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ON THE FORMULATION OF THE TRACTION PROBLEM FOR THE FLOW THEORY OF PLASTICITY

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1. INTRODUCTION

In this paper, an abstract ordinary differential equation with retarded argument is deduced. This equation is a model of incremental, rate independent strain theory.

The considerations are based on the incremental stres-strain relations, see K. Washizu [1]. To solve the boundary-value problem for increments, an abstract variational problem is solved with a quadratic functional.

The definition of the experience of the body is fundamental.

The behaviour of the abstract ordinary differential equation is such that the methods of contractive mappings, compact mappings or weakly continuous mappings are not applicable. The author will try to solve a regularized model in another paper, basing his considerations on the paper of J. Kratochvíl, O. W. Dillon [2], where the flow theory of plasticity is shown to be a singular limit of the theory with internal state variables.

This paper had its rise after discussions in the seminar of mechanics held at the University in Prague and it is my pleasure to thank ing. M. Hlaváček, ing. V. Kafka and dr. J. Kratochvíl for their suggestions.

2. STRESS-STRAIN RELATIONS

For the formulation of stress-strain relations, we shall suppose sufficient regularity of displacements and of the stress tensor. We shall consider only bounded bodies Ω with sufficiently regular boudary.

The deformation of Ω is described by small displacement theory and the strain tensor ε_{ij} is defined by

$$\varepsilon_{ij} = \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{2.1}$$

The components of the strain tensor ε_{ij} are supposed once continuously differentiable in time $t \in \langle 0,1 \rangle$ with the exception of a finite number of points where the one side derivatives are supposed to exist.

The usual symmetric stress tensor σ_{ij} is considered and σ_{ij}, $\partial\sigma_{ij}/\partial x_k$ are continuously differentiable in t in the above sense.

Let F_i be the components of the body-forces, once continuously differentiable in time.

The equilibrium equation

$$\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0\,, \quad ^{*)} \tag{2.2}$$

as well as

$$\frac{\partial \dot{\sigma}_{ij}}{\partial x_j} + \dot{F}_i = 0 \tag{2.3}$$

is valid.

Let us consider once continuously differentiable function $f(\sigma)$ of

$$\sigma = (\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{21}, \ldots, \sigma_{33}) \in R_9,\ \sigma_{ij} = \sigma_{ji}\,,$$

such that $f(\sigma) \geqq 0$, $f(\theta) = 0$ and

$$\left| \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{ij}} \right| \leqq c < \infty\,. \tag{2.4}$$

Let $h(\sigma)$ be a continuous function, $h(\sigma) \geqq 0$, such that

$$|h(\sigma)| \leqq c < \infty\,. \tag{2.5}$$

Let $\chi_1^\varepsilon(s)$, $\chi_2^\varepsilon(s)$ be bounded, Lipschitzian-functions on the interval $\langle 0, \infty \rangle$, such that

$$0 \leqq \chi_1^\varepsilon(s) \leqq 1, \quad \chi_1^\varepsilon(0) = 0, \quad \chi_1^\varepsilon(s) = 1 \quad \text{for} \quad s \geqq \varepsilon > 0\,,$$
$$0 \leqq \chi_2^\varepsilon(s) \leqq 1, \quad \chi_2^\varepsilon(0) = 1, \quad \chi_2^\varepsilon(s) = 0 \quad \text{for} \quad s \geqq \varepsilon > 0\,.$$

Put $n(\sigma)(x, t) = \max\limits_{0 \leqq \tau \leqq t} f(\sigma(x, t))$ and call $n(\sigma)$: experience.

