XXV), (§ 3), (5, 1) ("flight schedule")
$t^*(\varphi_a)$	flight time from the periapsis to the point with the true anomaly φ_a (for the orbit XXII, XXIII), (4, 9)
T	orbital period (4, 6)
$T_{\mathcal{O},\mathcal{P},\mathcal{T}}$	or T_i, orbital period on the orbit (XIV), (6, 7), the parking flightpath and the tajectory
T_p	transfer time of the orbital flight from $\mathcal{P}$ to $\mathcal{T}$ (XXIV), (§ 6)
T_{pj}	transfer time on the orbit denoted by $\mathcal{O}_j$ (§ 3)
$\mathcal{T}$	final trajectory (§ 2)
v	orbital velocity (XXX), (§ 4)
$v_{M,M\mathrm{II}}$	orbital velocity at the point M (V), (6, 3), escape velocity at the point M (VI), (6, 5)
v_n	launch velocity realising the transfer of the object A from the parking flightpath $\mathcal{P}$ to the transfer orbit $\mathcal{O}$ (VII), (6, 4)
Δv	impulse change of velocity at the moment of the transfer from $\mathcal{P}$ to $\mathcal{O}$ (VIII), (6, 6)
ε	eccentricity of a conic section (4, 3)
$\varepsilon_{\mathcal{O},\mathcal{P},\mathcal{T}}$	eccentricity of the orbit (XIII), the parking flightpath (2, 1) and the trajectory (ibid.)

ϑ	tangent angle; the angle between the velocity vector and the zenithal direction (XX, XXIX), (§ 4)
ϑ_M	tangent angle corresponding to the point M (IV), (6, 1), (7, 1)
μ	constant of the central body; gravitational parameter (4, 1); numerically for the earth field (8, 5)
$\tau_{\mathscr{P}}$*)	or $\tau_{\mathscr{P}j}$, moment of the passage of the body A through the launch point of the parking flightpath $\mathscr{P}$ (II), (§ 2) (to the orbit $\mathscr{O}_j$, respectively (§ 3))
$\tau_{\mathscr{T}}$*)	or $\tau_{\mathscr{T}j}$, moment of rendezvous of the body A (flying along the orbit $\mathscr{O}_j$ (§ 3)) with the body B (§ 2) (reaching by the body A of the trajectory $\mathscr{T}$, respectively (§ 3))
φ	vectorial angle (polar coordinates) (2, 1), (§ 4), (I)
φ_a	true anomaly (for the orbit (XV), for the trajectory (XXI)) (4, 7), (5, 1)
$\varphi_{\mathscr{O},\mathscr{P},\mathscr{T}}$	or φ_i, vectorial angle of the periapsis of the conic section $\mathscr{O}$ (XVI), $\mathscr{P}$, $\mathscr{T}$ (2, 1); for φ_0 cf. (4, 8)
$\varphi_{\mathscr{P}j,\mathscr{T}j}$	vectorial angle of the tangent point of the orbit $\mathscr{O}_j$ with the conical sections $\mathscr{P}$, $\mathscr{T}$ (§ 3, c)
$\varphi_{\mathscr{O}j}$	sequence ($j = 1, 2, \ldots$) of vectorial angles of the tangent points of $\mathscr{P}$ with $\mathscr{O}$ (§ 6)
$\Phi_{M,N}$	vectorial angles of (variable) points M (I), (§ 6) and N (XVII), (§ 6)
$\Phi_{\mathscr{P},\mathscr{T}}$**)	vectorial angle of the (fixed) launch point (tangent point of $\mathscr{O}$ with $\mathscr{P}$) to the orbit (§ 2, α) and of the (fixed) transfer point (tangent point of $\mathscr{O}$ with $\mathscr{T}$) to the trajectory (§ 2, β)

The list does not include auxiliary quantities $\mathscr{K}$, $\mathscr{L}$, $\mathscr{M}$ (§ 6) used ad hoc only as well as the quantities appearing only in § 8.

1. FORMULATION OF THE PROBLEM

The present paper realises the suggestion which is expressed in the final sentence in [2, § 5]:

The requirement of the contact of the conic section $\mathscr{O}$ with two semi-confocal conic sections $\mathscr{P}$, $\mathscr{T}$ which appears in studying transfer flightpaths of space ships has been solved [11] from the *geometrical* view point by a grapho-analytical method which is analytically transformed to the successive *approximations.* The present paper shows that the *kinematic* approach allows to determine *exactly* the double cotangential transfer path $\mathscr{O}$. The crucial idea is to express the flight quadric in the central field in the dependence on a fixed starting point and the flight direction on $\mathscr{P}$ with a variable scalar parameter of the magnitude of the orbital velocity, which

*) Values in brackets ($\tau_{\mathscr{P},\mathscr{T}}$) or (($\tau_{\mathscr{P},\mathscr{T}}$)) denote the approximations in the iteration process (§ 3, f).

**) For values in brackets ($\Phi_{\mathscr{P},\mathscr{T}}$) or (($\Phi_{\mathscr{P},\mathscr{T}}$)) cf. the preceding footnote.

realises the transfer from $\mathscr{P}$ to $\mathscr{O}$. The requirement of contact of $\mathscr{O}$ and $\mathscr{T}$ makes it possible to express explicitly the orbital velocity and hence all the elements of the double osculating path $\mathscr{O}$.

The exact formula (6, 4) for the launch velocity in the dependence on the starting point is used to find (successive approximations, § 3) the starting point on $\mathscr{P}$ from which the object on $\mathscr{P}$ is transferred to $\mathscr{O}$ so that the moment of its arrival at the tangent point with $\mathscr{T}$ coincides with the moment when the body moving along the path $\mathscr{T}$ passes through this point. Consequently, we deal with a rendezvous realised by a double tangential passage which requires technically only two changes of the magnitude of the velocity without a change of the flight direction.

Usual terms

flightpath (*path*), *orbit*, *trajectory*

are specified in the paper so that the original *parking* flight $\mathscr{P}$ of the rocket is denoted by the *flightpath* (path), the *transfer* (*double cotangential*) curve by the *orbit* $\mathscr{O}$ and the *final* quadric by the *trajectory* $\mathscr{T}$.

Besides a rendezvour of two space ships, the practical realisation of the suggested manoeuvre may serve e.g. to launch a research probe into the head of a comet. The probe awaits on a parking flightpath in the solar field the arrival of the comet and (after determining more precisely the path of the comet) it is transferred by the launch manoeuvre into the "body" of the comet.

The procedure given in the paper may be used for the transfer between any types of non-degenerate quadrics. Nevertheless, in the paper we show the case of periodic starting pathes as well as final trajectories (ellipses), the corresponding launch orbits being not only elliptical but, as the case may be, also hyperbolical or even parabolical.

The case of coincidence of both the starting and the final ellipses, i.e., the problem of double tangential rendezvous of bodies moving along the same ellipse with a given time difference of the passage through the same positions is solved in [4].

