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INITIAL CONDITION IN THE THEORY OF NEUTRON TRANSPORT

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INTRODUCTION

The problem of finding the distribution of the density of neutrons as a function of the spatial and time coordinates, of the angle and the energy provided the initial density is known, frequently occurs in the theory of transport of neutrons in a medium.

A good mathematical approximation of the problem is represented by

$$(1) \qquad \left\{\frac{\partial}{\partial t} + \sqrt{(2E)}\,\boldsymbol{\omega}\nabla + \sqrt{(2E)}\,\Sigma_u(\boldsymbol{x}, \boldsymbol{\omega}, E, t)\right\} \varphi(\boldsymbol{x}, \omega, E, t) =$$

$$= \int_\Omega \mathrm{d}\omega \int_0^\infty \mathrm{d}E' \sqrt{(2E)}\, \Sigma(\boldsymbol{x}, \omega' \to \omega, E' \to E, t)\, \varphi(\boldsymbol{x}, \omega', E', t) + \sqrt{(2E)}\, S(\boldsymbol{x}, \omega, E, t)\,,$$

$$\varphi(\boldsymbol{x}, \omega, E, t = 0) = \psi(\boldsymbol{x}, \omega, E)\,.$$

Here the following notation is used:

$\boldsymbol{x}, \boldsymbol{\omega}, E, t$ ... coordinates of location, angle, energy and time, respectively

Ω ... surface area of the unit sphere

φ ... neutron density

ψ ... initial neutron density

S ... density of the source of neutrons

Σ ... macroscopic differential effective cross-section of the medium for neutron scattering

Σ_u ... total macroscopic effective cross-section of the medium for neutrons

Problem (1) in the above formulation has not yet been solved generally. It has been discussed only in some particular cases.

For example, Mika [1] considers a bounded medium, zero source of neutrons, effective cross-sections independent of spatial, angular and time coordinates and, besides, the quadratic integrability of $\sqrt{(2E)}\, \Sigma(E' \to E)$. Under these assumptions he finds the solution of (1) and proves its uniqueness. Marti [2] solves the problem under the assumptions of a bounded medium whose characteristics Σ, Σ_u are independent of time and bounded. Unfortunately, the last assumption is rarely satisfied in concrete cases. Vidav [3] considers a bounded convex body, $S = 0$, $\partial\Sigma/\partial t = 0$.

The time independence of the characteristics of the medium is a common feature of the above mentioned papers. Problem (1) is then usually transformed to an eigenvalue problem for a certain operator in a Banach space. However, this method fails if we assume the effective cross-sections to be non-constant functions of time (e.g. when describing a transient state in the reactor — the time variation of the temperature of the moderator etc.).

Our paper deals with the last case in an infinite absorbing and non-multiplying medium. Existence and uniqueness of the solution of the initial value problem is proved by an iterative method. It should be mentioned that our method is well known in the theory of iterative processes [4]: actually, it is the method of successive approximations.

DEFINITION AND NOTATION

The definition domain of the functions characterizing the medium and the neutron density will be assumed to be the set M of quadruplets $(\boldsymbol{x}, \boldsymbol{\omega}, E, t)$:

$$M \equiv E_3 \times \Omega \times (0, \infty) \times \langle 0, \infty) .$$

We assume that the macroscopic differential effective cross-section satisfies the condition of detailed balance

$$(2) \quad E e^{-E/(kT)}\, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \to \boldsymbol{\omega}', E \to E', t) = E' e^{-E'/(kT)}\, \Sigma(\boldsymbol{x}, -\boldsymbol{\omega} \to -\boldsymbol{\omega}', E' \to E, t)$$

on the set $M \times \Omega \times (0, \infty)$, T being the temperature of the medium and k the Boltzmann constant.