If $\chi_1(s) = 0$ for $0 \leqq s < p$, $\chi_1(s) = 1$ for $s > p$, $\chi_2(0) = 1$ and 0 elsewhere, the incremental stress-strain relations are

$$\dot{\varepsilon}_{ij} = -\frac{\nu}{E}\sigma_{kk}\delta_{ij} + \frac{1+\nu}{E}\dot{\sigma}_{ij} + \chi_1(f(\sigma))\,\chi_2(n(\sigma) - f(\sigma))\,. \tag{2.6}$$
$$h(\sigma)\frac{\partial f}{\partial \sigma_{ij}}(\sigma)\, r\left(\frac{\partial f}{\partial \sigma_{kl}}(\sigma)\,\dot{\sigma}_{kl}\right),$$

*) Throughout the whole paper, the summation convention is used.

where

$$r(\lambda) = \tfrac{1}{2}(\lambda + |\lambda|) \quad \text{and} \quad 0 \leqq \nu < \tfrac{1}{2}, 0 < E\,.$$

If we put

$$\dot{\varepsilon}^p_{ij} = \chi_1(f(\sigma))\, \chi_2(n(\sigma) - f(\sigma))\, h(\sigma)\, (\partial f/\partial \sigma_{ij})\,(\sigma)\, r\,(\partial f/\partial \sigma_{kl})\,(\sigma)\, \dot{\sigma}_{kl})$$

the experience is given as

$$n(\sigma)\,(x, t) = F\left(\int_0^t \sigma_{ij}\dot{\varepsilon}^p_{ij}\, \mathrm{d}\tau\right),$$

where F is an increasing positive function.

3. SPACES $C(\langle 0,1\rangle, L_2(\Omega))$, $C^{(1)}(\langle 0,1\rangle, L_2(\Omega))$, $H_0^{(0,1)}(\Omega \times (0,1))$

Put $Q = \Omega \times (0, 1)$. Let us define $H^{(0,1)}(Q)$ as the subspace of $L_2(Q)$ such that for $u \in H^{(0,1)}(Q)$, $\partial u/\partial t \in L_2(Q)$ in the sence of distributions.

Put

$$\|u\|_{H^{(0,1)}} \overset{\mathrm{df}}{=} \left\{\int_Q \left(u^2 + \left(\frac{\partial u}{\partial t}\right)^2\right) \mathrm{d}x\, \mathrm{d}t\right\}^{1/2}. \tag{3.1}$$

It follows from Theorem 2.2, § 2., Chap. 2, J. Nečas [3] that $u \in H^{(0,1)}(Q) \Rightarrow u$ is absolutely continuous in $t \in \langle 0, 1\rangle$ for almost all $x \in \Omega$. Let us define $H_0^{(0,1)}(Q)$ as the subspace of $H^{(0,1)}(Q)$ such that $u(x, 0) = 0$.

The next lemma 3.1 follows as in Theorem 3.1, § 3, Chap. 2 of [3] and as in Theorem 1.2, § 1, Chap. 1 of [3], if we define $\mathscr{E}_0(\bar{Q})$, the space of infinitely differentiable functions on $\bar{Q}$ equal to zero for $t = 0$:

Lemma 3.1. *$\mathscr{E}_0(\bar{Q})$ is dense in $H_0^{(0,1)}(Q)$ and for all $t \in \langle 0,1\rangle$:*

$$\int_\Omega u^2(x, t)\, \mathrm{d}x \leqq c\, \|u\|^2_{H_0^{(0,1)}}\,. \tag{3.2}$$

Let $C(\langle 0,1\rangle, L_2(\Omega))$ be the set of functions $u(x, t)$ such that for every

$$t \in \langle 0,1\rangle : \int_\Omega u^2(x, t)\, \mathrm{d}x < \infty$$

and such that $u(\cdot, t)$ is continuous on $\langle 0,1\rangle$ in the L_2 norm. Put

$$\|u\|_{C(\langle 0,1\rangle, L_2(\Omega))} = \max_{t \in \langle 0,1\rangle}\left(\int_\Omega u^2(x, t)\, \mathrm{d}x\right)^{1/2}. \tag{3.3}$$

Let $C(\bar{Q})$ be the set of continuous functions on $\bar{Q}$.