2. MATHEMATICAL FORMULATION

Consider three non-degenerate conic sections

$$(2,1) \qquad r(1 + \varepsilon_i \cos(\varphi - \varphi_i)) = p_i\,; \quad i = \mathscr{O}, \mathscr{P}, \mathscr{T},$$

where the subscripts $\mathscr{P}$ and $\mathscr{T}$ correspond to the (starting) *parking* flightpath $\mathscr{P}$ and the final *trajectory* $\mathscr{T}$ respectively while $\mathscr{O}$ correspond to the desired transfer orbit $\mathscr{O}$. In addition to the elements p_i, ε_i, φ_i, $i = \mathscr{P}, \mathscr{T}$ we assume that the moments $t_{\mathscr{P},\mathscr{T}}$ of the passage of the launched body A on the path $\mathscr{P}$ and of the body B (which is the target of the rendezvous) on the trajectory $\mathscr{T}$ through the periapses are prescribed. Then our task is to determine:

α) the vectorial angle $\Phi_{\mathscr{P}}$ of the launch point, i.e., of the tangent point of the quadrics $\mathscr{P}$, $\mathscr{O}$, the moment $\tau_{\mathscr{P}}$ at which the body A passes through this point and the impulse change of the orbital velicity;

β) the vectorial angle Φ_T of the rendezvous point, i.e., of the tangent point of the quadrics $\mathscr{O}, \mathscr{T}$, and the moment $\tau_{\mathscr{T}}$ at which the both bodies A, B meet at this point.

Auxiliary quantities for finding the values $\Phi_{\mathscr{P},\mathscr{T}}$, $\tau_{\mathscr{P},\mathscr{T}}$ are apparently the elements $p_{\mathscr{O}}$, $\varepsilon_{\mathscr{O}}$, $\varphi_{\mathscr{O}}$, $t_{\mathscr{O}}$ which specify the shape, the location and the time of the periapsis passage of the transfer orbit $\mathscr{O}$. Here $t_{\mathscr{O}}$ may be a fictitious time in case that the periapsis of the transfer orbit does not belong to the launch section.

3. PLAN OF SOLUTION

Considering the character of our task, we cannot hope to express explicitly the quantities $\Phi_{\mathscr{P},\mathscr{T}}$, $\tau_{\mathscr{P},\mathscr{T}}$. We choose the following scheme of solution:

a) We find the *dependence of time on the vectorial angle* of the body A on the path $\mathscr{P}$ and of the body B on the tajectory $\mathscr{T}$ (the "flight schedule").

b) We discuss the sections of the parking flightpath $\mathscr{P}$ from which the cotangential transfer to the trajectory $\mathscr{T}$ is at all possible. At the same time we determine the sections of the final trajectory to which the cotangential transfer is possible.

c) In the admissible sections where the launch is possible we choose a sequence of points by their vectorial angles φ_{pj}, $j = 1, 2, \ldots$ We determine the cotangential transfer orbits $\mathscr{O}_j$, $j = 1, 2, \ldots$ whose tangent points with the path $\mathscr{P}$ are the points φ_{pj}; then we calculate the vectorial angles $\varphi_{\mathscr{T}j}$, $j = 1, 2, \ldots$ of the tangent points of $\mathscr{O}_j$ and $\mathscr{T}$ and determine the flight time T_{pj} through the section of the orbit $\mathscr{O}_j$ between the angles $\varphi_{\mathscr{P}j}$, $\varphi_{\mathscr{T}j}$. The values T_{pj}, $j = 1, 2, \ldots$ will be called the *transfer times*.

d) Adding the transfer times T_{pj} to the moments τ_{pj} of the passage of the body A through the points with vectorial angles $\varphi_{\mathscr{P}j}$ we obtain the moments $\tau_{\mathscr{T}j}$ at which the body A which is launched at he points $\varphi_{\mathscr{P}j}$ into the cotangential orbit $\mathscr{O}_j$ reaches the final trajectory $\mathscr{T}$. The diagram of the dependence of $\tau_{\mathscr{T}j}$ on $\varphi_{\mathscr{T}j}$ will be called the *transfer flight schedule* (from the parking flightpath to the final trajectory).

e) If both the *transfer flight schedule* and *the final trajectory flight schedule* are considered in the same coordinates then every point of intersection of both fligh schedules represents a possibility to realise the rendezvous of the body A with the body B. The coordinates of the points of intersection are the values Φ_T, τ_T which were found in β), § 2. We find their approximation from the graph.

f) We estimate the values (Φ_P), (τ_P) corresponding to the approximation Φ_T, τ_T from e), determine the corresponding transfer orbit $(\mathscr{O})$ and evaluate the corresponding (Φ_T), (τ_T). If the values (Φ_T), (τ_T) do not satisfy the final trajectory flight schedule with the necessary accuracy we pass from the estimate (Φ_P), (τ_P) to an estimate $((\Phi_P))$, $((\tau_P))$ etc. until we reach the desired accuracy. Hence we find the pair Φ_P, τ_P (which is to be found according to α), § 2) or the elements $p_\mathcal{O}$, $\varepsilon_\mathcal{O}$, $t_\mathcal{O}$ of the transfer orbit $\mathcal{O}$. This essentially immediate procedure is practically eliminated if a computer is used with a suitable tabulation (Cf. § 8 below.)

4. INITIAL CONDITIONS

From the number of possibilities how to prescribe the initial conditions of the flight along an ellipse we choose five values:

r ... *radius vector*
φ ... *vectorial angle*
ϑ ... *tangent angle* (i.e., the complementary or explementary angle to that between the vector of velocity and the local horizon)
v ... *orbital velocity*
t ... *the corresponding time*

If need be, we can evaluate from these quantities: the *parameter*

$$p = (rv \sin \vartheta)^2/\mu\,, \tag{4,1}$$

the *semi-major axis*

$$a = r/(2 - rv^2/\mu)\,, \tag{4,2}$$

the *eccentricity*

$$\varepsilon = \sqrt{[1 + (v^2 - 2\mu/r)(rv \sin \vartheta/\mu)^2]}\,, \tag{4,3}$$

the *half distance of foci*

$$e = a\varepsilon\,, \tag{4,4}$$

the *semi-minor axis*

$$b = a\sqrt{(1 - \varepsilon^2)} = \sqrt{(a^2 - e^2)} \tag{4,5}$$

and the *orbital period*

$$T = 2\pi ab/(rv \sin^2 \vartheta) = 2\pi\sqrt{(a^3/\mu)} = 2\pi\sqrt{\{[p/(1 - \varepsilon^2)]^3/\mu\}}\,. \tag{4,6}$$

The *true anomaly* φ_a of a point (r, φ) is found uniquely from the pair of relations

$$\begin{aligned} \varepsilon \sin \varphi_a &= v \cos \vartheta\,.\,\sqrt{(p/\mu)}\,, \\ 1 + \varepsilon \cos \varphi_a &= v \sin \vartheta\,.\,\sqrt{(p/\mu)} \end{aligned} \tag{4,7}$$

and hence the location of the *line of apsides* oriented with respect to the periapsis

$$\varphi_0 = \varphi - \varphi_a\,. \tag{4,8}$$

The *moment of the passage through the periapsis*

(4,9) $$t(\varphi) - t^*(\varphi_a)$$

is found from the given value $t(\varphi) = t$ and from the evaluated value (Kepler's equation for an ellipse)

(4,10) $$t^*(\varphi_a) = T(E - \varepsilon \sin E)/(2\pi)\,,$$

where

$$\mathrm{tg}\,(E/2) = \mathrm{tg}\,(\varphi/2) \,.\, \sqrt{[(1 - \varepsilon)/(1 + \varepsilon)]}\,.$$

In the sequel we follow the steps a) to d), § 3.

