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Vladimír Fiřt

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PLASTICITY CONDITIONS OF POLYCRYSTALLINE MATERIAL WITH ACICULAR STRUCTURE [PRELIMINARY COMMUNICATION]

VLADIMÍR FIŘT

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Plasticity conditions or conditions of strength, respectively, of the polycrystalline elastic isotropic material, the structure of which is created by an intergrowth of irregularly grouped acicular crystals, were derived under the following assumptions:

1. The material behaves as a space truss with elastic joints, the last being the very places of an intergrowth or contact, respectively, of acicular crystals, which we consider to be rods.

2. Bending deformations of the crystals are negligible.[1])

The result of these assumptions in represented by the axial stress of each crystal in the course of elastic strains of the investigated body.

The criterions for the origin of plastic strains in a point are defined by the following equations:

$$
\begin{aligned}
(1) \qquad & \max N_c = {}^{(t)}N_c && (\text{for } N_c > 0, \text{ traction}), \\
& \max |N_c| = \min\left(|{}^{(u)}N_c|, |{}^{(cr)}N_c|\right) && (\text{for } N_c < 0, \text{ compression}),
\end{aligned}
$$

where N_c is the axial force in the crystal c, ${}^{(t)}N_c$ is the ultimate tension force, under whose influence the crystal tears in two or grows loose at the place of the intergrowth, ${}^{(u)}N_c$ is the ultimate compressive force by which the crystal breaks or gets smashed and ${}^{(cr)}N_c$ is the critical axial force, by which the crystal buckles, as a result of shear deformations of the crystal joint.

According to the Hooke's law, the axial force N_c is given by the expression

$$(2) \qquad N_c = k_c e_c$$

in which k_c is the constant of proportionality and e_c is the unit deformation (the unit extension $e_c > 0$ or the unit linear contraction $e_c < 0$) of the crystal axis c of

[1]) The suggestion to introduce these assumptions was made by S. MODRÝ.

the intensity

(3) $$e_c = e_1 l^2 + e_2 m^2 + e_3 n^2 ,$$

where e_1, e_2, e_3 are the principal strains and l, m, n are the direction cosines, determining the crystal axis direction c in Haigh's space.

The value of the constant k_c was obtained from the equation

(4) $$\Pi_D = \Pi_N$$

in which Π_D represents the total strain work of the volume unit of a cubic form with the corner $h = 1$ and Π_N is the axial forces work of all crystals contained in this volume unit.

If we identify the corners of the unit cube with the directions of the principal stresses $\sigma_1, \sigma_2, \sigma_3$, we obtain

(5) $$k = \frac{\sigma_1 e_1 + \sigma_2 e_2 + \sigma_3 e_3}{\frac{1}{s}\sum_1^s (e_1 l^2 + e_2 m^2 + e_3 n^2)^2} \frac{1}{sd}$$

in which was made use of the relation

$$\sum_1^s k_c d_c e_c^2 \doteq kd \sum_1^s e_c^2 ,$$

where k is the average value of the constants k_c, d is the average length of the crystal and s is the amount of crystals in the volume unit.

The crystal axes of an isotropic body are oriented irregularly, so that no direction prevails. In this case, the denominator of the first factor in Eq. (5) is the mean value f_0 of the function

(6) $$f = (e_1 l^2 + e_2 m^2 + e_3 n^2)^2$$

which is defined in the space having the basis $\varphi_1, \varphi_2, \varphi_3$ on the surface (Φ), determined by the equation

(7) $$\varphi_3 = \arccos \sqrt{(1 - \cos^2 \varphi_1 - \cos^2 \varphi_2)} ,$$

where $\varphi_1, \varphi_2, \varphi_3$ are the angles between the crystal axis and the axes $\sigma_1, \sigma_2, \sigma_3$ ($l = \cos \varphi_1$, $m = \cos \varphi_2$, $n = \cos \varphi_3$). According to (6) and (7), f is the function of two variables φ_1, φ_2.

Realizing that the number s is large, we can apply the surface integral instead of the sum

(8) $$f_0 = \frac{1}{\Phi} \iint_{\Phi} f[\varphi_1, \varphi_2, \varphi_3(\varphi_1, \varphi_2)] \, d\Phi .$$

Transferring the integral in the Eq. (8) into the integral of the plane area F, which is the projection of the surface Φ into the plane φ_1, φ_2 (Fig. 1), we obtain from the Eq. (7)

$$\mathrm{d}\Phi = R(\varphi_1, \varphi_2)\,\mathrm{d}F\,, \tag{9}$$

where $\mathrm{d}F = \mathrm{d}\varphi_1\,\mathrm{d}\varphi_2$ and

$$R(\varphi_1, \varphi_2) = \frac{1}{2}\sqrt{\left(4 + \frac{\sin^2 2\varphi_1 + \sin^2 2\varphi_2}{(\cos^2\varphi_1 + \cos^2\varphi_2)\left[1 - (\cos^2\varphi_1 + \cos^2\varphi_2)\right]}\right)}\,. \tag{10}$$