Lemma 3.2. *$C(\bar{Q})$ is dense in $C(\langle 0,1\rangle, L_2(\Omega))$.*

Proof. Let $u \in C(\langle 0,1\rangle, L_2(\Omega))$. Clearly $u(\cdot, t)$ is uniformly continuous in t on $\langle 0,1\rangle$. Let $\varepsilon > 0$. Then there exists $\delta > 0$ such that $\|\varphi(t_1) - \varphi(t_2)\| < \varepsilon$ if $|t_1 - t_2| < \delta$. Let $h = 1/p$ and $t_j = jh$, $j = 0,1, \ldots, p$, $h < \delta$.
Let $\{v_i\}_{i=1}^{\infty}$ be a set of continuous functions on $\bar{\Omega}$, orthonormal and complete in $L_2(\Omega)$. For each j, let

$$u_h(\cdot, t_j) = \sum_{i=1}^{n_j} v_i(u(\cdot, t_j), v_i)\,,$$

where n_j is such that $\|u(\cdot, t_j) - u_h(\cdot, t_j)\|_{L_2(\Omega)} < \varepsilon$.

Put

$$u_h(\cdot, t) = u_h(\cdot, t_j) + ((t - t_j)/h)\,(u_h(\cdot, t_{j+1}) - u_h(\cdot, t_j)) \quad \text{for} \quad t_j \leqq t \leqq t_{j+1}\,.$$

We have for

$$t \in \langle t_j, t_{j+1}\rangle : \|u_h(\cdot, t) - u(\cdot, t)\|_{L_2(\Omega)} \leqq$$
$$\leqq ((h - t + t_j)/h)\,\|u_h(\cdot, t_j) - u(\cdot, t)\|_{L_2(\Omega)} + ((t - t_j)/h)\,\|u_h(\cdot, t_{j+1}) - u(\cdot, t)\|_{L_2(\Omega)} < 2\varepsilon$$

q.e.d.
We define

$$\dot{u}(\cdot, t) = \lim_{h \to 0} ((u(\cdot, t + h) - u(\cdot, t))/h$$

in $L_2(\Omega)$ and the space $C^{(1)}(\langle 0,1\rangle, L_2(\Omega))$ as the subspace of $C(\langle 0,1\rangle, L_2(\Omega))$ such that $\dot{u} \in C(\langle 0,1\rangle, L_2(\Omega))$ with the norm

$$\|u\|_{C(\langle 0,1\rangle, L_2(\Omega))} + \|\dot{u}\|_{C(\langle 0,1\rangle, L_2(\Omega))}\,.$$

We define $C_0^{(1)}(\langle 0,1\rangle, L_2(\Omega))$ as the subspace of u from $C^{(1)}(\langle 0,1\rangle, L_2(\Omega))$ such that $u(\cdot, 0) = 0$. Let $C_0^{(0,1)}(\bar{Q})$ be the space of continuous functions u on $\bar{Q}$ with continuous derivatives $\partial u/\partial t$ on $\bar{Q}$ and such that $u(x, 0) = 0$.

Lemma 3.3. *$C_0^{(0,1)}(\bar{Q})$ is dense in $C_0^{(1)}(\langle 0,1\rangle, L_2(\Omega))$.*

Proof. Let $u \in C_0^{(1)}(\langle 0,1\rangle, L_2(\Omega))$. There exists $h_n \in C(\bar{Q})$ by the preceding lemma such that $h_n \to \dot{u}$ in $C(\langle 0,1\rangle, L_2(\Omega))$.