5. DEPENDENCE OF TIME ON THE VECTORIAL ANGLE

To determine the relation between the time and the vectorial angle (the flight schedules)

(5,1) $$t(\varphi) = t(\varphi_a + \varphi_i) = t^*(\varphi_a) + t_i\,; \quad i = P, T^*)$$

of the original path and the final trajectory we use Kepler's equation.

6. TRANSFER ORBITS

Let Φ_M denote an arbitrary one from the chosen values $\varphi_{\mathcal{O}j}, j = 1, 2, \ldots$ to which the starting point M corresponds on the parking flightpath $\mathcal{P}$. For the other elements at this point we find the *tangent angle* ϑ_M:

(6,1) $$\mathrm{tg}\,\vartheta_M = \mathrm{cotg}\,(\Phi_M - \varphi_{\mathcal{P}}) + [\mathrm{cosec}\,(\Phi_M - \varphi_{\mathcal{P}})]/\varepsilon_{\mathcal{P}}\,; \quad \vartheta_M \in (0, \pi)\,,$$

the *radius vector*

(6,2) $$r_M = p_{\mathcal{P}}/(1 + \varepsilon_{\mathcal{P}} \cos\,(\Phi_M - \varphi_{\mathcal{P}}))$$

and the corresponding *orbital velocity* on the parking path

(6,3) $$v_M = \sqrt{[\mu(2/r_M - 1/a_{\mathcal{P}})]}\,.$$

Hence we calculate the necessary *initial velocity* to the transfer orbit

(6,4)
$$v_n = v_{M\,\mathrm{II}} \sqrt{\frac{(p_{\mathcal{T}} - r_M(1 + \varepsilon_{\mathcal{T}} \cos\,(\varphi_{\mathcal{T}} - \Phi_M)))\, p_{\mathcal{T}}}{p_{\mathcal{T}}^2 + (\varepsilon_{\mathcal{T}}^2 - 1)\,(r_M \sin \vartheta_M)^2 - 2p_{\mathcal{T}}\varepsilon_{\mathcal{T}} r_M \sin \vartheta_M \,.\, \sin\,(\vartheta_M + \Phi_M - \varphi_{\mathcal{T}})}}$$

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*) We substitute a more or less arbitrary sequence for φ_a with respect to the desired accuracy of the graph constructed (e.g. $\varphi_a = 0°, 10°, 20°, \ldots$) without being limited by the pair or relations (4, 7) which actually express a property of the orbit.

with

(6,5) $$v_{M\mathrm{II}} = \sqrt{(2\mu/r_M)}$$

denoting the *parabolic velocity* at the launch point M, so that the necessary *instantaneous change* to the launch velocity is

(6,6) $$\Delta v = v_n - v_M .$$

The four values Φ_M, r_M, ϑ_M, v_n determine the shape as well as the location of the transfer orbit whose elements

(6,7) $$p_{\mathcal{C}},\ a_{\mathcal{C}},\ \varepsilon_{\mathcal{C}},\ e_{\mathcal{C}},\ b_{\mathcal{C}},\ T_{\mathcal{C}},\ \varphi_{\mathcal{C}}$$

are found from the formulas (4,1) to (4,8).

The vectorial angle Φ_N of the point N of the rendezvous of the launched body with the object moving along the final trajectory is given uniquely by the relation

(6,8) $$\operatorname{tg}(\Phi_N/2) = \mathcal{K}/(\mathcal{L} + \mathcal{M})$$

where

$$\mathcal{K} = p_{\mathcal{C}}\varepsilon_{\mathcal{T}} \sin \varphi_{\mathcal{T}} - p_{\mathcal{T}}\varepsilon_{\mathcal{C}} \sin \varphi_{\mathcal{C}} ,$$

$$\mathcal{L} = p_{\mathcal{C}}\varepsilon_{\mathcal{T}} \cos \varphi_{\mathcal{T}} - p_{\mathcal{T}}\varepsilon_{\mathcal{C}} \cos \varphi_{\mathcal{C}} ,$$

$$\mathcal{M} = p_{\mathcal{T}} - p_{\mathcal{C}} .$$

The time of flight through the transfer section (*transfer time* T_p) is found by means of double use of Kepler's equation (or, as the case may be, its modifications for hyperbolic or parabolic paths) or from Lambert's equation.

7. LAUNCHING DOMAINS (KINEMATIC APPROACH)

The presented formula for the launching velocity (6,4) allows the kinematic approach to the problem of transfer orbits:

A real launching velocity and, consequently, also a real transfer orbit exists if and only if the fraction in (6,4) is non-negative. If its value is zero, the launching section degenerates to a point and the notion of contact becomes senseless. For given final trajectory and parking path, the expression under the sign of square root is a function of the vectorial angle Φ_M

(7,1)
$$f(\Phi_M) = \frac{p_{\mathcal{T}} - p_{\mathcal{P}}[1 + \varepsilon_{\mathcal{T}} \cos(\Phi_M - \varphi_{\mathcal{T}})]/[1 + \varepsilon_{\mathcal{P}} \cos(\Phi_M - \varphi_{\mathcal{P}})]}{p_{\mathcal{T}} + \dfrac{\varepsilon_{\mathcal{T}}^2 - 1}{p_{\mathcal{T}}}\left(\dfrac{p_{\mathcal{P}} \sin \vartheta_M}{1 + \varepsilon_{\mathcal{P}} \cos(\Phi_M - \varphi_{\mathcal{P}})}\right)^2 - \dfrac{2\varepsilon_{\mathcal{T}} p_{\mathcal{P}} \sin \vartheta_M \sin(\vartheta_M + \Phi_M - \varphi_{\mathcal{T}})}{1 + \varepsilon_{\mathcal{P}} \cos(\Phi_M - \varphi_{\mathcal{P}})}},$$

where ϑ_M is the tangent angle given uniguely (6,1) by the vectorial angle Φ_M. Zero launch velocity $v_n = 0$ – which appears for $f = 0$ – evidently corresponds to the intersections of the parking flighpath with the final trajectory. The infinite value of the launch velocity $v_n \to \infty$ means $f \to \infty$ and hence zero value of the denominator in f and points to a (fictitious) straight-line orbital flight, i.e., to the launch point at the tangent point of common tangents $t_{I,II}$ with the parking path. The behaviour of $f(\Phi_M)$ yields the following launching possibilities:

If $f > 0$ holds for all $\Phi_M \in \langle 0, 2\pi \rangle$ ($\mathscr{P}$ and $\mathscr{T}$ have no common points), then the double osculating transfer is possible from any point of the parking flightpath.

If f changes its sign according to the value of Φ_M then there are four intervals of the vectorial angles Φ_M with the following properties:

It is $f \leqq 0$ in two of them, f being strictly decreasing in one of the intervals from zero under all bounds while in the other it is strictly increasing from minus infinity to zero. Neither of the both intervals allows the cotangential transfer. In the other two intervals, $f \geqq 0$. In one of them, f increases from zero and comes back to zero again (the domain of the inner launching) while in the other f decreases from plus infinity and then increases to plus infinity again, remaining positive all the time (the domain of the outer launching).

