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CONTRIBUTION TO THE REISSNERIAN ALGORITHMI IN THE THEORY OF BENDING OF ELASTIC PLATES

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In the recent years great attention has been given to the refined theories of plates. It is known that Kirchhoff's classical theory has originated by introducing simplifying assumptions. It appears, however, that in some cases (e.g. plates of greater thickness, regions in the vicinity of concentrated loads, discontinuous loads, boundaries with corner points, holes whose dimensions are comparable with the thickness of the plate etc.) this theory is no longer suitable, and some simplifying assumptions have to be given up. The endeavour for refinement has resulted in a series of new theories concerning the beding of plates, generally known as the refined theories. Without analysing these theories more deeply, they can be divided in two groups: The first group starts from some statical or kinematical assumptions, of course, from a smaller number than in the case of Kirchhoff's theory. The second group starts from the threedimensional elasticity equations, under the assumption that the sought solution can be developed into an infinite series according to the variable z (Fig. 1), and that thus a dimensional reduction of the problem can be obtained. Though this method makes an analysis of higher approximations possible, however, because of numerical difficulties usually only two terms of the series are taken into account, which then leads to an equation of the sixth order.

When using only a finite number of terms in the series, the question arises, how to introduce the individual approximations in order to obtain the best approach to the exact threedimensional solution [1], [2], [3]. One of the possible proceedings starts from the variational principle, and determines the coefficients of the individual approximations from the condition that the solution for a given load should fulfil Lagrange's or Castigliano's principle. In such a case, though an optimum solution is obtained, however, this depends on the load. For each load the solution has to be repeated. That is why an approach would be desirable looking for an optimum solution not for a single load only, but for a whole class of loads.

Starting from this idea, BABUŠKA and PRÁGER have derived the method of the so called Reissnerian algorithmus [1]. They have shown that there exists such optimum sequence of functions $\varphi_i^0(z/h)$, $\psi_i^0(z/h)$, where, when the displacements are given in the form of series

(1a, b, c)
$$\tilde{u}_N(x, y, h\zeta) = \sum_{i=1}^{N} (-1)^{i-1} h^{2i-1} a_i(x, y)\, \varphi_i^0(\zeta)\,,$$
$$\tilde{v}_N(x, y, h\zeta) = \sum_{i=1}^{N} (-1)^{i-1} h^{2i-1} b_i(x, y)\, \varphi_i^0(\zeta)\,,$$
$$\tilde{w}_M(x, y, h\zeta) = \sum_{i=1}^{M+1} (-1)^{i-1} h^{2i-2} c_i(x, y)\, \psi_i^0(\zeta)\,,$$

the energetic error for the whole class of loads will be asymptotically (for $h \to 0$) minimum (a_i, b_i, c_i are the solutions of a system of differential equations resulting from Lagrange's variational principle). Further they have shown that the asymptotic behaviour of the series (1) do not change, provided that instead of the independent functions a_i, b_i, c_i, we introduce $\partial \Delta^i w/\partial x$, $\partial \Delta^i w/\partial y$, $\Delta^i w$. It is important that in equation (1c) there should be one member more than in the series (1a) and (1b), i.e. $N = M$.

In this paper several variants resulting from the assumption of approximation of the displacements according to the equations (1) are derived using Lagrange's variational principle. The corresponding differential equations, together with the boundary conditions are analysed. In the conclusion some variants of the Reissnerian algorithmi are compared with some known refined theories using the example of a square plate.

DERIVATION OF DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS FOR SOME VARIANTS OF THE REISSNERIAN ALGORITHMI

Let us consider the expression (1) of the displacements $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, where the so called optimum functions $\varphi_i^0(\zeta)$, $\psi_i^0(\zeta)$ are according to [1]

$$\varphi_1^0 = -\gamma\zeta\,, \qquad \psi_1^0 = \gamma\,,$$
$$\varphi_2^0 = \frac{10 - 3\gamma}{10}\,\zeta - \frac{\gamma + 2}{6}\,\zeta^3\,, \qquad \psi_2^0 = \frac{10 + 3\gamma}{10} + \frac{\gamma - 2}{2}\,\zeta^2\,,$$

(2)
$$\varphi_3^0 = \frac{157\gamma - 140}{4200}\,\zeta + \frac{4 - 3\gamma}{60}\,\zeta^3 - \frac{\gamma + 4}{120}\,\zeta^5\,,$$
$$\psi_3^0 = -\frac{157\gamma + 140}{4200} + \frac{4 + 3\gamma}{20}\,\zeta^2 + \frac{\gamma - 4}{24}\,\zeta^4\,,$$
$$\psi_4^0 = \frac{569\gamma - 1020}{252000} + \frac{174 - 157\gamma}{8400}\,\zeta^2 + \frac{3\gamma - 2}{240}\,\zeta^4 + \frac{\gamma - 6}{720}\,\zeta^6\,,$$

where

(3)
$$\gamma = \frac{\lambda + 2\mu}{\lambda + \mu} = 2(1 - \nu)\,.$$

In the equations (2) and (3) λ and μ are Lamé's constants, ν is Poisson's ratio, ζ is the reduced coordinate z/h, when the plate thickness is equal to $2h$ (Fig. 1). Further we shall consider that the loading of the plate is

$$(4)\qquad \begin{aligned} \sigma_z(x, y, \pm h) &= \pm \tfrac{1}{2} p(x, y)\,, \\ \tau_{zx}(x, y, \pm h) &= \tau_{zy}(x, y, \pm h) = 0\,. \end{aligned}$$