Put

$$H_n(x, t) = \int_0^t h_n(x, \tau)\,d\tau \quad \text{and let} \quad \int_0^t \dot{u}(\cdot, \tau)\,d\tau$$

be the Riemann abstract integral. Since for every $v \in L_2(\Omega)$:

$$(u(\cdot, t), v)'_{L_2} = (\dot{u}(\cdot, t), v)_{L_2}, \quad \text{it is} \quad (u(\cdot, t), v)_{L_2} =$$
$$= \left(\int_0^t \dot{u}(\cdot, \tau)\,d\tau, v\right)_{L_2}, \; u(\cdot, t) = \int_0^t \dot{u}(\cdot, \tau)\,d\tau$$

and $H_n \to u$ in $C(\langle 0,1\rangle, L_2(\Omega))$, q.e.d.

4. THE FUNCTIONAL FOR INCREMENTS AND THE FORMULATION OF THE FIRST PROBLEM OF PLASTICITY

Let $F_i \in C_0^{(1)}(\langle 0,1\rangle, L_2(\Omega))$, $g_i \in C_0^{(1)}(\langle 0,1\rangle, L_2(\partial\Omega))$.
We suppose

$$\int_\Omega F_i(x, t)\,\mathrm{d}x + \int_{\partial\Omega} g_i(x, t)\,\mathrm{d}s = 0,\ i = 1, 2, 3\ ,$$

$$\int_\Omega (F(x, t) \times x)_i\,\mathrm{d}x + \int_{\partial\Omega} (g(x, t) \times x)_i\,\mathrm{d}s = 0, \quad i = 1, 2, 3\ ,$$

where $(F \times x)$ is the vector product. Lef $H^1(\Omega)$ be the Sobolev space of L_2 functions with the first L_2 derivatives in the sence of distributions.

Let S be the subspace of $\sigma \in [L_2(\Omega)]^9$ with $\sigma_{ij} = \sigma_{ji}$ and the scalar product:

$$\int_\Omega \sigma_{ij}\tau_{ij}\,\mathrm{d}x\ .$$

Let K be the subspace of S defined by

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad u_i \in H^1(\Omega)\,.$$

Let H be the orthogonal complement of K in S.

Lemma 4.1. *K is closed and $S = K \dotplus H$.*

Proof. Let $u_i \in H^1(\Omega)$ such that

$$\int_\Omega u_i\,\mathrm{d}x = 0, \quad \int_\Omega (u \times x)_i\,\mathrm{d}x = 0\,.$$

For such u, we have Korn's inequality:

$$\int_\Omega \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}\,\mathrm{d}x \leqq c\int_\Omega \varepsilon_{ij}\varepsilon_{ij}\,\mathrm{d}x\ , \tag{4.1}$$

see I. Hlaváček, J. Nečas [4]. Therefore K is closed, because $\varepsilon_{ij} = 0$, $i, j = 1, 2, 3$ iff $u = a + (b \times x)$, see the same paper, q.e.d.

Lemma 4.2. *The operator $\sigma \mapsto n(\sigma)$ is a Lipschitzian, bounded operator from*

$$[H_0^{(0,1)}(Q)]^9 \cap S \mapsto C(\langle 0,1\rangle, L_2(\Omega))\,.$$

Proof. Since $\sigma_{ij}(x, t)$ are absolutely continuous in $t \in \langle 0,1\rangle$ for almost all $x \in \Omega$, the experience $n(\sigma)(x, t)$ is defined for almost all $x \in \Omega$. Let $t_1 < t_2$. We have

$$(n(\sigma)(x, t_2) - n(\sigma)(x, t_1))^2 \leqq \max_{\tau \in \langle t_1, t_2\rangle} (f(\sigma(x, \tau)) - f(\sigma(x, t_1)))^2 \leqq$$

$$\leqq c\left(\int_{t_1}^{t_2} \|\dot{\sigma}(x, t)\|_{R_9} \, d\tau\right)^2 \leqq c(t_2 - t_1) \int_0^1 \|\dot{\sigma}(x, \tau)\|_{R_9}^2 \, d\tau\,,$$

hence $n(\sigma) \in C(\langle 0,1\rangle, L_2(\Omega))$. Since $n(\sigma)(x, 0) = 0$ by the same argument as above, we obtain that $n(\sigma)$ is a bounded operator.