This describes all the possibilities of launching on the parking flightpath. If we wanted to describe from the kinematic view point also the domain of rendezvous on the final trajectory, it would be sufficient to repeat formally our argument replacing the parking path by the final trajectory and the final trajectory by the parking path, i.e., interchanging the subscripts $\mathscr{P}$, $\mathscr{T}$ in the relation (1).

In the conclusion of this section let us observe that for a positive value of f, the inequality $f(\Phi_M) < 1, > 1$ and $= 1$ indicates an *elliptical*, *hyperbolical* and *parabolical* orbital flight, respectively.

8. NUMERICAL EXAMPLE

In the end we shall present a numerical application of the method to a concrete case of an elliptical parking path $\mathscr{P}$ which intersects the elliptical final trajectory $\mathscr{T}$. We shall consider the same pair of ellipses as in [2]

$$a_{\mathscr{P}} = 14\,000{\cdot}0 \text{ km}, \qquad a_{\mathscr{T}} = 12\,000{\cdot}0 \text{ km},$$
$$b_{\mathscr{P}} = 12\,124{\cdot}4 \text{ km}, \qquad b_{\mathscr{T}} = 11\,313{\cdot}7 \text{ km},$$
$$p_{\mathscr{P}} = 10\,500{\cdot}0 \text{ km}, \qquad p_{\mathscr{T}} = 10\,666{\cdot}7 \text{ km},$$
$$e_{\mathscr{P}} = 7\,000{\cdot}0 \text{ km}, \qquad e_{\mathscr{T}} = 4\,000{\cdot}0 \text{ km},$$
$$\varepsilon_{\mathscr{P}} = 0{,}50000, \qquad \varepsilon_{\mathscr{T}} = 0{,}33333,$$
$$\varphi_{\mathscr{P}} = 205°00'00'', \qquad \varphi_{\mathscr{T}} = 0°00'00''$$

with the additional choice of the moments of passage through the periapsis

$$t_{\mathscr{P}} = -4^h08^m56{\cdot}2^s\,, \qquad t_{\mathscr{T}} = 0^h00^m00{\cdot}0^s\,.$$

The following possibilities of the cotangential transfer between both ellipses were found in [2]:

The domain of launching with outer contact on the parking flightpath is the interval of vectorial angles

(8,1) $$(-49°47'10'', \; 80°02'21'')$$

which allows launching to the final trajectory in the domain

(8,2) $$(128°11'28'', \; 262°04'04'')\,.$$

The inner contact is possible on the parking path in the interval

(8,3) $$(106°14'28'', \; 286°14'26'')$$

with the possible contact on the final trajectory in the range

(8,4) $$(-73°45'34'', \; 106°14'28'')\,.$$

The kinematic interpretation in the earth field with the gravitational parameter [10]*)

(8,5) $$\mu = \varkappa m = 3{,}986032\,.\,10^5\ \mathrm{km}^3\ \mathrm{s}^{-2}$$

yields for the corresponding orbital periods

(8,6) $$T_{\mathscr{P}} = 4^h34^m45{\cdot}6^s\,, \quad T_{\mathscr{T}} = 3^h38^m02{\cdot}2^s\,.$$

Text to Table 1 (see next page)

Relation of the parking path $\mathscr{P}$ and the transfer orbits $\mathscr{O}$. The first and the last columns of the table indicate the index j of the vectorial angle (§ 3, c) $\varphi_{\mathscr{P}j}$. It distinguishes the positions on the parking path (starting points M from § 6) to which the particular lines of all five tables refer.

Col. I: vectorial angle Φ_M (the beginning of § 6)
II: flight schedule $\tau_{\mathscr{O}}$ (5,1)
III: radius vector $r_{\mathscr{P}\mathscr{O}}$ (6,2)
IV: tangent angle ϑ (6,1)
V: orbital velocity v (6,3)
VI: escape velocity v_{II} (6,5)
VII: launch velocity v_n (6,4)
VIII: instantaneous change of velocity $\Delta v_{\mathscr{P}}$ (6,6) realising the start from the parking path $\mathscr{P}$ to the transfer orbit $\mathscr{O}$, i.e., the difference of values from Cols. VII and V. The tabulated values arise by rounding-up; e.g., for $j = 1$ the difference $4774{\cdot}363 - 3612{\cdot}107 = 1162{\cdot}256$ is rounded-up in the table to $4774{\cdot}36 - 3612{\cdot}11 \approx 1162{\cdot}26$, contrary to $1162{\cdot}25$ which follows from the tabulated values.

*) We use the value 58 (8) (Clarce V. C., Kaula W. M., Kozai Y.).

Table 1

j	I Φ_M			II $\tau_{\mathbb{C}}$			III $r_{\mathscr{P}\mathbb{C}}$	IV ϑ			V v	VI v_{II}	VII v_n	VIII $\Delta v_{\mathscr{P}}$	j
	°	′	″	h	m	s	km	°	′	″	m/s	m/s	m/s	m/s	
1	0			−2	38	15·35	19 201·02	68	52	21·94	3 612·11	6 443·52	4 774·36	1 162·26	1
2	20			−2	01	26·82	20 920·39	85	2	15·81	3 104·03	6 173·06	4 054·91	950·88	2
3	40			−1	22	28·28	20 308·02	104	3	6·89	3 283·91	6 265·44	4 450·54	1 166·63	3
4	60			−0	49	26·78	17 783·83	115	54	26·50	4 044·25	6 695·34	6 291·02	2 246·77	4
5	120			+0	01	20·32	10 061·54	115	30	54·13	7 124·70	8 901·29	4 536·64	−2 588·06	5
6	140			+0	09	9·82	8 668·31	110	30	39·43	7 968·46	9 589·99	6 139·55	−1 828·91	6
7	160			+0	15	11·01	7 757·36	104	38	19·70	8 619·52	10 137·44	7 077·47	−1 542 05	7
8	180			+0	20	11·67	7 225·66	98	16	25·05	9 047·55	10 503·80	7 617·14	−1 430·41	8
9	200			+0	24	43·13	7 008·89	91	39	58·30	9 234·20	10 665·00	7 809·16	−1 425·05	9
10	220			−4	05	36·39	7 080·42	85	0	45·96	9 171·77	10 610·99	7 652·28	−1 519·50	10
11	240			−4	00	53·61	7 449·05	78	29	59·06	8 862·82	10 345·11	7 105·94	−1 756·88	11
12	260			−3	55	27·37	8 159·85	72	20	38·69	8 320·28	9 884·26	6 018·80	−2 301·48	12
13	280			−3	48	39·49	9 296·89	66	50	50·19	7 568·23	9 260·12	3 484·32	−4 083·91	13
14	340			−3	06	41·88	16 242·64	61	19	29·82	4 539·76	7 005·79	6 463·04	1 923·28	14
15	13	26	37	−2	14	10·09	20 582·75	78	53	28·32	3 203·14	6 223·49	4 179·61	976·47	15
16	13	26	38	−2	14	10·06	20 582·77	78	53	29·20	3 203·14	6 223·48	4 179·60	976·47	16
17	13	26	39	−2	14	10·03	20 582·79	78	53	30·09	3 203·13	6 223·48	4 179·60	976·47	17
18	13	26	40	−2	14	10·00	20 582·81	78	53	30·98	3 203·13	6 223·48	4 179·59	976·46	18
19	13	26	41	−2	14	9·97	20 582·83	78	53	31·87	3 203·12	6 223·47	4 179·58	976·46	19