Then in the case when on the cylindrical surface of the plate homogeneous boundary conditions, expressed either in displacements or in stresses, are given, the potential energy of the system is expressed by the quadratic functional [4]

$$(5)\qquad \begin{aligned} V = \tfrac{1}{2} \iiint_{-h}^{h} \Big\{ \lambda \Theta^2 + 2\mu \Big[\Big(\frac{\partial u}{\partial x}\Big)^2 + \Big(\frac{\partial v}{\partial y}\Big)^2 + \Big(\frac{\partial w}{\partial z}\Big)^2 \Big] + \\ + \mu \Big[\Big(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\Big)^2 + \Big(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\Big)^2 + \Big(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\Big)^2 \Big] \Big\} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \, - \\ - \iint [w(x, y, h) + w(x, y, -h)] \, \tfrac{1}{2} p(x, y) \, \mathrm{d}x \, \mathrm{d}y < \infty\,, \end{aligned}$$

where

$$(6)\qquad \Theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\,.$$

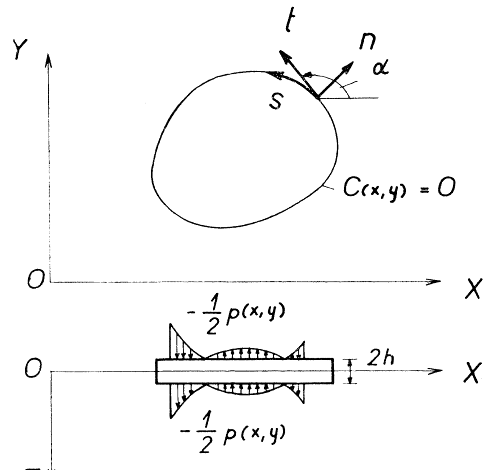

Fig. 1.

On the basis of Lagrange's variational principle we obtain Lamé's differential equations and the boundary conditions. When substituting the approximative values $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, expressed by means of (1) into (5), using Lagrange's principle we can derive the differential equations and boundary conditions corresponding to the N-th approximation. Starting from equation (1) we shall gradually increase the number of members in the series. For the sake of completness we shall begin with the simplest assumption $N = 1$, $M = 0$, i.e. expect the R eissnerian algorithmi

$$(7a, b, c)\qquad \begin{aligned} \tilde{u}(x, y, h\zeta) &= \varphi_1^0(\zeta)\, h \frac{\partial w}{\partial x}\,, \\ \tilde{v}(x, y, h\zeta) &= \varphi_1^0(\zeta)\, h \frac{\partial w}{\partial y}\,, \\ \tilde{w}(x, y, h\zeta) &= \psi_1^0(\zeta)\, w\,. \end{aligned}$$

When substituting (7abc) into equation (5) the latter can be integrated according to ζ. We obtain

$$\tilde{V} = D \frac{2(1-\nu)^3}{1-2\nu} \iint \left\{ \nu(\Delta w)^2 + (1-2\nu)\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 w}{\partial x \, \partial y}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]\right\} \mathrm{d}x \, \mathrm{d}y - 2(1-\nu)\iint wp \, \mathrm{d}x \, \mathrm{d}y \,, \tag{8}$$

where

$$D = \frac{2Eh^3}{3(1-\nu^2)} \tag{9}$$

is the flexural rigidity of the plate, E is Young's modulus and Δ the Laplace operator $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$. After Lagrange's principle

$$\delta\tilde{V} = D \frac{4(1-\nu)^3}{1-2\nu} \iint \left[\nu \Delta w \delta \Delta w + (1-2\nu)\left(\frac{\partial^2 w}{\partial x^2}\,\delta\,\frac{\partial^2 w}{\partial x^2} + 2\,\frac{\partial^2 w}{\partial x \, \partial y}\,\delta\,\frac{\partial^2 w}{\partial x \, \partial y} + \frac{\partial^2 w}{\partial y^2}\,\delta\,\frac{\partial^2 w}{\partial y^2}\right)\right] \mathrm{d}x \, \mathrm{d}y - 2(1-\nu)\iint p\delta w \, \mathrm{d}x \, \mathrm{d}y = 0 \,. \tag{10}$$

By an integration per partes it is

$$\iint \left[(1-\nu)\,\Delta^2 w - \frac{1-2\nu}{2(1-\nu)^2}\,\frac{p}{D}\right] \delta w \, \mathrm{d}x \, \mathrm{d}y - (1-\nu)\oint_c \frac{\partial \Delta w}{\partial n}\,\delta w \, \mathrm{d}s + (1-2\nu)\oint_c \frac{\partial^2 w}{\partial n \, \partial t}\,\frac{\partial \delta w}{\partial t}\,\mathrm{d}s + \oint_c \left[\nu \Delta w + (1-2\nu)\frac{\partial^2 w}{\partial n^2}\right]\frac{\partial \delta w}{\partial n}\,\mathrm{d}s = 0 \,. \tag{11}$$

With regard to the arbitrariness of δw we obtain the resulting differential equation

$$\Delta^2 w = \frac{1-2\nu}{2(1-\nu)^3} \cdot \frac{p}{D} \,. \tag{12}$$

When realizing that in the line integrals in (11) $\partial w/\partial t = \partial w/\partial s$ and $\partial^2 w/\partial n \, \partial t = \partial^2 w/\partial n \, \partial s - (\partial\alpha/\partial s)(\partial w/\partial s)$ can be substituted, we obtain the homogeneous boundary conditions

$$(1-\nu)\frac{\partial \Delta w}{\partial n} + (1-2\nu)\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial n \, \partial s} - \frac{\partial \alpha}{\partial s}\frac{\partial w}{\partial s}\right) = 0 \,, \tag{13a, b}$$

$$\nu \Delta w + (1-2\nu)\frac{\partial^2 w}{\partial n^2} = 0 \,,$$

where $\Delta w = \partial^2 w/\partial n^2 + \partial^2 w/\partial t^2 = \partial^2 w/\partial n^2 + \partial^2 w/\partial s^2 + (\partial\alpha/\partial s)(\partial w/\partial n)$ and $\partial\alpha/\partial s$ is the curvature of the boundary C. When comparing the differential equation (12) with the classical theory of thin plates, and when considering that $\tilde{w}(x, y, 0) = 2(1 - \nu) w(x, y)$, we see that we can obtain Sophie Germain's equation only in the case $\nu = 0$. The two obtained boundary conditions represents two conditions, that both the bending moment $M_n(s)$ and the generalized shear force $\bar{Q}_n(s)$ vanish. However the boundary conditions will be discussed in full detail later.