Substituting the expressions (6), (9) and (10) in the Eq. (8), we obtain

$$\begin{aligned} f_0 J_6 = {} & J_1 e_1^2 + J_2 e_2^2 + \left[J_1 + J_2 + J_6 + 2(J_3 - J_4 - J_5)\right] e_3^2 + \\ & + 2J_3 e_1 e_2 + 2(J_4 - J_1 - J_3)\,e_1 e_3 + 2(J_5 - J_2 - J_3)\,e_2 e_3 \end{aligned} \tag{11}$$

of which

$$J_j = \int_0^{\pi/2} \mathrm{d}\varphi_1 \int_{\pi/2-\varphi_1}^{\pi/2} \Psi_j(\varphi_1, \varphi_2)\,R(\varphi_1, \varphi_2)\,\mathrm{d}\varphi_2\,, \quad j = 1, 2, \ldots, 6\,, \tag{12}$$

where

$$\begin{aligned} &\Psi_1 = \cos^4\varphi_1\,, \quad \Psi_2 = \cos^4\varphi_2\,, \quad \Psi_3 = \cos^2\varphi_1\cos^2\varphi_2\,, \\ &\Psi_4 = \cos^2\varphi_1\,, \quad \Psi_5 = \cos^2\varphi_2\,, \quad \Psi_6 = 1 \end{aligned} \tag{13}$$

and $J_6 = \frac{1}{4}\Phi$, because, with respect to the symmetry, the integration is carried out in the triangle area as shown in Fig. 1.

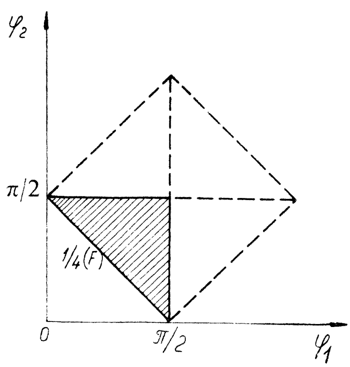

Fig. 1.

The integrals J_j, $j = 1, 2, \ldots, 6$, are infinite integrals, because the function R is not bound in the neighbourhood of the straight line $\varphi_2 = \frac{1}{2}\pi - \varphi_1$. The numerical method was used for the evaluation of these integrals, by an uneven division of the area $\frac{1}{4}F$ into 181 plane parts. We have obtained after a rounding-off

$$f_0 = \tfrac{1}{5}(e_1^2 + e_2^2 + e_3^2) + \tfrac{2}{15}(e_1 e_2 + e_1 e_3 + e_2 e_3)\,. \tag{14}$$

In regard to Eqs. (1) and (2), there are

$$\begin{aligned} &k\,.\,\max e \leqq {}^{(t)}N\,, \\ &k\,.\,\max |e| \leqq \min\left(|{}^{(u)}N|, |{}^{(cr)}N|\right), \end{aligned} \tag{15}$$

where ${}^{(t)}N$, ${}^{(u)}N$, ${}^{(cr)}N$ are the average values of the ultimate axial forces in crystals.

With $e_1 \geqq e_2 \geqq e_3$ and $e_1 > 0$, there is $\max e = e_1$ and with respect to (5) and (14), the first of inequalities (15), after the substitution of the generalized Hooke's law, is expressed by

$$\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)}{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + A(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right] \leqq \sigma_{YP}^{(+)}, \tag{16}$$

where ν is the Poisson's ratio,

$$A = \frac{\frac{2}{3} - \frac{16}{3}\nu + 4\nu^2}{1 - \frac{4}{3}\nu + \frac{8}{3}\nu^2} \tag{17}$$

and

$$\sigma_{YP}^{(+)} = \tfrac{1}{5}sd(1 - \tfrac{4}{3}\nu + \tfrac{8}{3}\nu^2)\,{}^{(t)}N \tag{18}$$

is the yield-point in tension (the tensile strength for short material) at the uniaxial stress of $\sigma_1 > 0$ ($\sigma_2 = \sigma_3 = 0$).

In the same way, we obtain from the second of inequalities (15)

$$\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)}{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + A(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)}\left|\sigma_3 - \nu(\sigma_1 + \sigma_2)\right| \leqq \sigma_{YP}^{(-)}, \tag{19}$$

where

$$\sigma_{YP}^{(-)} = \tfrac{1}{5}sd(1 - \tfrac{4}{3}\nu + \tfrac{8}{3}\nu^2)\min\left(\left|{}^{(u)}N\right|, \left|{}^{(cr)}N\right|\right) \tag{20}$$

s the yield-point in compression (the compressive strength for short material) at the uniaxial stress of $\sigma_3 < 0$ ($\sigma_1 = \sigma_2 = 0$).

Values of the ratio $|\sigma_3|/\sigma_{YP}^{(-)}$ at the plastic limit (at strength limit), calculated from the Eq. (19) for three states of stress, are given in Table I.

Table I. Limiting values of the ratio $|\sigma_3|/\sigma_{YP}^{(-)}$

ν	0·0	0·1	0·2	0·25	0·3	0·4	0·5
$\sigma_1 = \sigma_2 = \sigma_3$	1·6667	1·8657	1·9841	2·0000	1·9841	1·8657	1·6667
$\sigma_1 = 0,\ \sigma_2 = \sigma_3$	1·3333	1·3543	1·3393	1·3333	1·3444	1·4925	2·0000
$\sigma_1 = 0,\ \sigma_2 = -\sigma_3$	0·6667	0·7463	0·7937	0·8000	0·7937	0·7463	0·6667

Author's address: Ing. *Vladimír Fiřt* C.Sc., Ústav teoretické a aplikované mechaniky ČSAV, Vyšehradská 49, Praha 2.