Table 2

j	IX f	X p	XI a	XII ε	XIII e	XIV T			j
	1	km	km	1	km	h	m	s	
1	0·5490150	18 344·21	21 287·86	0·3718583	7 916·07	8	35	10·61	1
2	4314804	17 918·40	18 399·01	1616214	2 973·67	6	53	57·11	2
3	5045719	19 285·56	20 495·43	2429629	4 979·63	8	6	40·81	3
4	8828689	25 407·12	75 914·22	8156703	61 920·98	57	49	18·52	4
5	2597543	4 257·21	6 796·08	6112110	4 153·84	1	32	55·67	5
6	4098607	6 233·24	7 344·29	3889484	2 856·55	1	44	23·75	6
7	4874147	7 079·11	7 566·90	2538954	1 921·20	1	49	10·68	7
8	5258855	7 442·37	7 620·17	1527486	1 163·97	1	50	19·97	8
9	5361505	7 509·29	7 555·13	0779004	588·55	1	48	55·41	9
10	5200794	7 309·10	7 376·66	0956976	705·93	1	45	5·20	10
11	4718170	6 749·77	7 051·58	2068821	1 458·84	1	38	13·04	11
12	3707923	5 494·56	6 484·23	3906743	2 533·22	1	26	36·33	12
13	1415800	2 225·54	5 415·12	7674721	4 155·95	1	6	5·72	13
14	8510597	21 281·27	54 527·35	7808417	42 577·23	35	11	55·87	14
15	4510289	17 877·57	18 746·66	2153123	4 036·38	7	5	44·37	15
16	4510276	17 877·57	18 746·63	2153096	4 036·33	7	5	44·32	16
17	4510264	17 877·57	18 746·61	2153068	4 036·27	7	5	44·27	17
18	4510252	17 877·57	18 746·58	2153042	4 036·22	7	5	44·22	18
19	0·4510239	17 877·57	18 746·56	0·2153015	4 036·16	7	5	44·17	19

Transfer orbits $\mathcal{O}$ in the dependence on the launch point (the shape of the orbit, without the localisation)

Col. IX: the dependence $f(\Phi_M)$ (7,1) whose value determines the type of the orbit (the end of § 7)
X: parameter p (4,1)
XI: semi-major axis a (4,2)
XII: eccentricity ε (4,3)
XIII: the half distance of foci e (4,4)
XIV: orbital period T (4,6)

Text to Table 3 (see next page)

Transfer orbits $\mathcal{O}$ in the dependence on the launch point (relation to the parking path $\mathcal{P}$ and the localisation)

Col. XV: true anomaly $\varphi_{a\mathcal{O}}$ of the launch point with respect to the orbit $\mathcal{O}$ (4,7)
XVI: vectorial angle $\varphi_{\mathcal{O}}$ of the line of apsides (4,8)
XVII: vectorial angle Φ_N of the tangent point of the transfer orbit with the final trajectory $\mathcal{T}$ (6,8)
XVIII: radius vector $r_{\mathcal{O}\mathcal{T}}$ of the tangent point of the transfer orbit $\mathcal{O}$ with the final trajectory $\mathcal{T}$ evaluated from the equation of the orbit $\mathcal{O}$; the relation (2,1) for i $= \mathcal{O}$
XIX: the orbital velocity $v_{\mathcal{O}\mathcal{T}}$ at the tangent point with the trajectory $\mathcal{T}$; according to (6,3) $v_{\mathcal{O}\mathcal{T}} = \sqrt{[\mu(2/r_{\mathcal{O}\mathcal{T}} - 1/a_{\mathcal{O}})]}$ with $a_{\mathcal{O}}$ from Col. XI
XX: tangent angle $\vartheta_{\mathcal{O}\mathcal{T}} \in (0, \pi)$ at the tangent point with the final trajectory $\mathcal{T}$ evaluated from the equation of the orbit $\mathcal{O}$; according to (6,1) $\text{tg}\, \vartheta_{\mathcal{O}\mathcal{T}} = \text{cotg}\,(\Phi_N - \varphi_{\mathcal{O}}) + (\text{cosec}\,(\Phi_N - \varphi_{\mathcal{O}}))/\varepsilon_{\mathcal{O}}$; $\vartheta_{\mathcal{O}\mathcal{T}} \in (0, \pi)$
XXI: true anomaly $\varphi_{a\mathcal{T}}$ of the tangent poin of the transfer orbit $\mathcal{O}$ with the final trajectory $\mathcal{T}$ measured on the orbit $\varphi_{a\mathcal{T}} = \Phi_N - \varphi_{\mathcal{O}}$ (+360°). When compared with the difference of values in Cols. XVII, XVI see the note to Col. VIII.

Table 3

j	XV $\varphi_{a\mathbb{C}}$			XVI $\varphi_{\mathbb{C}}$			XVII Φ_N			XVIII $r_{\mathbb{C}\mathcal{T}}$	XIX $v_{\mathbb{C}\mathcal{T}}$	XX $\vartheta_{\mathbb{C}\mathcal{T}}$			XXI $\varphi_{a\mathcal{T}}$			j
	°	′	″	°	′	″	°	′	″	km	m/s	°	′	″	°	′	″	
1	96	53	31·59	263	6	28·41	210	51	27·26	14 942·46	5 884·50	103	28	7·43	307	44	58·85	1
2	152	36	17·14	227	23	42·86	190	4	39·22	15 877·57	5 342·77	94	57	43·28	322	40	56·36	2
3	258	2	25·19	141	57	34·81	169	19	18·78	15 862·71	5 550·52	84	45	7·63	27	21	43·98	3
4	301	42	15·64	118	17	44·36	148	41	11·11	14 913·84	6 942·87	76	23	2·74	30	23	26·75	4
5	199	17	38·10	280	42	21·89	88	8	27·81	10 552·60	4 110·24	71	45	29·13	167	26	5·92	5
6	223	45	35·33	276	14	24·67	68	28	24·12	9 504·23	5 441·07	74	33	18·38	152	13	59·45	6
7	249	51	25·07	270	8	34·94	49	0	55·29	8 753·11	6 196·75	78	19	59·37	138	52	20·35	7
8	281	19	24·21	258	40	35·79	29	42	6·43	8 271·72	6 638·40	82	42	5·01	131	1	30·65	8
9	336	25	2·96	223	34	57·04	10	27	10·97	8 033·36	6 817·46	87	23	31·88	146	52	13·93	9
10	70	16	31·46	149	43	28·54	351	10	59·85	8 023·73	6 732·04	92	12	3·27	201	27	31·32	10
11	116	59	6·60	123	0	53·40	331	48	31·29	8 244·53	6 337·86	96	56	22·39	208	47	37·89	11
12	146	43	41·93	113	16	18·07	312	15	20·21	8 713·58	5 478·81	101	23	41·55	198	59	2·14	12
13	172	20	5·73	107	39	54·26	292	28	7·20	9 461·39	3 263·38	105	16	53·48	184	48	12·94	13
14	66	35	31·11	273	24	28·89	231	33	45·76	13 454·79	7 206·98	108	13	44·05	318	9	16·87	14
15	127	37	9·41	245	49	27·59	196	53	30·64	15 662·18	5 444·03	98	5	38·64	311	4	3·05	15
16	127	37	12·48	245	49	25·52	196	53	29·66	15 662·19	5 444·03	98	5	38·17	311	4	4·14	16
17	127	37	15·54	245	49	23·45	196	53	28·80	15 662·20	5 444·02	98	5	37·68	311	4	5·34	17
18	127	37	18·60	245	49	21·40	196	53	27·74	15 662·21	5 444·01	98	5	37·23	311	4	6·35	18
19	127	37	21·68	245	49	19·32	196	53	26·66	15 662·22	5 444·01	98	5	36·78	311	4	7·34	19