Let us consider further the first approximation of the Reissnerian algoritmus, with one function $w(x, y)$ $/N = M = 1/$

(14a, b, c)
$$\tilde{u} = \varphi_1^0 h \frac{\partial w}{\partial x},$$
$$\tilde{v} = \varphi_1^0 h \frac{\partial w}{\partial y},$$
$$\tilde{w} = \psi_1^0 w - \psi_2^0 h^2 \Delta w .$$

By substituting (14) into (5) and integrating by ζ, and by applying Green's theorem, we obtain again $\delta \tilde{V}$. Thus it follows that

(15)
$$\Delta^2 w - \frac{24 - 28\nu + 9\nu^2}{25(1 - \nu)} h^2 \Delta^3 w = \frac{1}{2(1 - \nu) D} (p - \tfrac{4}{5} h^2 \Delta p)$$

and the three boundary conditions are

(16a, b, c)
$$\frac{\partial \Delta w}{\partial n} = 0 ,$$
$$- (1 - \nu) \frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial n \partial s} - \frac{\partial \alpha}{\partial s} \frac{\partial w}{\partial s}\right) + \frac{h^2}{25(1 - \nu)} (24 - 28\nu + 9\nu^2) \frac{\partial \Delta^2 w}{\partial n} - \frac{2}{5} \frac{h^2}{(1 - \nu) D} \frac{\partial p}{\partial n} = 0 ,$$
$$\nu \Delta u + (1 - \nu)^2 \frac{\partial^2 w}{\partial n^2} - \frac{h^2}{25(1 - \nu)} (24 - 28\nu + 9\nu^2) \Delta^2 w - \frac{2}{5} \frac{h^2}{(1 - \nu) D} p = 0 .$$

Let's remark that equation (15) was derived in a slightly different way in [5]. Further instead of (14a, b, c) we consider two free function $w(x, y)$ and $t(x, y)$, see also [6]

(17a, b, c)
$$\tilde{u} = \varphi_1^0 h \frac{\partial t}{\partial x},$$
$$\tilde{v} = \varphi_1^0 h \frac{\partial t}{\partial y},$$
$$\tilde{w} = \psi_1^0 w - \psi_2^0 h^2 \Delta w .$$

After performing the calculation with regard to the arbitrariness of δw or δt we obtain a system of two equations

$$(18a, b)\quad \Delta(w - t) + h^2 \frac{2}{3}\frac{1 - \nu}{1 - 2\nu} \Delta^2 t - h^2 \frac{1}{15}\frac{12 - 31\nu + 24\nu^2}{(1 - \nu)(1 - 2\nu)} \Delta^2 w = 0\,,$$

$$-\Delta(w - t) - h^2 \frac{1}{15}\frac{12 - 31\nu + 24\nu^2}{(1 - \nu)(1 - 2\nu)} \Delta^2 t + h^2 \frac{2}{15}\frac{12 - 31\nu + 24\nu^2}{(1 - \nu)(1 - 2\nu)} \Delta^2 w -$$

$$- h^4 \frac{1}{150}\frac{20 + 20\gamma + 9\gamma^2}{(1 - \nu)^2} \Delta^3 w = \frac{1}{2}\frac{1 + \nu}{Eh(1 - \nu)}\left(p - \frac{4}{5} h^2 \Delta p\right),$$

or after elimination of t

$$(19)\quad \Delta^2 w - \frac{h^2}{30}\frac{48 - 124\nu + 77\nu^2 - 2\nu^3}{(1 - \nu)(1 - 2\nu)} \Delta^3 w + \frac{2h^4}{75}\frac{24 - 28\nu + 9\nu^2}{1 - 2\nu} \Delta^4 w =$$

$$= \frac{1}{2(1 - \nu) D}\left(p - \frac{2}{15} h^2 \frac{11 - 17\nu}{1 - 2\nu} \Delta p + \frac{8}{15} h^4 \frac{1 - \nu}{1 - 2\nu} \Delta^2 p\right).$$

It is obvious, that by introducing two functions, the equations become rather complicated, and that's why we don't give here the corresponding boundary conditions.

Let's consider further two members in the series (1), i.e. again like in the case (7) except the Reissnerian algorithmi ($N = 2$, $M = 1$)

$$(20a, b, c)\qquad \tilde{u} = \varphi_1^0 h \frac{\partial w}{\partial x} - \varphi_2^0 h^3 \frac{\partial \Delta w}{\partial x}\,,$$

$$\tilde{v} = \varphi_1^0 h \frac{\partial w}{\partial y} - \varphi_2^0 h^3 \frac{\partial \Delta w}{\partial y}\,,$$

$$\tilde{w} = \psi_1^0 w - \psi_2^0 h^2 \Delta w\,.$$

The corresponding differential equation will be

$$(21)\quad \Delta^2 w - \frac{4}{5} h^2 \Delta^3 w + \frac{4 - 4\nu + 85\nu^2}{525} h^4 \Delta^4 w = \frac{1}{2(1 - \nu) D}\left(p - \frac{4}{5} h^2 \Delta p\right).$$