Concerning the dependence on the angle, it is usual to put

$$\Sigma(\boldsymbol{x}, \boldsymbol{\omega} \to \boldsymbol{\omega}', E \to E', t) = \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E \to E', t) \geqq 0$$

which we shall follow in the sequel.

The assumption of a non-multiplying medium implies

$$\int_\Omega \mathrm{d}\boldsymbol{\omega}' \int_0^\infty \mathrm{d}E'\, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E \to E', t) \leqq \Sigma_u(\boldsymbol{x}, \boldsymbol{\omega}, E, t) .$$

Further, denote by $C\{B; M\}$ the set of functions φ such that

$$g_1(\boldsymbol{x}, \boldsymbol{\omega}, E, t) = \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \, . \, \boldsymbol{\omega}', E' \to E, t) \, . \, \varphi(\boldsymbol{x}, \boldsymbol{\omega}', E', t) \, ,$$

$\partial g_1/\partial E$, $\partial g_1/\partial \omega_i$ and $\partial g_1/\partial x_i$ $(i = 1, 2, 3)$ are continuous functions on the set M while the function

$$g_2(\boldsymbol{x}, \boldsymbol{\omega}, E, t) = \int_\Omega \mathrm{d}\omega' \int_0^\infty [\Sigma(\boldsymbol{x}, \boldsymbol{\omega} \, . \, \boldsymbol{\omega}', E \to E', t)/\Sigma_u(\boldsymbol{x}, \boldsymbol{\omega}, E, t)] \, . \, [e^{E'/(kT)}/E'] \, .$$
$$. \, \varphi(\boldsymbol{x}, \boldsymbol{\omega}', E', t) \, \mathrm{d}E'$$

is bounded on M (B being a constant, $T(t) \geqq B/k$ for $t \in \langle 0, \infty))$.

EXISTENCE AND UNIQUENESS

First of all we shall be interested in the number of possible solutions of Problem (1).

Theorem 1. *Let the following assumptions be fulfilled:*

(a) *Functions $\Sigma_u(\boldsymbol{x}, \boldsymbol{\omega}, E, t)$, $\partial \Sigma_u/\partial E$, $\partial \Sigma_u/\partial x_i$ and $\partial \Sigma_u/\partial \omega_i$ $(i = 1, 2, 3)$ are continuous on M.*

(b) *To every quadruplet $(\boldsymbol{x}, \boldsymbol{\omega}, E, t)$ there is a non-degenerated neighbourhood U in M, $(\boldsymbol{x}, \boldsymbol{\omega}, E, t) \in U$ so that $\Sigma(\boldsymbol{x}_1, \boldsymbol{\omega}_1 \, . \, \boldsymbol{\omega}', E_1 \to E', t_1)$ as a function of variables $\boldsymbol{x}_1, \boldsymbol{\omega}_1, E_1, t_1$ is continuous on U for almost all pairs $(\boldsymbol{\omega}', E') \in \Omega \times (0, \infty)$.*

(c) *Functions $T(t)$, $\mathrm{d}T/\mathrm{d}t$ are continuous on the interval $\langle 0, \infty)$ and, moreover, $kT \geqq B_1$, $\mathrm{d}T/\mathrm{d}t \geqq 0$ where B_1 is a positive constant.*

(d) *To every $l \in (0, \infty)$ there are constants $F \geqq 0, K > 0, 1 > A_1 \geqq 0$ so that*

$$\int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \, . \, \boldsymbol{\omega}', E \to E', t) \leqq A_1 \, \Sigma_u(\boldsymbol{x}, \boldsymbol{\omega}, E, t)$$

on $M_1 \equiv E_3 \times \Omega \times (K, \infty) \times \langle 0, l \rangle$,

$$\sqrt{(2E)} \, \Sigma_u(\boldsymbol{x}, \boldsymbol{\omega}, E, t) \leqq F$$

on $M_2 \equiv E_3 \times \Omega \times (0, K \rangle \times \langle 0, l \rangle$.