Table 4

j	XXII $t_{\mathcal{OP}}$			XXIII $t_{\mathcal{OT}}$			XXIV T_p			XXV $\tau_{\mathcal{OT}}$			XXVI $\tau_{\mathcal{TO}}$			XXVII $T_{\mathcal{OT}}$			j
	h	m	s	h	m	s	h	m	s	h	m	s	h	m	s	h	m	s	
1	1	17	30·56	8	0	51·63	6	43	21·07	4	05	5·72	2	22	40·84	1	42	24·88	1
2	2	44	31·03	6	22	47·10	3	38	16·07	1	37	49·25	2	0	28·12	−0	22	38·87	2
3	6	26	43·91	0	22	10·92	2	2	7·82	0	39	39·54	1	36	53·61	−0	57	14·07	3
4	57	10	15·95	0	17	57·48	0	57	0·05	0	07	23·27	1	14	54·18	−1	07	30·91	4
5	1	1	54·31	0	36	5·05	1	7	6·41	1	08	26·73	0	30	52·63	0	37	34·10	5
6	1	16	0·94	0	36	6·92	1	4	29·73	1	13	39·55	0	22	5·30	0	51	34·25	6
7	1	24	32·88	0	35	23·25	1	0	1·05	1	15	12·06	0	14	53·24	1	00	18·82	7
8	1	31	20·70	0	35	47·58	0	54	46·85	1	14	57·52	0	8	40·40	1	06	17·12	8
9	1	42	48·80	0	42	53·20	0	48	59·81	1	13	42·94	0	2	59·78	1	10	43·16	9
10	0	17	34·56	1	0	3·68	0	42	29·12	−3	23	7·27	3	35	31·10	−6	58	38·37	10
11	0	25	45·37	1	0	33·45	0	34	48·08	−3	26	5·53	3	29	49·61	−6	55	55·14	11
12	0	27	34·51	0	52	40·84	0	25	6·33	−3	30	21·04	3	23	35·14	−6	53	56·18	12
13	0	26	19·28	0	37	18·74	0	10	59·46	−3	37	40·03	3	16	19·81	−6	53	59·84	13
14	0	37	23·08	34	51	27·75	34	14	4·67	31	07	22·79	2	41	23·38	28	25	59·41	14
15	2	5	26·45	6	27	38·89	4	22	12·44	2	8	2·35	2	8	2·20	0	0	0·15	15
16	2	5	26·54	6	27	38·84	4	22	12·30	2	8	2·24	2	8	2·18	0	0	0·06	16
17	2	5	26·62	6	27	38·80	4	22	12·18	2	8	2·15	2	8	2·17	−0	0	0·02	17
18	2	5	26·70	6	27	38·76	4	22	12·06	2	8	2·06	2	8	2·15	−0	0	0·09	18
19	2	5	26·78	6	27	38·71	4	22	11·96	2	8	1·99	2	8	2·13	−0	0	0·14	19

Time conditions of the rendezvous of the bodies on the transfer orbit $\mathcal{O}$ and on the final trajectory $\mathcal{T}$

Col. XXII: time of (fictitious) flight $t_{\mathcal{OP}}$ from the periapsis of the launch orbit $\mathcal{O}$ to the launch point on the parking path $\mathcal{P}$

XXIII: time of fictitious flight $t_{\mathcal{OT}}$ from the periapsis of the transfer orbit $\mathcal{O}$ to the tangent point with the final trajectory $\mathcal{T}$

XXIV: transfer time T_p; i.e., the difference of values from Cols. XXIII and XXII, possibly adding ($j = 3$ to 9) the orbital periods from Col. XIV

XXV: moments $\tau_{\mathcal{OT}}$ at which the body moving along the transfer orbit $\mathcal{O}$ reaches the final trajectory $\mathcal{T}$, i.e. the sum of values from Cols. II, XXIV

XXVI: moments $\tau_{\mathcal{TO}}$ at which the body moving along the final trajectory $\mathcal{T}$ reaches the tangent points with the transfer orbits $\mathcal{O}$

XXVII: time $T_{\mathcal{OT}}$ between the passages of the body from the transfer orbit $\mathcal{O}$ and the body moving along the final trajectory $\mathcal{T}$ through the tangent point of $\mathcal{O}$ and $\mathcal{T}$; i.e., the difference of values from Cols. XXV and XXVI

Table 5

j	XXVIII $r_{\mathcal{T}\mathcal{O}}$	XXIX $\vartheta_{\mathcal{T}\mathcal{O}}$			XXX $v_{\mathcal{T}\mathcal{O}}$	XXXI $\Delta v_{\mathcal{O}}$	XXXII $r_{\mathcal{O}\mathcal{T}} - r_{\mathcal{T}\mathcal{O}}$	XXXIII $\vartheta_{\mathcal{O}\mathcal{T}} - \vartheta_{\mathcal{T}\mathcal{O}}$	j
	km	°	′	″	m/s	m/s	m	″	
1	14 942·46	103	28	7·40	4 487·19	−1 397·31	−0·183	0·03	1
2	15 877·57	94	57	43·28	4 122·22	−1 220·56	061	00	2
3	15 862·71	84	45	7·51	4 127·92	−1 422·60	977	12	3
4	14 913·84	76	23	2·72	4 498·58	−2 444·29	488	02	4
5	10 552·59	71	45	29·13	6 506·08	2 395·84	183	00	5
6	9 504·23	74	33	18·38	7 117·74	1 676·67	244	00	6
7	8 753·11	78	19	59·37	7 606·58	1 409·83	006	00	7
8	8 271·72	82	42	5·01	7 947·35	1 308·95	122	00	8
9	8 033·36	87	23	31·88	8 125·28	1 307·81	427	00	9
10	8 023·73	92	12	3·28	8 132·60	1 400·56	214	− 01	10
11	8 244·53	96	56	22·40	7 967·33	1 629·47	006	− 01	11
12	8 713·58	101	23	41·57	7 633·69	2 154·88	− 006	− 02	12
13	9 461·39	105	16	53·58	7 144·37	3 880·99	000	− 09	13
14	13 454·79	108	13	44·06	5 102·33	−2 104·65	793	− 01	14
15	15 662·18	98	5	38·56	4 205·14	−1 238·89	− 732	08	15
16	15 662·19	98	5	38·12	4 205·13	−1 238·89	− 305	04	16
17	15 662·20	98	5	37·75	4 205·13	−1 238·89	977	− 07	17
18	15 662·21	98	5	37·28	4 205·12	−1 238·89	732	− 05	18
19	15 662·22	98	5	36·81	4 205·12	−1 238·89	0·549	−0·03	19

Relation of the transfer orbit $\mathcal{O}$ and the final trajectory $\mathcal{T}$