The line integrals can be arranged in a form, from which we can determine four boundary conditions

$$(22)\quad \left[-2D(1 - \nu) \frac{\partial \Delta w}{\partial n} + D \frac{4}{5} h^2 (1 - \nu) \frac{\partial \Delta^2 w}{\partial n}\right] \delta w + \left[D2(1 - \nu)^2 \frac{\partial^2 w}{\partial n\, \partial t} + \right.$$

$$\left. + D \frac{4}{5} h^2 \nu (1 - \nu) \frac{\partial^2 \Delta w}{\partial n\, \partial t}\right] \frac{\partial \delta w}{\partial s} + \left[D2(1 - \nu)\left(\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 u}{\partial t^2}\right) - \right.$$

$$\left. - D \frac{4}{5} h^2 (1 - \nu)^2 \Delta^2 w + D \frac{4}{5} h^2 \nu (1 - \nu) \frac{\partial^2 \Delta w}{\partial n^2}\right] \frac{\partial \delta w}{\partial n} - D \frac{4}{5} h^2 \nu (\nu - 2) \frac{\partial \Delta w}{\partial n} \Delta \delta w +$$

$$+ D \frac{4}{5} h^2 \nu (1 - \nu) \frac{\partial^2 w}{\partial n\, \partial t} \cdot \frac{\partial \Delta \delta w}{\partial s} + D \frac{4}{5} h^2 \left(\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial t^2}\right) \frac{\partial \Delta \delta w}{\partial n} + O(h^4) = 0\,.$$

We have abstained here from giving the higher powers of h, neither will the equations be analysed now. The further approximation of the Reissnerian algorithmi will be $(N = M = 2)$

(23a, b, c)

$$\tilde{u} = \varphi_1^0 h \frac{\partial w}{\partial x} - \varphi_2^0 h^3 \frac{\partial \Delta w}{\partial x},$$

$$\tilde{v} = \varphi_1^0 h \frac{\partial w}{\partial y} - \varphi_2^0 h^3 \frac{\partial \Delta w}{\partial y},$$

$$\tilde{w} = \psi_1^0 w - \psi_2^0 h^2 \Delta w + \psi_3^0 h^4 \Delta^2 w .$$

After laborious calculations we obtain

(24)
$$\Delta^2 w - \frac{4}{5} h^2 \Delta^3 w + \frac{27}{175} h^4 \Delta^4 w - \frac{3}{16(1-\nu)} \left[\frac{76519 - 146738\nu + 58599\nu^2}{2205000} + \right.$$
$$\left. + \frac{(1+\nu)^2}{648} \right] h^6 \Delta^5 w = \frac{1}{2(1-\nu) D} \left(p - \frac{4}{5} h^2 \Delta p + \frac{27}{175} h^4 \Delta^2 p \right).$$

When introducing a further approximation, again outside the Reissnerian algorithmi $(N = 3, M = 2)$

(25a, b, c)

$$\tilde{u} = \varphi_1^0 h \frac{\partial w}{\partial x} - \varphi_2^0 h^3 \frac{\partial \Delta w}{\partial x} + \varphi_3^0 h^5 \frac{\partial \Delta^2 w}{\partial x},$$

$$\tilde{v} = \varphi_1^0 h \frac{\partial w}{\partial y} - \varphi_2^0 h^3 \frac{\partial \Delta w}{\partial y} + \varphi_3^0 h^5 \frac{\partial \Delta^2 w}{\partial y},$$

$$\tilde{w} = \psi_1^0 w - \psi_2^0 h^2 \Delta w + \psi_3^0 h^4 \Delta^2 w ,$$

and when substituting these terms into (5), we can see that the obtained differential equation will have at the powers h^0, h^2, h^4 the same coefficients as in (24), and only the member with the highest derivative will have a changed coefficient. The boundary conditions are determined from the condition, that the expression in the line integral equals to zero

(26)
$$2(1-\nu) \left\{ -2(1-\nu) D \frac{\partial \Delta w}{\partial n} \delta \left(w - \frac{2}{5} \frac{2-\nu}{1-\nu} h^2 \Delta w \right) - \left[-2(1-\nu)^2 D \frac{\partial^2 w}{\partial n \, \partial t} - \right. \right.$$
$$\left. - \frac{4}{5} \nu(1-\nu) D h^2 \frac{\partial^2 \Delta w}{\partial n \, \partial t} \right] \delta \frac{\partial}{\partial s} \left(w + \frac{2}{5} \frac{\nu}{1-\nu} h^2 \Delta w \right) - \left[-2(1-\nu) D \left(\frac{\partial^2 w}{\partial n^2} + \right. \right.$$
$$\left. \left. \left. + \nu \frac{\partial^2 w}{\partial t^2} \right) + \frac{4}{5} \nu(1-\nu) D h^2 \frac{\partial^2 \Delta w}{\partial t^2} \right] \delta \frac{\partial}{\partial n} \left(w + \frac{2}{5} \frac{\nu}{1-\nu} h^2 \Delta w \right) + O(h^4) \right\} = 0 .$$

In this equation we again do not give the members with higher powers of h. Further the stressess are expressed trough the differentiation of the displacements given by equation (25a, b, c). It is easy to prove, that the equation (26) can be written in the form

$$(26') \quad 2(1-\nu)\Big[Q_n\delta\left(w - \frac{2}{5}\frac{2-\nu}{1-\nu}h^2\Delta w\right) - M_{ns}\,\delta\frac{\partial}{\partial s}\left(w + \frac{2}{5}\frac{\nu}{1-\nu}h^2\Delta w\right) - - M_n\delta\frac{\partial}{\partial n}\left(w + \frac{2}{5}\frac{\nu}{1-\nu}h^2\Delta w\right) + O(h^4)\Big] = 0\,,$$

where

$$(27a, b, c) \qquad Q_n = -2(1-\nu)\,D\frac{\partial\Delta w}{\partial n}\,,$$

$$M_{ns} = -2(1-\nu)^2 D\left(\frac{\partial^2 w}{\partial n\,\partial s} - \frac{\partial\alpha}{\partial s}\frac{\partial w}{\partial s}\right) - \frac{4}{5}\nu(1-\nu)\,h^2 D\left(\frac{\partial^2\Delta w}{\partial n\,\partial s} - \frac{\partial\alpha}{\partial s}\frac{\partial\Delta w}{\partial s}\right),$$

$$M_n = -2(1-\nu)\,D\left[\frac{\partial^2 w}{\partial n^2} + \nu\left(\frac{\partial^2 w}{\partial s^2} + \frac{\partial\alpha}{\partial s}\frac{\partial w}{\partial n}\right)\right] + \frac{4}{5}\nu(1-\nu)\,h^2 D\left(\frac{\partial^2\Delta w}{\partial s^2} + \frac{\partial\alpha}{\partial s}\frac{\partial\Delta w}{\partial n}\right).$$

The introducing of further members in series (1) does not influence the coefficients of the powers h^0 and h^2 in the equation (26).