We have

$$(n(\sigma_1)(x, t) - n(\sigma_2)(x, t))^2 \leqq \max_{0 \leqq \tau \leqq 1} (f(\sigma_1(x, \tau)) -$$

$$- f(\sigma_2(x, \tau)))^2 \leqq c \max_{0 \leqq \tau \leqq 1} \|\sigma_1(x, \tau) - \sigma_2(x, \tau)\|_{R_9}^2 \leqq$$

$$\leqq c \int_0^1 \|\dot{\sigma}_1(x, \tau) - \dot{\sigma}_2(x, \tau)\|_{R_9}^2 \, d\tau\,,$$

hence

$$\|n(\sigma_1)(\cdot, t) - n(\sigma_2)(\cdot, t)\|_{L_2(\Omega)} \leqq c\|\sigma_1 - \sigma_2\|_{H_0^{(0,1)}(Q)}\,,$$

q.e.d.

Remark. We have abso proved that Lemma 4.2 holds for the mapping $\sigma \to f(\sigma)$.

Let $\sigma \in [C_0^{(1)}(\langle 0,1\rangle, L_2(\Omega))]^9$. We define for every $t \in \langle 0,1\rangle$ the operator $G(\sigma) = \dot{\tau}$ from $[C_0^{(1)}(\langle 0,1\rangle, L_2(\Omega))]^9$ to $[L_2(\Omega)]^9$, where $\dot{\tau}$ is an element satisfying the following conditions for every $u \in H^1(\Omega)$:

(4.2)
$$0 = \int_\Omega \dot{\tau}_{ij}\varepsilon_{ij} \, dx - \int_\Omega \dot{F}_i u_i \, dx - \int_{\partial\Omega} \dot{g}_i u_i \, ds \quad [1]$$

$$\left(\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)$$

and for every $h \in H$:

(4.3)
$$\int_\Omega \Big[-\frac{\nu}{E} \dot{\tau}_{kk}(x) h_{kk}(x) + \frac{1+\nu}{E} \dot{\tau}_{ij}(x) h_{ij}(x) +$$

$$+ \chi_1^\varepsilon(f(\sigma(x, t)))\, \chi_2^\varepsilon(n(\sigma)(x, t) - f(\sigma(x, t)))\, h(\sigma(x, t))$$

$$\frac{\partial f}{\partial \sigma_{ij}}(\sigma(x, t))\, h_{ij}(x) \frac{\partial f}{\partial \sigma_{kl}}(\sigma(x, t))\, \dot{\tau}_{kl}(x)\Big] dx = 0\,. \quad [2]$$

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[1]) (4.2) are equations of equilibrium fulfilled in the sense of virtual stresses.

[2]) (4.3) means the stationary value of the functional of complementary energy.

Since

$$\int_\Omega \left[-\frac{\nu}{E}(h_{kk})^2 + \frac{1+\nu}{E} h_{ij} \, . \, h_{ij} + \chi_1^\varepsilon(f(\sigma))\chi_2^\varepsilon(n(\sigma) - f(\sigma))\, h(\sigma) \left(\frac{\partial f}{\partial \sigma_{kl}}(\sigma) h_{kl} \right)^2 \right] \mathrm{d}x \geqq \frac{1-2\nu}{E} \int_\Omega h_{ij} h_{ij} \, \mathrm{d}x \,, \tag{4.4}$$

we obtain

Theorem 4.1. *There exists a unique solution of* (4.2), (4.3) *and, independently of* σ:

$$\|\dot{\tau}\|_{[L_2(\Omega)]^9} \leqq c(\|\dot{F}\|_{[L_2(\Omega)]^3} + \|\dot{g}\|_{[L_2(\partial\Omega)]^3}) \, . \tag{4.5}$$

Proof. We can find $\sigma^\circ \in [C_0^{(1)}(\langle 0,1 \rangle, L_2(\Omega))]^9$ as the solution of the linear problem (4.2), (4.3) where $h(\sigma) = 0$; such solution exists and is unique, see for example I. Hlaváček, J. Nečas [4]. But if we put $\dot{\tau} = \sigma^\circ + z$, we obtain a unique z from the Riesz theorem; (4.5) is obvious, q.e.d.