Col. XXVIII: radius vector $r_{\mathcal{T}\mathcal{O}}$ of the tangent point of the transfer orbit $\mathcal{O}$ with the final trajectory $\mathcal{T}$ evaluated from the equation of the trajectory $\mathcal{T}$; relation (2,1) for $i = \mathcal{T}$

XXIX: tangent angle $\vartheta_{\mathcal{O}\mathcal{T}}$ at the tangent point with the final trajectory $\mathcal{T}$ evaluated from the equation of the trajectory $\mathcal{T}$; according to (6,1) $\operatorname{tg} \vartheta_{\mathcal{T}\mathcal{O}} = \operatorname{cotg} (\Phi_N - \varphi_{\mathcal{T}}) + (\operatorname{cosec} (\Phi_N - \varphi_{\mathcal{T}}))/\varepsilon_{\mathcal{T}}$; $\vartheta_{\mathcal{T}\mathcal{O}} \in (0\ \pi)$

XXX: orbital velocity $v_{\mathcal{T}\mathcal{O}}$ on the final trajectory $\mathcal{T}$ at the tangent point with the transfer orbit $\mathcal{O}$; acording to (6,3) $v_{\mathcal{T}\mathcal{O}} = \sqrt{[\mu(2/r_{\mathcal{T}\mathcal{O}} - 1/a_{\mathcal{T}})]}$

XXXI: instantaneous change of velocity $\Delta v_{\mathcal{O}}$ realising the insertion from the transfer orbit $\mathcal{O}$ to the final trajectory $\mathcal{T}$; i.e., the difference of the values from Cols. XXX and XIX with the same note as in Col. VIII

XXXII: $r_{\mathcal{O}\mathcal{T}} - r_{\mathcal{T}\mathcal{O}}$ (the difference of values from Cols. XVIII and XXVIII), i.e., the difference of two numerical values obtained for the same quantity. This is a test of accuracy. To obtain non-zero differences, it was necessary to use more digits compared with Cols. XVIII, XXVIII; note also the change from *kilometers* to *meters*

XXXIII: $\vartheta_{\mathcal{O}\mathcal{T}} - \vartheta_{\mathcal{T}\mathcal{O}}$ (the difference of values from Cols. XX and XXIX), again a test of accuracy as in the preceding column. Again the note to Col. VIII is to be considered.

The numerical solution for chosen bodies is presented in Tables 1 to 5, the results being illustrated by two figures. The author expresses his gratitude to Ing M. Volf for the program (FORTRAN) and calculation (MINSK 22).

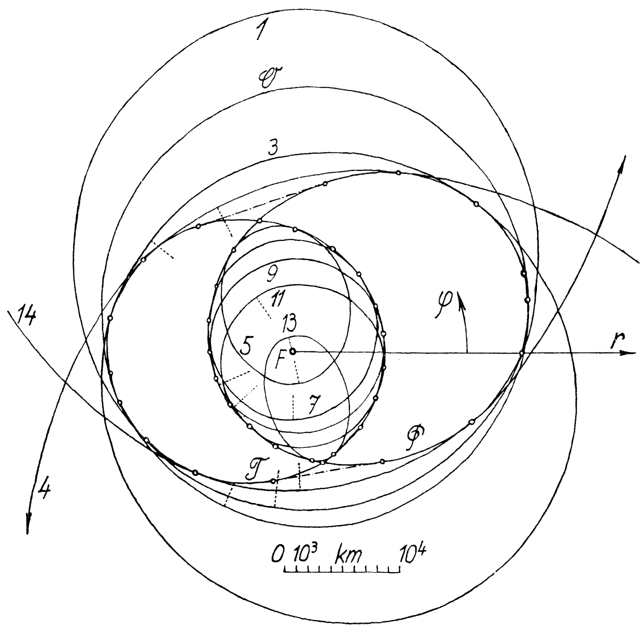


Figure 1. A system of double cotangential orbits between the parking flightpath $\mathscr{P}$ and the final trajectory $\mathscr{T}$.

The numbers inserted indicate the indices from the first and the last column of the tables. The ellipse $\mathscr{O}$ is the common graph corresponding to the values of $j = 15$ to 19 which cannot be distinguished in the scale of the figure. The tangent points of the orbits with both $\mathscr{P}$ and $\mathscr{T}$ (including those corresponding to the values $j = 6, 8, 10, 12$ whose ellipses are not introduced for the sake of clarity) are indicated by circles. The same holds for the tangent points (1), (2) of both common tangents (drawn by *dot-and-dashed lines*) as well as for the points of intersection of $\mathscr{P}$ and $\mathscr{T}$ whose vectorial angles were found in [2] to be 106°14′28″ and 286°14′26″ respectively. *Dotted* segments indicate the ends of the periapsis radius vectors, the periapsis radius vector of the trajectory $\mathscr{T}$ coincides with the polar axis. The transfer sections of the orbits $j = 5, 6$ intersect the body of the earth.

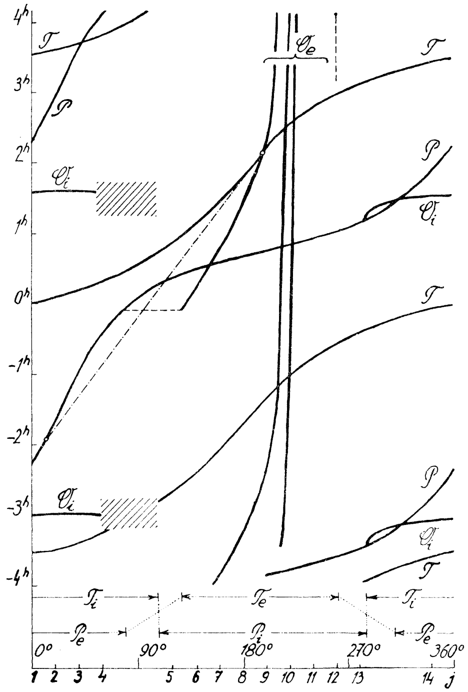
Figure 2. Flight schedules and orbital transfer.

Notation $\mathscr{P}_{e,i}$ indicates the ranges of a possible launch from the parking path; the subscript distinguishes the outer and inner osculation of the orbit with the parking path (e or i, respectively). The ranges of the dotted sections of the outer launch $\mathscr{P}_e$ corresponding to *hyperbolical* orbits are limited by unit values of the function f (which is tabulated in column IX; the selection given in the tables does not include these values). Similarly, by $\mathscr{T}_{e,i}$ the launch section on the final trajectory are denoted. The corresponding endpoints are connected by dotted lines. The curves $\mathscr{P}$ and $\mathscr{T}$ are the flight schedules of the body on the parking path and the final trajectory, respectively; they correspond to columns II & XVII. The curves $\mathscr{O}_{e,i}$ show the contact of the transfer orbits with the final trajectory distinguishing again the outer and inner osculation by subscripts (columns XVII & XXV). The shaded area indicates the range where the launch sections pass through the earth body. The right boundary of the shading ($\varphi^* = 106°14'28''$) corresponds to the fictitious case of a degenerate transfer ellipse (the osculation becomes senseless) on the connecting line of the point of intersection $\mathscr{P} \times \mathscr{T}(\varphi^*)$ with the central point F. Formally, the corresponding launch time is