Should we want to obtain more accurate results, it would be necessary to involve more members and to introduce, apart from the moments and the shear forces, moments and forces of higher orders. Since the numerical solution of equation (24) is rather laborious we shall abstain here from further refinements.

In our paper we shall deal with relatively thin plates, where the expresions of the order h^4/a^4 [a is one of the plane dimensions of the plate] can be neglected. We shall analyse the so called internal problem and the boundary effect will be disregarded [7], [8]. Further when we neglect the members containing h^{2n} for $n \geqq 2$ the equation (24) can be written

$$(28) \qquad \Delta^2 w - \frac{4}{5}h^2\Delta^3 w = \frac{1}{2(1-\nu)\,D}\left(p - \frac{4}{5}h^2\Delta p\right).$$

The statical boundary conditions corresponding to equation (28) can be obtained from (26')

$$(29a, b, c) \qquad \begin{aligned} Q_n &= 0\,,\\ M_{ns} &= 0\,,\\ M_n &= 0\,. \end{aligned}$$

It can be seen that if the members are maintained up to the power h^2, we receive the three Poisson's boundary conditions for the unloaded free edge. For the kinematical boundary conditions of a clamped plate it holds that

(30a, b, c)
$$w - \frac{2}{5}\frac{2 - \nu}{1 - \nu} h^2 \Delta w = 0 ,$$
$$\frac{\partial}{\partial s}\left(w + \frac{2}{5}\frac{\nu}{1 - \nu} h^2 \Delta w \right) = 0 ,$$
$$\frac{\partial}{\partial n}\left(w + \frac{2}{5}\frac{\nu}{1 - \nu} h^2 \Delta w \right) = 0 .$$

For the sake of completness, let us further mention that when deriving the differential equations and the natural boundary conditions, corresponding to the case of a free edge, we had started from the Lagrange variational principle without using further subsidiary conditions. The introduction of approximations for $\tilde{u}$, $\tilde{v}$, $\tilde{w}$ in the form of finite series (1) results approximate solutions, that do not fulfil exactly the boundary conditions on the surfaces $z = \pm h$. Thus e.g. only the second Reissnerian approximation (23) satisfies the condition $\sigma_z(x, y, \pm h) = \pm\frac{1}{2}p(x, y)$, however, only with an accuracy to h^0, and the member with h^2 gives an error. Similarly in $\tau_{zn}(x, y, \pm h)$ the members with h^0 and h^2 vanish, but in the case of h^4 there remains a non-zero value. Should we also want to satisfy exactly the boundary conditions on the surfaces $z = \pm h$, it would be necessary to introduce these conditions as subsidiary conditions of our variational problem.

COMPARISONS OF SOME VARIANTS OF REISSNERIAN ALGORITHMI WITH SOME KNOWN REFINED THEORIES

Let us consider a square plate with sides a, and loaded in the following manner:

(31)
$$p = p_0 \sin\frac{n\pi x}{a} \sin\frac{n\pi y}{a} \quad (n = 1, 2, 3, \ldots) .$$

We suppose further that the boundary conditions are fulfilled by the choice of the solution in the form

(32)
$$w = w_0 \sin\frac{n\pi x}{a} \sin\frac{n\pi y}{a} \quad (n = 1, 2, 3, \ldots) .$$

The unknown constant w_0 can be found by applying Navier's method substituting (31) and (32) into the corresponding equation. For comparison let us present here first Kirchhoff's theory. It holds, that

(33)
$$\Delta^2 w = \frac{p}{D} ,$$

and we receive the solution

$$w_{\text{Kirch}} = \frac{p_0 a^4}{4n^4\pi^4 D} \sin\frac{n\pi x}{a} \sin\frac{n\pi y}{a}\,. \tag{34}$$

We consider the assumptions concerning the displacements after equations (7). By substituting into (12) for $\nu = 3/10$ it will be

$$w_0 = \frac{p_0 a^4}{4n^4\pi^4 D}\cdot\frac{200}{343}\,,$$

and the mean vertical deflection is

$$\tilde{w}_{\text{mean}} = \frac{7}{5}\cdot\frac{200}{343}\, w_{\text{Kirch}} \doteq 0{,}8 w_{\text{Kirch}}\,.$$

When the first approximation of the Reissnerian algorithmi ($N = M = 1$) with one function is considered according to (14), we receive from (15)

$$w_0 = \frac{5}{7}\frac{p_0 a^4}{4n^4\pi^4 D}\cdot\frac{1 + 2/5\pi^2\varepsilon^2}{1 + 1641/3140\pi^2\varepsilon^2} \doteq \frac{5}{7}\frac{p_0 a^4}{4n^4\pi^4 D}\left(1 - \frac{77}{628}\pi^2\varepsilon^2\right), \tag{35}$$

where we have denoted

$$\varepsilon = n\,\frac{2h}{a}\,. \tag{36}$$

The fraction in (35) has been divided and since we are interested in the accuracy of the order ε^2, the members of higher powers could be neglected. Further when comparing the vertical deflections, according to various theories, one has to decide upon a uniform basis. As we are considering relatively thin plates, we shall start from the average value of deflection along the height $2h$. From (14c) we obtain ($\nu = 3/10$)

$$\tilde{w}_{\text{mean}} = \frac{7}{5}\left(1 + \frac{33}{70}\pi^2\varepsilon^2\right) w\,. \tag{37}$$

