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PŘEDBĚŽNÁ SDĚLENÍ

GENERAL STRESS-STRAIN RELATIONSHIPS OF ANISOTROPIC BODIES AND THE CONCEPT OF THE TRANSFORMED STRAIN

[ADVANCED NOTE]

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The author presents the general stress-strain relationships and the law of the deformation theory of plasticity of anisotropic bodies.

The general relation between the stress and strain components may be expressed in the following form

$$\sigma_{ij} = f_{ij}(B_{klmn}\varepsilon_{mn}), \tag{1}$$

where B_{klmn} are the components of the fourth-rank tensor of anisotropy.

Introducing the transformed strain tensor of the rank two

$$\beta_{kl} = B_{klmn}\varepsilon_{mn}, \tag{2}$$

we may consider the general function

$$\sigma_{ij} = f_{ij}(\beta_{kl}) \tag{3}$$

of two coaxial tensors σ_{ij} and β_{kl}.

The preceding function may be, under certain conditions, developed into absolutely convergent power series as follows

$$\sigma_{ij} = A_0\delta_{ij} + A_1\beta_{ij} + A_2\beta_{i\alpha}\beta_{\alpha j} + A_3\beta_{i\alpha}\beta_{\alpha\beta}\beta_{\beta j} + \dots \tag{4}$$

where A_0, A_1, A_2, A_3, etc. are scalar coefficients and δ_{ij} is the Kronecker delta.

The left-hand side of (4) being a symmetrical tensor of the second rank, it follows from the tensorial dimensionality that the absolutely convergent series of the terms on the right-hand side is also represented by symmetrical tensors of rank two, which may be expressed according to the Cayley-Hamilton theorem in terms of three principal tensors

$$\delta_{ij}, \quad \beta_{ij} = B_{ijkl}\varepsilon_{kl}, \quad \beta_{i\alpha}\beta_{\alpha j} = B_{i\alpha kl}B_{\alpha jmn}\varepsilon_{kl}\varepsilon_{mn}$$

and by functions of the three principal transformed strain invariants

$$(5) \qquad I_\beta = \beta_{ij}\delta_{ij} = B_{ijkl}\delta_{ij}\varepsilon_{kl}\,,$$

$$(6) \qquad II_\beta = \beta_{ij}\beta_{ij} = B_{ijkl}B_{ijmn}\varepsilon_{kl}\varepsilon_{mn}\,,$$

$$(7) \qquad III_\beta = \beta_{ij}\beta_{i\alpha}\beta_{\alpha j} = B_{ijkl}B_{i\alpha mn}B_{\alpha jpq}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\,.$$

Then we have the following constitutive stress-strain relation

$$(8) \qquad \sigma_{ij} = \Phi_0\delta_{ij} + \Phi_1 B_{ijkl}\varepsilon_{kl} + \Phi_2 B_{i\alpha kl}B_{\alpha jmn}\varepsilon_{kl}\varepsilon_{mn}\,.$$

The scalar functions of the invariants Φ_0, Φ_1, Φ_2 follow from three equations which are analoguous to those for isotropic materials

$$(9) \qquad \sigma_{ij}\delta_{ij} = 3\Phi_0 + \Phi_1 B_{ijkl}\delta_{ij}\varepsilon_{kl} + \Phi_2 B_{ijkl}B_{ijmn}\varepsilon_{kl}\varepsilon_{mn}\,,$$

$$(10) \qquad \begin{aligned}\sigma_{ij}\sigma_{ij} = {} & 3\Phi_0^2 + \Phi_1^2 B_{ijkl}B_{ijmn}\varepsilon_{kl}\varepsilon_{mn} + \\ & + \Phi_2^2 B_{i\alpha kl}B_{\alpha jmn}B_{i\beta pq}B_{\beta jrs}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\varepsilon_{rs} + \\ & + 2\Phi_0\Phi_1 B_{ijkl}\delta_{ij}\varepsilon_{kl} + 2\Phi_0\Phi_2 B_{ijkl}B_{ijmn}\varepsilon_{kl}\varepsilon_{mn} + \\ & + 2\Phi_1\Phi_2 B_{ijkl}B_{i\alpha mn}B_{\alpha jpq}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\,,\end{aligned}$$