$$\mathscr{T} = 2\pi \sqrt{(a^3/\mu)} =$$

$$= 2\pi \sqrt{[(p_{\mathscr{T}}/(1 + \varepsilon_{\mathscr{T}} \cos \varphi^*))^3/\mu]} \approx$$

$$\approx 1^h 13^m .$$



The beginning of the section $\mathscr{O}_i$ on $\mathscr{P}$ for $\varphi^{**} = 286°14'26''$ corresponds to the launch at the point of intersection $\mathscr{P} \times \mathscr{T}(\varphi^{**})$ with zero transfer time when the notion of osculation becomes senseless. The beginning of the section $\mathscr{O}_e$ for $\varphi = 128°11'28''$ [1]) corresponds to the limit case of launch with zero transfer time from the point $\varphi = 80°02'21''$ [2]) (connected in the graph by a dashed horizontal segment) by the straight-line flight with infinite velocity along the common tangent of the quadrics $\mathscr{P}$, $\mathscr{T}$ *between* the tangent points. With increasing φ the points of the curve $\mathscr{O}_e$ tend to the asymptote (vertical dashed line in the graph) $\varphi = 262°04'04''$ [3]) whose infinite point corresponds to the limit case of infinite straight-line flight from the point $\varphi = 310°12'50''$ [4]) along the common tangent of the ellipses $\mathscr{P}$, $\mathscr{T}$ *outside* of its tangent points.

The introduced point of intersection of $\mathscr{T} \times \mathscr{O}_e$ determines the position and the moment of rendezvous of the body moving along the tangential transfer orbit with the object on the final trajectory. It is connected by a dot-and-dashed line with the corresponding starting point on the curve of flight schedule $\mathscr{P}$. Here we deal with the case tabulated as $j = 17$ which coincides graphically with $j = 15, 16, 18, 19$. A shift of the curves $\mathscr{O}_{e,i}$ in the direction of the time axis by whole multiples of the orbital period $T_{\mathscr{P}}$ corresponds to the possibility of start during various circulations on the parking path. The points of intersection of $\mathscr{O}_{e,i}$ with $\mathscr{T}$ are then the images (at least theoretically possible) of rendezvous of the body on the transfer orbit with the body on the final trajectory.

[1]) Left boundary point of the interval (2).

[2]) Right boundary point of the interval (1).

[3]) Right boundary point of the interval (2).

[4]) Left boundary point of the interval (1).

The *first four tables* are sufficient for the solution of the rendezvous. Tab. 5, besides showing the impulse changes realising the common flight of both bodies is a testing view on the numerical accuracy of the calculation: the radius vectors as well as the flight directions at the tangent point of the transfer orbit and the final trajectory, are calculated on the one hand from the equation of the transfer orbit, on the other hand from the equation of the trajectory. Numerical differences of thousand kilometers lengths do not reach even one meter and the angle differences of directions one second. This shows altogether an accuracy which is much better than the approximative character of the approach to the problem of two bodies together with the technical possibilities of both the localisation and the non-instantaneous impulse changes of the orbital velocity might require.

The choice of the transfer orbit is made according to column XXVII of Tab. 4. The change of signs*) between the lines $j = 1, 2$ shows that in the interval $(0°, 20°)$ there exists a vectorial angle of the launch point of the parking flightpath which leads to a cotangential transfer orbit realising the rendezvous. Its iteration is given, with a finer step of argument Φ_M, by the lines $j = 15$ to 19. The numerical differences between the five orbital flights are far below the technically accessible accuracy of the project. As the final solution let us introduce the orbital launch $j = 17$:

The object on the parking flightpath has at the moment $\tau_{\mathcal{C}} = -2^h 14^m 10{\cdot}03^s$ the polar coordinates $r_{\mathcal{P}\mathcal{C}} = 20\,582{,}79$ km, $\Phi_M = 13°26'39''$ while the tangent angle is $\vartheta_M = 78°53'30{\cdot}09''$ and the orbital velocity 3203·13 m/s (the escape velocity at this point is 6223·48 m/s; the value $f \approx 0{\cdot}45$ shows that the transfer orbit is elliptical). By an instantaneous increase of velocity equal to 976·47 m/s the object passes with the velocity 4179·60 m/s to the bicotangential transfer orbital ellipse

$$a_{\mathcal{C}} = 18\,746{\cdot}61 \text{ km}, \quad b_{\mathcal{C}} = 18\,306{\cdot}93 \text{ km}. \quad p_{\mathcal{C}} = 17\,877{\cdot}57 \text{ km},$$

$$e_{\mathcal{C}} = 4\,036{\cdot}27 \text{ km}, \quad \varepsilon_{\mathcal{C}} = 0{,}2153068, \quad T_{\mathcal{C}} = 7^h 05^m 44{\cdot}27^s,$$

$$\varphi_{\mathcal{C}} = 245°49'23{\cdot}45''.$$

After the time interval $T_p = 4^h 22^m 12{\cdot}18^s$, i.e., at the moment $\tau_{\mathcal{C}\mathcal{T}} = 2^h 08^m 02{\cdot}15^s$, it reaches the final trajectory at the point $r_{\mathcal{C}\mathcal{T}} = 15\,662{\cdot}20$ km, $\Phi_N = 196°53'27{\cdot}74''$ with the orbital velocity $v_{\mathcal{C}\mathcal{T}} = 5444{\cdot}02$ m/s, tangent angle ${}_{\mathcal{C}\mathcal{T}} = 98°05'37{\cdot}68''$, true anomaly $\varphi_{a\mathcal{T}} = 311°04'05{\cdot}34''$ meeting the object moving along the trajectory. By decreasing its orbital velocity by 1238·89 m/s to the value 4205·13 m/s the object passes from the launch orbit to the final trajectory on which the rendezvous is realised.

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*) The sign change between the lines $j = 4, 5$ accompanies the *discontinuous* transfer between the outer and inner launching by excluding the vectorial angles from the interval (80°02′02″, 106°14′28″) — i.e., the right endpoint of the interval (1) and the left endpoint of the interval (3) without giving a possibility of realising the rendezvous of the two bodies. Nor do the sign changes between the pairs $j = 9, 10$ and $j = 13, 14$ which correspond to multiple circulations along the final trajectory yield a possibility of the rendezvous.

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Souhrn

KOSMICKÉ SETKÁNÍ USKUTEČNĚNÉ DVOJDOTYKOVÝM PŘECHODOVÝM ORBITEM

KAREL MIŠOŇ

Vyšetřuje se převedení umělého kosmické tělesa ze známé parkovací dráhy do dané komplanární cílové trajektorie tak, aby se zde setkalo s tělesem obíhajícím trajektorií a obě tělesa pak pokračovala společným letem v trajektorii. Převedení se uskutečňuje keplerovským naváděcím orbitem dotýkajícím se jak parkovací dráhy, tak i cílové trajektorie. Jde tedy o stanovení okamžiků a příslušných impulsových změn rychlosti (beze změny jejího směru), převádějících umělé kosmické těleso z parkovací dráhy do naváděcího orbitu a odtud pak do cílové trajektorie. Ze tří disjunktiv (cílová trajektorie je celá uvnitř, resp. vně, resp. protíná parkovací dráhu) v zásadě odlišných je poslední (thematicky nejširší) možnost numericky i graficky konkretisována příkladem vztaženým na zemské gravitační pole.

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