By substituting (32) and (35) into (37) and taking into accont (34) we obtain

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}}\left(1 + \frac{11\,697}{35\,628}\pi^2\varepsilon^2\right) \doteq w_{\text{Kirch}}\left(1 + \frac{24{,}6}{70}\pi^2\varepsilon^2\right). \tag{38}$$

Assuming the first approximation of the Reissnerian algorithmi with two free functions according to (17), we obtain after cumbersome calculations

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}}\left(1 + \frac{1691}{4200}\pi^2\varepsilon^2\right) \doteq w_{\text{Kirch}}\left(1 + \frac{28{,}2}{70}\pi^2\varepsilon^2\right). \tag{39}$$

The assumption of equation (20) and the differential equation (21) leads to

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}}\left(1 + \frac{33}{70}\,\pi^2\varepsilon^2\right). \tag{40}$$

When considering after (23) the second Reissnerian algorithm ($N = M = 2$) with one free function w we see, that $\tilde{w}_{\text{mean}}$ will be determined with the accuracy of ε^2 by the equation (40), and even the further members do not change the coefficient $\frac{33}{70}\pi^2$.

Let it be added that the bending moment M_x with an accuracy of ε^2 will be

$$M_x = M_{x,\text{Kirch}}\left(1 + \frac{3}{65}\,\pi^2\varepsilon^2\right). \tag{41}$$

We compare these results with the values obtained after some well known refined theories.

In an older variant [9] A. J. LURJE received the equation

$$\Delta^2 w - \frac{8 - 3\nu}{1 - \nu}\,\frac{h^2}{10}\,\Delta^3 w = \frac{p}{D} \tag{42}$$

(all the other equations will be given in our notation). By integrating $\tilde{w}(x, y, z)$ we obtain the average value

$$\tilde{w}_{\text{mean}} = w - \frac{1 - \nu}{1 + \nu}\,\frac{h^2}{6}\,\Delta w\,. \tag{43}$$

When substituting, we obtain

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}}\left(1 + \frac{1421}{3672}\,\pi^2\varepsilon^2\right) \doteq w_{\text{Kirch}}\left(1 + \frac{27{,}2}{70}\,\pi^2\varepsilon^2\right). \tag{44}$$

In the later formulation of LURJE's theory it is

$$\Delta^2 w - \frac{1}{5}\,h^2\Delta^3 w = \frac{p}{D}\,, \tag{45}$$

and the avarage value of the deflection can be obtained by integrating the equations given in [10]

$$\tilde{w}_{\text{mean}} \doteq w - \frac{1}{3}\,\frac{3 - 2\nu}{1 - \nu}\,h^2\Delta w\,. \tag{46}$$

By substituting and rearrangement it will be

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}}\left(1 + \frac{33}{70}\,\pi^2\varepsilon^2\right). \tag{47}$$

According to the well known Reissnerian theory it is [11]

$$\Delta^2 w_A = \frac{1}{D}\left(p - \frac{2}{5}\frac{2 - \nu}{1 - \nu} h^2 \Delta p\right). \tag{48}$$

We should like to point out that in equation (48) w_A may be comprehended as weighted average deflection, resulting from the energetic balance

$$\int_{-h}^{h} \tau_{nz} \tilde{w} \, dz = Q_n w_A , \tag{49}$$

where Q_n is the shear force

$$Q_n = \int_{-h}^{h} \tau_{nz} \, dz . \tag{50}$$

By substituting into (48) we obtain for $\nu = 3/10$

$$w_A = w_{\text{Kirch}}\left(1 + \frac{34}{70} \pi^2 \varepsilon^2\right). \tag{51}$$

Let us note that according the latter theory, the bending moment M_x is

$$M_x = M_{x,\text{Kirch}}\left(1 + \frac{3}{65} \pi^2 \varepsilon^2\right). \tag{52}$$

Another theory was presented by HENCKY [12]. According to this theory, the vertical deflection is not dependent on z, and for our case it is enough to consider only one differential equation

$$\Delta^2 w = \frac{1}{D}\left(p - \frac{2}{3} h^2 \Delta p\right). \tag{53}$$

After substituting the loading (31), the deflection can be easily established

$$\tilde{w}_{\text{mean}} = w_{\text{Kirch}}\left(1 + \frac{33,3}{70} \pi^2 \varepsilon^2\right). \tag{54}$$

After MUŠTARI's refined theory it is [13]

$$\Delta^2 w + \frac{\nu}{1 - \nu} \frac{h^2}{10} \Delta^3 w = \frac{1}{D}\left(p - \frac{2}{5}\frac{2 - \nu}{1 - \nu} h^2 \Delta p\right) \tag{55}$$

and

$$\tilde{w}(x, y, z) = w + \frac{\nu z^2}{2(1 - \nu)} \Delta w . \tag{56}$$

After integrating, substituting and rearranging we obtain

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}}\left(1 + \frac{34}{70}\pi^2\varepsilon^2\right). \tag{57}$$

In AMBARCUMJAN's book [14] we can find several alternative methods for the case of anisotropic plates. One of these theories in the case of an isotropic plate leads to the equation (p. 332)

$$\Delta^2 w = \frac{1}{D}\left[p - \frac{2}{5}\frac{2 - \nu + \nu^3}{1 - \nu} h^2 \Delta p\right] \tag{58}$$

and in the case of the assumed independence of w on z it will be

$$\tilde{w}_{\text{mean}} = w_{\text{Kirch}}\left(1 + \frac{1727}{3500}\pi^2\varepsilon^2\right) \doteq w_{\text{Kirch}}\left(1 + \frac{34,5}{70}\pi^2\varepsilon^2\right). \tag{59}$$

Further ([14] p. 338) the results of the solutions corresponding to different supposed forms of the distribution of the shear stress τ_{nz}, and the shear strain γ_{nz} along the height of the plate are given, and the slight sensibility of the solutions on the form of the assumed distribution has been found. For the coefficients of $\pi^2\varepsilon^2$ the values 40/70, 38,1/70 and 33,3/70 were obtained ([14], eq. 6.19).