Theorem 4.2. $G(\sigma)(\cdot, \cdot) \in [C(\langle 0,1 \rangle, L_2(\Omega))]^9$.

Proof. Let $t_0 \in \langle 0,1 \rangle$ and $t_n \to t_0$, $t_n \in \langle 0,1 \rangle$. But all functions

$$\chi_1^\varepsilon(f(\sigma)(x, t_n))), \chi_2^\varepsilon(n(\sigma)(x, t_n) - f(\sigma(x, t_n))) \,,$$

$$h(\sigma(x, t_n)), \frac{\partial f}{\partial \sigma_{ij}}(\sigma(x, t_n))$$

are such that (we shall write it for example for

$$\frac{\partial f}{\partial \sigma_{ij}}(\sigma(x, t_n)) : \frac{\partial f}{\partial \sigma_{ij}}(\sigma(x, t_n)) \to \frac{\partial f}{\partial \sigma_{ij}}(\sigma(x, t_0))$$

almost everywhere in Ω after choosing a subsequence of t_n, if necessary. Because of (4.4) and (2.4), we obtain easily (for details see Proposition 6.2, § 6, Chap. 3, J. Nečas [3]) that $G(\sigma)(\cdot, t_n) \to G(\sigma)(\cdot, t_0)$, q.e.d.

Put

$$\int_0^t G(\sigma)(\cdot, \tau) \, \mathrm{d}\tau$$

to be the Riemann abstract integral.

The first problem of "regularized" plasticity (because of $\chi_1^\varepsilon, \chi_2^\varepsilon$*) is to find*

$$\sigma \in [C_0^{(1)}(\langle 0,1 \rangle, L_2(\Omega))]^9$$

satisfying the equation

$$\sigma(\cdot, t) = \int_0^t G(\sigma)(\cdot, \tau) \, \mathrm{d}\tau \tag{4.6}$$

or the equivalent differential equation

$$\sigma(\cdot, t) = G(\sigma)(\cdot, t), \quad \sigma(\cdot, 0) = 0 . \tag{4.7}$$

Let us consider the non-regularized case: let, as above, $\dot{\tau} \in [L_2(Q)]^9$ be the solution of (4.2), (4.3) for $\sigma \in [H_0^{(0,1)}(Q)]^9$. As above, we obtain Theorem 4.1.
We have

Theorem 4.3. $\tau(x, t) \in [L_2(Q)]^9$ *and*

$$\|\dot{\tau}\|_{[L_2(Q)]^9} \leqq (c \max_{t\in\langle 0,1\rangle} \|\dot{F}\|_{L_2(\Omega)} + \max_{t\in\langle 0,1\rangle} \|\dot{g}\|_{L_2(\partial\Omega)}) . \tag{4.8}$$

Proof. Let us define $\chi_1^\varepsilon(s) = 0$ for $0 \leqq s \leqq p - \varepsilon$, $\chi_1^\varepsilon(s) = (s - (p - \varepsilon))/\varepsilon$ for $p - \varepsilon \leqq s \leqq p$ and $\chi_1^\varepsilon(s) = 1$ for $s \geqq p$, $\chi_2^\varepsilon(s) = 1 - s/\varepsilon$, $0 \leqq s \leqq \varepsilon$, $\chi_2^\varepsilon(s) = 0$ for $\varepsilon \leqq s$. We have for example

$$\lim_{n\to\infty} \int_\Omega |\chi_2^{1/n}(n(\sigma) - f(\sigma)) - \chi_2^0(n(\sigma) - f(\sigma))| \, dx = 0$$