Then there is at most one solution of Problem (1) in the set $C\{B_1; M\}$.

Before proceeding to the proof of Theorem 1, let us introduce the following

Proposition 1. *Let $\Sigma_u(\boldsymbol{x}, \boldsymbol{\omega}, E, t)$, $f(\boldsymbol{x}, \boldsymbol{\omega}, E, t)$, $\partial \Sigma_u/\partial E$, $\partial f/\partial E$, $\partial \sum_u/\partial x_i$, $\partial f/\partial x_i$, $\partial \Sigma_u/\partial \omega_i$ and $\partial f/\partial \omega_i$ $(i = 1, 2, 3)$ be functions continuous on M, $g(\boldsymbol{x}, \boldsymbol{\omega}, E)$, $\partial g/\partial E$, $\partial g/\partial x_i$ and $\partial g/\partial \omega_i$ $(i = 1, 2, 3)$ functions continuous on the set $E_3 \times \Omega \times (0, \infty)$.*

Then a)

$$\text{(1a)} \quad \varphi(\boldsymbol{x}, \boldsymbol{\omega}, E, t) = \int_0^t \mathrm{d}t_1 \sqrt{(2E)} \,.\, f(\boldsymbol{x} - \sqrt{(2E)} \,.\, \boldsymbol{\omega}(t - t_1), \boldsymbol{\omega}, E, t_1) \,.$$

$$. \exp\left(\int_t^{t_1} \mathrm{d}t_2 \sqrt{(2E)} \,.\, \Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \boldsymbol{\omega}(t - t_2), \boldsymbol{\omega}, E, t_2)\right) +$$

$$+ g(\boldsymbol{x} - \sqrt{(2E)}\, \boldsymbol{\omega} t, \boldsymbol{\omega}, E) \,.$$

$$. \exp\left(- \int_0^t \mathrm{d}t_1 \sqrt{(2E)}\, \Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \boldsymbol{\omega}(t - t_1), \boldsymbol{\omega}, E, t_1)\right),$$

$\partial\varphi/\partial t$, $\partial\varphi/\partial E$, $\partial\varphi/\partial x_i$ and $\partial\varphi/\partial\omega_i$ $(i = 1, 2, 3)$ being continuous functions on M;

***b)** at the same time φ is the only solution of the problem*

$$\text{(1b)} \quad \left\{\frac{\partial}{\partial t} + \sqrt{(2E)}\, \boldsymbol{\omega}\nabla + \sqrt{(2E)}\, \Sigma_u\right\} \varphi = \sqrt{(2E)} f$$

$$\varphi(\boldsymbol{x}, \boldsymbol{\omega}, E, t = 0) = g(\boldsymbol{x}, \boldsymbol{\omega}, E)$$

on M (in other words, relation (1a) and problem (1b) are equivalent on M).

The contents of Proposition 1 is a well known fact from the theory of partial differential equations and hence its proof is omitted (see [5]).

Proof of Theorem 1. Let $\varphi_1, \varphi_2 \in C\{B_1; M\}$ be two distinct solutions of Problem (1). Then the function $\chi = \varphi_1 - \varphi_2$ obviously solves the problem

$$\text{(3)} \quad \{\partial/\partial t + \sqrt{(2E)}\, \boldsymbol{\omega} \,.\, \nabla + \sqrt{(2E)}\, \Sigma_u\}\, \chi =$$

$$= \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \sqrt{(2E)}\, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t)\, \chi(\boldsymbol{x}, \boldsymbol{\omega}', E', t)$$

$$\chi(\boldsymbol{x}, \boldsymbol{\omega}, E, t = 0) = 0$$

and belongs to the class $C\{B_1; M\}$. Assumption (a) of Theorem 1 guarantees that the assumptions of Proposition 1 are satisfied and hence Problem (3) is equivalent to the problem of solving the integral equation (for the function χ)