After RAYMONDI's theory [15] we obtain the equations

$$\Delta^2 w = \frac{1}{D}\left[p - \frac{h^2}{(1 - \nu)}\Delta p\right], \tag{60}$$

$$\Delta^2 t = \frac{1}{(1 - \nu) D}\Delta p, \tag{61}$$

where the deflection $\tilde{w}(x, y, z)$ is

$$\tilde{w}(x, y, z) = w(x, y) + z^2 t(x, y). \tag{62}$$

When substituting and rearranging, we obtain

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}}\left(1 + \frac{33,33}{70}\pi^2\varepsilon^2\right). \tag{63}$$

After RAYMONDI's simplified theory it is

$$\Delta^2 w = \frac{1}{D}\left[p - \frac{4h^2}{5(1 - \nu)}\Delta p\right], \tag{64}$$

where it is assumed that

$$\tilde{w}(x, y, z) = w(x, y). \tag{65}$$

By a substitution we get

$$\tilde{w}_{\text{mean}} = w_{\text{Kirch}} \left(1 + \frac{40}{70} \pi^2 \varepsilon^2 \right). \tag{66}$$

The variant of the refined theory, presented by KHAČATURJAN [16] leads to two differential equations, of which it is sufficient to take into consideration only one

$$\Delta^2 w = \frac{p}{D}, \tag{67}$$

and the deflection along the height will be

$$\tilde{w} = w - \left(\frac{8 - 3\nu}{1 - \nu} \frac{h^2}{10} - \frac{\nu}{1 - \nu} \frac{z^2}{2} \right) \Delta w . \tag{68}$$

By a substitution and rearrangement the average deflection will be

$$\tilde{w}_{\text{mean}} = w_{\text{Kirch}} \left(1 + \frac{33}{70} \pi^2 \varepsilon^2 \right). \tag{69}$$

The further theory of KHAČATURJAN [17] is a little different. It also leads to a system of two equations, of which, in the given case, one is sufficient

$$\Delta^2 w = \frac{1}{D} \left(p + \frac{2}{5} \frac{\nu}{1 - \nu} h^2 \Delta p \right), \tag{70}$$

and along the height the average deflection $\tilde{w}_{\text{mean}}$ can be determined from the equation

$$\tilde{w}_{\text{mean}} = \alpha w - \frac{2}{3} \frac{h^2}{1 - \nu} \Delta w + \frac{4}{15} \frac{\nu}{(1 - \nu)^2} \frac{h^2}{D} p . \tag{71}$$

In the last equation the coefficient α is in connection with the assumption of a non-linear distribution of the displacements $\tilde{u}$ and $\tilde{v}$ along the height of the plate. For a linear distribution $\alpha = 1$. Khačaturjan suggests to find the the constant α experimantally. The resulting equation will be

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}} \left(\alpha + \frac{33{,}33 - 6\alpha}{70} \pi^2 \varepsilon^2 \right), \tag{72}$$

which for $\alpha = 1$ yields a relatively low value. The SUŁOCKI arrangement of Lurje's equations [18] for the technical theory of plates acquires the relation (33) and the deflection $\tilde{w}$ is the same as that in Lurje's equation (43). We receive then

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}} \left(1 + \frac{40}{70} \pi^2 \varepsilon^2 \right). \tag{73}$$

When considering the average weighted value according to (49), the Sułocki arrangement of Lurje's equations yields

$$w_A \doteq w_{\text{Kirch}} \left(1 + \frac{41}{70} \pi^2 \varepsilon^2 \right). \tag{74}$$

In the given case the theories of V. PANC [19], M. ŠEREMETJEV and B. L. PELEKH [20] lead, also to the equation (64) which holds along with equation (65). That's why we obtain identical results with (66).

In conclusion let us present here the exact solution of the discussed case, when starting from the threedimensional equations of Lamé. On the basic of the equations given by B. F. VLASOV [21] after some rearrangements one may arrive to the equation

$$\tilde{w}(x, y, z) = w_{\text{Kirch}}(x, y) \cdot \frac{(\sqrt{(2)}\, \pi\varepsilon)^3}{12(1 - \nu)(\sinh \sqrt{(2)}\, \pi\varepsilon - \sqrt{(2)}\, \pi\varepsilon)} \tag{75}$$
$$\left\{ - \left[2(\nu - 1) \cdot \cosh \frac{\sqrt{(2)}\, \pi\varepsilon}{2} + \frac{\sqrt{(2)}\, \pi\varepsilon}{2} \sinh \frac{\sqrt{(2)}\, \pi\varepsilon}{2} \right] \cosh \frac{\sqrt{(2)}\, \pi\varepsilon}{h} z + \right.$$
$$\left. + \frac{\sqrt{(2)}\, \pi\varepsilon}{h} z \cosh \frac{\sqrt{(2)}\, \pi\varepsilon}{2} \sinh \frac{\sqrt{(2)}\, \pi\varepsilon}{h} z \right\}.$$

When integrating according to z, we may obtain the exact value of the average deflection

$$\tilde{w}_{\text{mean}} = w_{\text{Kirch}} \cdot \frac{(\sqrt{(2)}\, \pi\varepsilon)^3}{6(\sinh \sqrt{(2)}\, \pi\varepsilon - \sqrt{(2)}\, \pi\varepsilon)} \, \frac{1 - (3 - 2\nu)(\sinh \sqrt{(2)}\, \pi\varepsilon)/\sqrt{(2)}\, \pi\varepsilon}{2(1 - \nu)}. \tag{76}$$

When we expand the individual terms in (76) into series, we obtain for $\nu = 3/10$

$$\tilde{w}_{\text{mean}} \doteq w_{\text{Kirch}} \left(1 + \frac{33}{70} \pi^2 \varepsilon^2 \right). \tag{77}$$

The equation represents the exact solution of the threedimensional problem with an accuracy up to the quadratic term in ε. A comparison of the ratio 33/70, as that of an exact value, with the other coefficients, shows the accuracy of the various approximative theories (see table 1). This comparison shows that the Reissnerian algorithmi make it possible to obtain exact results up to the power ε^2 and a good agreement can be expected for other loadings as well.