by the Lebesgue theorem on the integrable majorant. Thus we obtain by Proposition 6.2, § 6., Chap. 3, J. Nečas [3] [3]) that for every $t \in \langle 0,1\rangle : \dot{\tau}_n(t) \to \dot{\tau}(t)$ in $L_2(\Omega)$. Hence by the Lebesgue theorem on the integrable majorant, (4.5) implies:

$$\int_0^1 \|\dot{\tau}_n(t) - \dot{\tau}(t)\|^2_{L_2(\Omega)} \, dt \to 0 \quad \text{for} \quad n \to \infty ,$$

therefore $\dot{\tau} \in [L_2(Q)]^9$ and (4.8) follows as above, q.e.d.

If we put $\tau(x, t) \overset{\text{df}}{=} \int_0^t \dot{\tau}(x, s) \, ds$, we obtain $\tau \in [H_0^{(0,1)}(Q)]^9$ and we can formulate *the first problem of plasticity* as follows: *to find* $\sigma \in [H_0^{(0,1)}(Q)]^9$ such that

$$\sigma(x, t) = \int_0^t G(\sigma)(x, \tau) \, d\tau \tag{4.7}$$

for almost all

$$x \in \Omega, \quad t \in \langle 0,1\rangle .$$

Let us remark that we can define $\dot{\tau}$ replacing (4.3) by

$$\int_\Omega \Big[-\frac{\nu}{E} \dot{\tau}_{kk} h_{kk} + \frac{1+\nu}{E} \dot{\tau}_{ij} h_{ij} + \chi_1(f(\sigma)) \, \chi_2(\dot{n}(\sigma) - $$
$$- f(\sigma)) \, h(\sigma) \frac{\partial f}{\partial \sigma_{ij}}(\sigma) \, h_{ij} r \left(\frac{\partial f}{\partial \sigma_{kl}}(\sigma) \, \dot{\tau}_{kl} \right) \Big] \, dx = 0 .$$

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[3]) We apply this theorem to the uniformly elliptic bilinear forms (4.3) the coefficients of which have limit in measure and are uniformly bounded which implies $\dot{\tau}_n(t) \to \dot{\tau}(t)$ in $L_2(\Omega)$.

It follows from the relation (4.7) that both definitions are equivalent: for almost all $x \in \Omega$, τ_{ij} is absolutely continuous in t and therefore for almost all t we have

$$\frac{\partial f}{\partial \sigma_{kl}} (\sigma(x, t)) \, \dot{\sigma}_{kl}(x, t) \geqq 0 \quad \text{if} \quad n(\sigma)(x, t) = f(\sigma(x, t))$$

and σ is a solution of (4.7).

References:

[1] *K. Washizu:* Variational methods in elasticity and plasticity. Pergamon Press, 1968.

[2] *J. Kratochvíl, O. W. Dillon:* Thermodynamics of Elastic-Plastic Materials as a Theory with Internal State Variables. J. Applied Physics, Vol. 40, n. 8, 3207—3218, 1969.

[3] *J. Nečas:* Les méthodes directes en théorie des équations elliptiques. Academia, Prague 1967.

[4] *I. Hlaváček, J. Nečas:* On Inequalities of Korn's Type. Archive for Rat. Mech. Anal. Vol. 36, No. 4, 1970, 305—311, 312—334.

Souhrn

FORMULACE PRVÉHO PROBLÉMU V PŘÍRŮSTKOVÉ TEORII PLASTICITY

JINDŘICH NEČAS

V práci je odvozena abstraktní obyčejná diferenciální rovnice se zpožděným argumentem, která je matematickým modelem přírůstkové teorie plasticity. Je zaveden pojem zkušenosti materiálu, veličiny udávající zpevnění.

V práci není podán důkaz existence či unicity řešení odvozené rovnice, protože tato rovnice se vymyká dosud studovaným typům. Tyto otázky zůstávají otevřeny.

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