$$\text{(3a)} \quad \chi(\boldsymbol{x}, \boldsymbol{\omega}, E, t) =$$

$$= \int_0^t \mathrm{d}t_1 \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \Big[\sqrt{(2E)}\, \Sigma(\boldsymbol{x} - \sqrt{(2E)}\, \boldsymbol{\omega}(t - t_1), \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t_1) \times$$

$$\times \exp\left(\int_t^{t_1} \mathrm{d}t_2 \sqrt{(2E)}\, \Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \boldsymbol{\omega}(t - t_2), \boldsymbol{\omega}, E, t_2)\right),$$

$$. \chi(\boldsymbol{x} - \sqrt{(2E)}\, \boldsymbol{\omega}(t - t_1), \boldsymbol{\omega}', E', t_1)\Big] \,.$$

Let us take now an arbitrary fixed $l \in (0, \infty)$ and restrict our consideration to the set $M_3 \equiv E_3 \times \Omega \times (0, \infty) \times \langle 0, l)$. With regard to (2), equation (3a) yields the inequality

$$(4) \quad |\chi(\boldsymbol{x}, \omega, E, t)| \leqq \int_0^t \mathrm{d}t_1 \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, |\chi(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_1), \omega', E', t_1)| \, E \, .$$

$$. \, e^{(E'-E)/[kT(t_1)]} \, . \, \frac{\Sigma(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_1), \omega \, . \, \omega', E \to E', t_1)}{\Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_1), \omega, E, t_1) \, . \, E'} \times$$

$$\times \frac{\partial}{\partial t_1} \exp\left(\int_1^{t_1} \mathrm{d}t_2 \sqrt{(2E)}\, \Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_2), \omega, E, t_2)\right).$$

Hence and from the assumptions (c), (d) of Theorem 1 we obtain on the set M_3 the estimate

$$(5) \qquad |\chi(\boldsymbol{x}, \omega, E, t)| \leqq A_1 E e^{-E/kT} \, . \, C \, ,$$

C being a constant.

Substituting (5) into the inequality (4), we obtain $|\chi| \leqq A_1^2 E e^{-E/[kT]} \, . \, C$ and, after n steps $|\chi(\boldsymbol{x}, \omega, E, t)| \leqq A_1^n E e^{-E/kT(t)} \, . \, C$.

This proves Theorem 1 for the set M_3 instead of M. However, since l was chosen arbitrarily, the assertion of Theorem 1 is true on the whole set M.

Let us now proceed to the problem of existence of solutions of Problem (1). For the sake of brevity, let us introduce the following notation:

$$F(\boldsymbol{x}, \omega, E, t) = \Bigg\{ \psi(\boldsymbol{x} - \sqrt{(2E)}\, \omega t, \omega, E) +$$

$$+ \int_0^t \mathrm{d}t_1 \sqrt{(2E)}\, S(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_1), \omega, E, t_1) \times$$

$$\times \exp\left(\int_0^{t_1} \mathrm{d}t_2 \sqrt{(2E)}\, \Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_2), \omega, E, t_2)\right)\Bigg\} \times$$

$$\times \exp\left(\int_t^0 \mathrm{d}t_1 \sqrt{(2E)}\, \Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_1), \omega, E, t_1)\right),$$

$$K(\boldsymbol{x}, \omega, E, t, \omega', E', t_1) = \sqrt{(2E)}\, \Sigma(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_1), \omega \, . \, \omega', E' \to E, t_1) \times$$

$$\times \exp\left(\int_t^{t_1} \mathrm{d}t_2 \sqrt{(2E)}\, \Sigma_u(\boldsymbol{x} - \sqrt{(2E)}\, \omega(t - t_2), \omega, E, t_2)\right).$$