Further it becomes obvious, that those theories that take into consideration the influence of the shear forces only, are less accurate than those that involve in their calculations also the stress σ_z, or the strain ε_z. The difference is about ε^2 and it becomes more pronounced for thicker plates.

The analysis of stresses could be carried out in a similar way, but this is not the scope of the present paper.

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Table 1.

Comparison of the results obtained from the solution of a square plate by various refined theories

$$w_{\text{ref}} \doteq w_{\text{Kirch}}(1 + \varrho\pi^2\varepsilon^2) \quad \varepsilon = n\frac{2h}{a}$$

Author		Reference	Equation	$1 + \varrho\pi^2\varepsilon^2$
Reissnerian algorithmi $[N = M]$ and related cases $[N \neq M]$	One free funct. $N = 1, M = 0$		(7), (12)	0,8
	one free funct. $N = M = 1$		(14), (15)	$1 + \frac{24,6}{70}\pi^2\varepsilon^2$
	two free funct. $N = M = 1$		(17), (19)	$1 + \frac{28,2}{70}\pi^2\varepsilon^2$
	one free funct. $N = 2, M = 1$		(20), (21)	$1 + \frac{33}{70}\pi^2\varepsilon^2$
	one free funct. $N = M = 2$		(23), (24)	$1 + \frac{33}{70}\pi^2\varepsilon^2$
Lurje — 1936		[9]	(42), (43)	$1 + \frac{27,2}{70}\pi^2\varepsilon^2$
Lurje — 1942		[10]	(45), (46)	$1 + \frac{33}{70}\pi^2\varepsilon^2$
Reissner — 1944		[11]	(48)	$1 + \frac{34}{70}\pi^2\varepsilon^2$
Hencky — 1947		[12]	(53)	$1 + \frac{33,3}{70}\pi^2\varepsilon^2$
Muštari — 1959		[13]	(55), (56)	$1 + \frac{34}{70}\pi^2\varepsilon^2$
Ambarcumjan — 1961		[14]	(58)	$1 + \frac{34,5}{70}\pi^2\varepsilon^2$
		[14]	6.19 pp. 338	$1 + \frac{40}{70}\pi^2\varepsilon^2$
		[14]	6.19 pp. 338	$1 + \frac{38,1}{70}\pi^2\varepsilon^2$
		[14]	6.19 pp. 338	$1 + \frac{33,3}{70}\pi^2\varepsilon^2$

Table 1 (continued)

Author	Reference	Equation	$1 + \varrho^2\pi^2\varepsilon^2$
Raymondi — 1963	[15]	(60), (61), (62)	$1 + \frac{33,3}{70}\pi^2\varepsilon^2$
	[15]	(64)	$1 + \frac{40}{70}\pi^2\varepsilon^2$
Khačaturjan — 1963	[16]	(67), (68)	$1 + \frac{33}{70}\pi^2\varepsilon^2$
	[17]	(70), (71)	$\alpha + \frac{33,3 - 6\alpha}{70}\pi^2\varepsilon^2$
Sułocki — 1964	[18]	(33), (43)	$1 + \frac{40}{70}\pi^2\varepsilon^2$
	[18]	(33), (49)	$1 + \frac{41}{70}\pi^2\varepsilon^2$
Panc — 1964	[19]	(64)	$1 + \frac{40}{70}\pi^2\varepsilon^2$
Šeremetjev-Pelekh — 1964	[20]	(64)	$1 + \frac{40}{70}\pi^2\varepsilon^2$
Vlasov B. F. — 1957 — exact solution	[21]	(75), (76)	$1 + \frac{33}{70}\pi^2\varepsilon^2$

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Súhrn

PRÍSPEVOK K REISSNEROVSKÝM ALGORITMOM V TEÓRII OHYBU PRUŽNÝCH DOSÁK

ALEXANDER HANUŠKA

V článku sú študované niektoré varianty spresnených teórií ohybu izotropných dosák, pričom sa vychádza z metódy tzv. Reissnerovských algoritmov zavedených v teórii pružnosti I. Babuškom a M. Prágerom. Na základe Lagrangeovho variačného princípu sú odvodené diferenciálne rovnice problému a im zodpovedajúce okrajové podmienky pre rôzne varianty Reissnerovských algoritmov. Ako príklad sa vyšetruje štvorcová doska, zaťažená symetrickým kopovitým zaťažením. Porovnanie priehybu podľa viacerých variánt Reissnerovských algoritmov s výsledkami riešení podľa niektorých známych spresnených teórií ukázalo veľmi dobrú zhodu riešení podľa Reissnerovských algoritmov vyšších priblížení s presným riešením.

Резюме

К РЕЙСНЕРОВСКИМ АЛГОРИФМАМ В ТЕОРИИ ИЗГИБА УПРУГИХ ПЛАСТИНОК

АЛЕКСАНДЕР ГАНУШКА (ALEXANDER HANUŠKA)

Исходя из метода так. наз. рейснеровских алгорифмов, были в статье исследованы некоторые варианты уточненных теорий изгиба изотропных пластинок. На основе вариационного принципа Лагранжа выведены дифференциальные уравнения и соответствующие им граничные условия. В качестве примера была исследована квадратная пластинка с коплевидной нагрузкой и дано сравнение прогиба, определенного по различным вариантам рейснеровских алгорифмов, с решениями по некоторым известным уточненным теориям. Сравнение показывает, что решения по высшим приближениям рейснеровских алгорифмов практически совпадают с точным трехмерным решением.

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