$$(11) \qquad \begin{aligned}\sigma_{ij}\sigma_{i\alpha}\sigma_{\alpha j} = {} & 3\Phi_0^3 + \Phi_1^3 B_{ijkl}B_{i\alpha mn}B_{\alpha jpq}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq} + \\ & + \Phi_2^3 B_{ijkl}B_{i\alpha mn}B_{\alpha\beta pq}B_{\beta\gamma rs}B_{\gamma\delta tu}B_{\delta jab}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\varepsilon_{rs}\varepsilon_{tu}\varepsilon_{ab} + \\ & + 3\Phi_0^2\Phi_1 B_{ijkl}\delta_{ij}\varepsilon_{kl} + 3\Phi_0\Phi_1^2 B_{ijkl}B_{ijmn}\varepsilon_{kl}\varepsilon_{mn} + \\ & + 3\Phi_0^2\Phi_2 B_{ijkl}B_{ijmn}\varepsilon_{kl}\varepsilon_{mn} + 3\Phi_0\Phi_2^2 B_{ijkl}B_{i\alpha mn}B_{\alpha\beta pq}B_{\beta jrs}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\varepsilon_{rs} + \\ & + 3\Phi_1^2\Phi_2 B_{ijkl}B_{i\alpha mn}B_{\alpha\beta pq}B_{\beta jrs}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\varepsilon_{rs} + \\ & + 3\Phi_1\Phi_2^2 B_{ijkl}B_{i\alpha mn}B_{\alpha\beta pq}B_{\beta\gamma rs}B_{\gamma jtu}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\varepsilon_{rs}\varepsilon_{tu} + \\ & + 6\Phi_0\Phi_1\Phi_2 B_{ijkl}B_{i\alpha mn}B_{\alpha jpq}\varepsilon_{kl}\varepsilon_{mn}\varepsilon_{pq}\,.\end{aligned}$$

The third term in (8) represents the second-order effects. In the case of infinitesimal deformation, (8) becomes

$$(12) \qquad \sigma_{ij} = \Phi_0\delta_{ij} + \Phi_1 B_{ijkl}\varepsilon_{kl}\,.$$

The invariant functions may be expressed from

$$(13) \qquad \sigma_{ij}\delta_{ij} = 3\Phi_0 + \Phi_1 B_{ijkl}\delta_{ij}\varepsilon_{kl}\,,$$

$$(14) \qquad \sigma_{ij}\sigma_{ij} = 3\Phi_0^2 + 2\Phi_0\Phi_1 B_{ijkl}\delta_{ij}\varepsilon_{kl} + \Phi_1^2 B_{i\alpha kl}B_{\alpha jmn}\varepsilon_{kl}\varepsilon_{mn}$$

after introducing the relations (5) and (6) as follows

$$(15) \qquad \Phi_0 = \frac{1}{3}\left(I_\sigma - I_\beta\sqrt{\frac{3II_\sigma - I_\sigma^2}{3II_\beta - I_\beta^2}}\right),$$

$$(16) \qquad \Phi_1 = \sqrt{\frac{3II_\sigma - I_\sigma^2}{3II_\beta - I_\beta^2}}\,,$$

where $I_\sigma = \sigma_{ij}\delta_{ij}$, $II_\sigma = \sigma_{ij}\sigma_{ij}$ are the invariants of the stress tensor.

After some rearrangements, the author has obtained the stress-strain relations of the deformation theory of plasticity for anisotropic bodies,

$$\sigma_{ij} - \sigma\delta_{ij} = \frac{2\sigma_i}{3\beta_i}(\beta_{ij} - \beta\delta_{ij}), \tag{17}$$

where $\sigma = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$ is the mean stress

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)]}$$

the effective stress, $\beta_{ij} = B_{ijkl}\varepsilon_{kl}$ the transformed strain components,

$$\beta_i = \frac{\sqrt{2}}{3}\sqrt{[(\beta_{11} - \beta_{22})^2 + (\beta_{22} - \beta_{33})^2 + (\beta_{33} - \beta_{11})^2 + 6(\beta_{12}^2 + \beta_{23}^2 + \beta_{31})^2]}$$

the transformed effective strain and $\beta = \frac{1}{3}(\beta_{11} + \beta_{22} + \beta_{33})$ the transformed mean strain.

Then, the concept of the transformed strain makes it possible to express the stress-strain relationships for anisotropic bodies in a manner analogous to that of the isotropic case.

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