Theorem 2. *Let the following assumptions be fulfilled:*

(a) *All assumptions of Theorem 1 are fulfilled.*

(b) *To every quadruplet* $(x, \omega, E, t) \in M$ *there is a non-degenerate neighbourhood* U *in* M, $(x, \omega, E, t) \in U$ *so that* $\Sigma(x_1, \omega_1\omega', E_1 \to E', t_1)$, $\partial\Sigma/\partial E_1$, $\partial\Sigma/\partial x_{1i}$, $\partial\Sigma/\partial\omega_{1i}$ *and* $K(x, \omega_1, E_1, t_1, \omega', E', t')$, $\partial K/\partial E_1$, $\partial K/\partial x_{1i}$, $\partial K/\partial\omega_{1i}$ $(i = 1, 2, 3)$ *as functions of variables* x_1, ω_1, E_1, t_1 *are continuous on* U *for almost all pairs* $(\omega', E') \in \Omega \times (0, \infty)$ *and triples* $(\omega', E', t') \in \Omega \times (0, \infty) \times \langle 0, \infty)$, *respectively. Furthermore, they have integrable majorants on* $\Omega \times (0, \infty)$ *and* $\Omega \times (0, \infty) \times \langle 0, \infty)$, *respectively.*

(c) $F(x, \omega, E, t) \in C\{B_1; M\}$

(d) *Functions*

$$\frac{\partial}{\partial x_i}\int_0^t \mathrm{d}t_1 \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' K(x, \omega, E, t, \omega', E', t_1)\, F(x - \sqrt{(2E)}\,\omega(t - t_1), \omega', E', t_1)\,.$$

$$\cdot \frac{e^{E/[kT(t_1)]}}{E}\,,$$

$$K_i(x, \omega, E, t) = \int_0^t \mathrm{d}t_1 \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \frac{\partial}{\partial x_i} K(x, \omega, E, t, \omega', E', t_1)\, \frac{E' e^{(E-E')/kT}}{E}$$

$(i = 1, 2, 3)$ *are bounded on* M.

Then the series $\sum_{m=0}^{\infty} \varphi_m(x, \omega, E, t)$ *where* $\varphi_0 = F$,

(6)
$$\varphi_m(x, \omega, E, t) =$$
$$= \int_0^t \mathrm{d}t_1 \int_\Omega \int \mathrm{d}\omega' \int_0^\infty \mathrm{d}E'\, K(x, \omega, E, t, \omega', E', t_1)\, \varphi_{m-1}(x - \sqrt{(2E)}\,\omega(t - t_1), \omega', E', t_1)$$

is uniformly convergent and solves Problem (1) *in the class* $C\{B_1; M\}$.

Proof. Let us again restrict our attention to the set $M_3 \equiv E_3 \times \Omega = (0, \infty) \times \times \langle 0, l)$, l being an arbitrary but fixed number from the interval $(0, \infty)$. First of all, it is apparent from the assumption (c) that $F \in C\{B_1; M_3\}$. This together with the assumptions (a) and (c) implies the existence of positive constants $A_1 < 1$, A_2, A_3, A_4 such that

$$\int_0^t \mathrm{d}t_1 \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E'\, K(x, \omega, E, t, \omega', E', t_1)\, \frac{E' e^{(E-E')/kT(t_1)}}{E} \leqq A_1\,,$$

$$|\varphi_1| \leqq A_2 E e^{-E/[kT(t)]}\,,$$

$$|K_i| \leqq A_3\,, \quad \left|\frac{\partial\varphi_1}{\partial x_i}\right| \leqq A_4 E e^{-E/[kT(t)]} \quad (i = 1, 2, 3).$$

By mathematical induction, it is easy to prove

Proposition 2. *For any positive integer m, functions φ_m, $\partial\varphi_m/\partial x_i$ $(i = 1, 2, 3)$ are continuous on the set M_3 and*

$$(7) \qquad |\varphi_m(x, \omega, E, t)| \leqq A_1^{m-1} A_2 E e^{-E/kT}$$

$$(8) \qquad \left|\frac{\partial \varphi_m}{\partial x_i}\right| \leqq \{A_1^{m-1} A_4 + (m-1)\, A_1^{m-2} A_2 A_3\}\, e^{-E/kT} \,.\, E\,.$$

Indeed, let $m = 1$. Then the inequalities (7) and (8) are obvious and it can be seen that φ_1, $\partial\varphi_1/\partial x_i$ are continuous functions on M_3 for $i = 1, 2, 3$.

