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ON HIDDEN VARIABLES

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Jauch and Piron ([1]) demonstrated that, if there are sufficiently many states without dispersion on a structure of events (yes-no experiments), this structure $\mathscr{E}$ must be a Boolean algebra. This problem was reconsidered in [2], for there had been some objections against the formulation and proof. The assumptions (needed for the proof) which the structure $\mathscr{E}$ must fulfil are too restrictive, and therefore Gudder ([3]) made an attempt to prove the same result under weaker conditions; however, the proof is only sketched.

Very weak asumptions on the structures of events are made in [4], but the formulation is different. In [1] the problem of hidden variables is formulated:

for a given state s is it possible to find states without dispersion s_i and numbers $\alpha_i > 0$, $\Sigma\alpha_i = 1$ so that $s = \Sigma\alpha_i s_i$?

and in [4] the formulation reads:

is it possible to extend a structure of events $\mathscr{E}$ in that manner that the extended structure $\mathscr{E}'$ becomes a Boolean algebra and every state on $\mathscr{E}$ is extendable to a state on $\mathscr{E}'$?

In both cases, the answer is negetive for hidden variables, since the affirmative answer, in either case, is possible only for $\mathscr{E}$ being itself a Boolean algebra and hence the considered system being classical ([5]).

The second formulation requires weaker assumptions about the structure $\mathscr{E}$ for its demonstration, but the first one gives us the possibility to consider α_i as hidden variables.

We shall show that it is possible to pass from the second formulation to the first one (in a simple manner) without adding new assumptions on $\mathscr{E}$.

We have a structure of events $\mathscr{E}$ and an order-determining set of states $\mathscr{S}$ on it (see [6]).

We shall suppose that every state $s \in \mathcal{S}$ is a weighted average of states without dispersion, i.e.

$$(1) \qquad s(a) = \int_D \sigma(a)\, \mathrm{d}\mu_s(\sigma)\,, \quad a \in \mathcal{E}$$

where D is the set of states without dispersion on $\mathcal{E}$.

This is the formulation corresponding to that from [1] where only special expressions, namely sums, are considered.

Every σ being without dispersion, it cannot assume other values but 0 and 1. Let D_a be the set of σ for which $\sigma(a) = 1$. It is, therefore,

$$(2) \qquad s(a) = \int_{D_a} \mathrm{d}\mu_s(\sigma) = \mu_s(D_a)\,.$$

In order that (1) have a meaning, we must suppose that a σ-algebra is given in D, the functions $\sigma(a)$ are measurable with regard to this algebra, and the measures μ_s are measures on this algebra.

However, making this assumptions (which are needed for the formulation) our situation is the same as that considered in [4] (from where all definitions, notations and lemmas are taken).

Let us denote by A_s the algebra generated by all D_a modulo the ideal of sets of μ_s-zero measure and let us consider the correspondence $a \to D_a$ and the relation $s(a) = \mu_s(D_a)$. $a \to D_a$ is a homomorphims (and even additive). Hence there is an extension of the state s to the algebra B by homomorphism α – this is lemma 4.4 from [4]. This being valid for every state, we can use lemma 4.5 from [4] and we obtain:

The structure $\mathcal{E}$ is imbedded in the Boolean algebra B.

Taking into account that B is generated by elements of $\mathcal{E}$ we conclude by theorem 17 (chap. X, §13 in [7]) that under assumption that $\mathcal{E}$ is a lattice (this is supposed in [1] from the beginning)

$$\mathcal{E} = B\,.$$

This is the formulation of the non-existence of hidden variables in [1].

Besides this main resultat, it is demonstrated in [1] that from the existence of states without dispersion the reducibility of lattice of yes-no experiments follows. The question arises if this can be proved under weaker conditions.

In a recent publication ([8]) it has been proved that hidden variables can be found for the measurement of a set of commutable events. However, the proofs of the nonexistence of hidden variables for all events simultaneously retain their value because they give us the distinction between classical and quantum systems.

References

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Souhrn

SKRYTÉ PARAMETRY

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Neexistence skrytých parametrů ve formulaci z [1] plyne jednoduše z výsledků v [4]; přitom není nutné činiti další předpoklady o struktuře ano – ne experimentů.

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