Let now $k > 1$ be integer and assume that Proposition 2 holds for $m = k$. The recursive formula (6) yields

$$|\varphi_{k+1}(x, \omega, E, t)| \leqq \int_0^t \mathrm{d}t_1 \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E'\, K(x, \omega, E, t, \omega', E', t_1)\, A_1^{k-1} A_2 E' e^{-E'/[kT(t_1)]}$$

$$\leqq A_1 \,.\, A_1^{k-1} A_2 E e^{-E/[kT(t)]} = A_1^k A_2 E e^{-E/[kT(t)]}\,,$$

$$\left|\frac{\partial \varphi_{k+1}}{\partial x_i}\right| \leqq A_3(A_1^{k-1} A_2 e^{-E/kT} \,.\, E) + A_1 E e^{-E/kT}[A_1^{k-1} A_4 + (k-1)\, A_1^{k-2} A_2 A_3] =$$

$$= E e^{-E/kT}(A_1^k A_4 + k A_1^{k-1} A_2 A_3)\,.$$

An immediate consequence of these inequalities is that also φ_{k+1}, $\partial\varphi_{k+1}/\partial x_i$ are continuous functions on M_3. Hence Proposition 2 is true also for $m = k + 1$ and, consequently, for any positive integer m.

Consider now the integral equation for the function φ of the form

$$(9) \qquad \varphi(x, \omega, E, t) = \int_0^t \mathrm{d}t_1 \int_\Omega \mathrm{d}\omega' \int_0^\infty \mathrm{d}E'\, K(x, \omega, E, t, \omega', E', t_1)\,.$$

$$.\, \varphi(x - \sqrt{(2E)}\, \omega(t - t_1), \omega', E', t_1) + F(x, \omega, E, t)\,.$$

Every solution of this equation which is of the class $C\{B_1; M\}$ is at the same time a solution of Problem (1), as it is easy to verify. Hence we shall consider equation (9). Obviously, the function $\varphi = \sum_{m=0}^{\infty} \varphi_m(x, \omega, E, t)$ is a solution of this equation, since according to Proposition 2, it is a uniformly convergent series of successive approximations of the solution of (9).

It remains to prove that $\varphi \in C\{B_1; M\}$.

Let

$$\chi(x, \omega, E, t) = \sum_{m=1}^{\infty} \varphi_m\,.$$

According to Proposition 2, functions

$$e^{E/kT}/E \,.\, \chi\,, \quad e^{E/kT}/E \,.\, \partial\chi/\partial x_i \qquad (i = 1, 2, 3)$$

are continuous and bounded on the set M_3. Therefore (with regard to the assumptions of Theorem 2) the functions

$$\int_{\Omega} \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t) \, \chi(\boldsymbol{x}, \boldsymbol{\omega}', E', t) \,, \tag{10}$$

$$\frac{\partial}{\partial x_i} \int_{\Omega} \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t) \, \chi(\boldsymbol{x}, \boldsymbol{\omega}', E', t)$$

$(i = 1, 2, 3)$ are also continuous on M_3. Now it is easy to see that

$$\frac{\partial}{\partial \omega_i} \int_{\Omega} \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t) \, \chi(\boldsymbol{x}, \boldsymbol{\omega}', E', t) = \tag{11}$$

$$= \int_{\Omega} \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \frac{\partial}{\partial \omega_i} \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t) \, \chi(\boldsymbol{x}, \boldsymbol{\omega}', E', t) \,,$$

$$\frac{\partial}{\partial E} \int_{\Omega} \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t) \, \chi(\boldsymbol{x}, \boldsymbol{\omega}', E', t) =$$

$$= \int_{\Omega} \mathrm{d}\omega' \int_0^\infty \mathrm{d}E' \, \frac{\partial}{\partial E} \Sigma(\boldsymbol{x}, \boldsymbol{\omega} \,.\, \boldsymbol{\omega}', E' \to E, t) \, \chi(\boldsymbol{x}, \boldsymbol{\omega}', E', t)$$

$(i = 1, 2, 3)$. Condition (c) together with the relation (11) and the fact that l is chosen arbitrarily implies that $\varphi \in C\{B_1; M\}$ which completes the proof of Theorem 2.

Note: It is easy to see that $\varphi \geqq 0$ for $F \geqq 0$ on M.

CONCLUDING REMARKS

The class $C\{B; M\}$ was defined in such a way to enable us to work with generalized functions. Hence Theorems 1 and 2 hold in a more general form.

The necessary explanation is given by the following examples:

1. Consider the case with no sources ($S = 0$) and let $\psi = Ee^{-E/kT}\psi_1$ where the functions ψ_1, $\partial\psi_1/\partial E$, $\partial\psi_1/\partial x_i$ and $\partial\psi_1/\partial\omega_i$ $(i = 1, 2, 3)$ are continuous and bounded on the set M. Furthermore, let assumptions (a) and (b) of Theorem 2 be satisfied, the functions K_i being bounded on M for all i. Then evidently all assumptions of Theorem 2 are satisfied and Problem (1) has consequently only one solution in the class $C\{B_1; M\}$.

2. Assume now $\psi = 0$, $S = S_1\delta(E - E_0)$, where the functions $S_1(\boldsymbol{x}, \boldsymbol{\omega}, E, t)$, $\partial S_1/\partial E$, $\partial S_1/\partial x_i$ and $\partial S_1/\partial \omega_i$ $(i = 1, 2, 3)$ are continuous and bounded on M, $E_0 \in$ $\in (0, \infty)$ (e.g., an external source of photoneutrons). Let again conditions (a), (b) of

Theorem 2 be satisfied and, moreover, let i) functions Σ, $\partial\Sigma/\partial E$, $\partial\Sigma/\partial x_i$ and $\partial\Sigma/\partial\omega_i$ ($i = 1, 2, 3$b be continuous on $M \times \Omega \times (0, \infty)$, ii) K_i ($i = 1, 2, 3$) be bounded on M. Then the majority of concrete models of effective cross-sections satisfy assumptions (c) and (d) of Theorem 2 as well. The method of successive approximations presented above yields then the solution of Problem (1) (which is unique in the set $C\{B_1; M\}$).

In practical computations, bounded material media are involved in most cases. On the boundary between the body and the vacuum, or even inside the body, discontinuities of the effective cross-sections, sources etc. may occur.

However, it is apparent that almost always it is possible to replace the discontinuous transition of the corresponding functions by a convenient continuous transition and hence to approximate, with an arbitrary accuracy, the given problem by a problem to which both Theorems apply.

The vacuum temperature is considered to be equal to that of the material body. Regarding more in detail equations (3a) and (9) we find that the contributions to the integrals on the right hand sides due to the vacuum are zero and hence the vacuum temperature introduced above has only formal character.

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Souhrn

POČÁTEČNÍ PODMÍNKA V TEORII TRANSPORTU NEUTRONŮ

JAN KYNCL

V článku je diskutována transportní rovnice pro funkci hustoty neutronů v nenásobícím prostředí při zadaném počátečním rozložení. Makroskopické účinné průřezy a zdroje jsou uvažovány obecně jako funkce prostorových, úhlových, energetických a časových souřadnic. Výsledky týkající se existence a jednoznačnosti řešení jsou shrnuty do dvou základních vět.

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