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CAUCHY SEQUENCES IN $\mathscr{L}$-GROUPS

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The relationship between Cauchy sequences in an $\mathscr{L}$-group G and Cauchy filters in the first countable filter modification γG of G (introduced by R. Beattie and H.-P. Butzmann in [4]) is investigated. In particular, an $\mathscr{L}$-group G (without the Urysohn axiom of convergence) and a Cauchy sequence S in G such that the corresponding elementary filter of sections of S fails to be a Cauchy filter in γG is constructed.

1.

In what follows, N denotes the positive integers, MON the set of all strictly monotone mappings of N into N and $FTON$ the set of all finite-to-one mappings of N into N (i.e., $\{n \in N;\ s(n) = k\}$ is a finite set whenever $s \in FTON$ and $k \in N$). Let G be a nonempty set; a sequence $S = \langle S(n) \rangle$ of points of G is a mapping of N into G, and for $s \in MON$ the composition $S \circ s$ denotes the subsequence of S the n-th term of which is $S(s(n))$; for $x \in G$, $\langle x \rangle$ denotes the constant sequence each term of which is x; if S, T are sequences in G, then $S \wedge T$ is defined by $(S \wedge T) . (2n-1) = S(n)$ and $(S \wedge T)(2n) = T(n)$, $n \in N$; if S is a sequence in G then the sets $\{S(n);\ n > k\}$, $k \in N$, form a base of the so-called *elementary (Fréchet) filter* $\mathscr{F}(S)$ of sections of S; by a sequential convergence on G we understand a subset $\mathfrak{G} \subset G^N \times G$ satisfying certain axioms of convergence (throughout the paper we assume that every constant sequence $\langle x \rangle$ converges to x, each subsequence of a convergent sequence converges to the same limit and, with the exception of Proposition 1 and Proposition 2, every convergent sequence has a unique limit), $(S, x) \in \mathfrak{G}$ means that S converges (i.e. $\mathfrak{G}$-converges) to x, and for $x \in G$ the set of all sequences converging to x is denoted by $\mathfrak{G}^{\leftarrow}(x)$. Let G be a group equipped with a sequential convergence $\mathfrak{G}$ such that $(ST^{-1}, xy^{-1}) \in \mathfrak{G}$ whenever $(S, x) \in \mathfrak{G}$ and $(T, y) \in \mathfrak{G}$. Then $(G, \mathfrak{G})$, or simply G, is said to be an *$\mathscr{L}$-group* (cf. [7]). We are mainly interested in abelian groups and in such cases the additive notation will be used.

Besides the basic axioms of convergence, we consider the following ones (cf. [6]):

$(\mathscr{F}\mathscr{L})$ if $(S, x) \in \mathfrak{G}$ and $\mathscr{F}(S) = \mathscr{F}(T)$, then $(T, x) \in \mathfrak{G}$;

$(\mathscr{M}\mathscr{L})$ if (S, x), $(T, x) \in \mathfrak{G}$, then $(S \wedge T, x) \in \mathfrak{G}$.

Starting with a filter convergence λ on a set X (we assume that for each $x \in X$ the ultrafilter $\dot{x}$ converges to x, and if a filter converges to x, then each finer filter converges to x), the most natural way to define a sequential convergence on X is to let a sequence S converge to a point x whenever the elementary filter $\mathscr{F}(S)$ λ-converges to x; denote by $\mathscr{L}(\lambda)$ the resulting sequential convergence. As shown in [4], [2], [1] and [3], among all known opposite functors (assigning to suitable sequential convergences certain filter convergences) the one introduced by R. Beattie and H.-P. Butzmann plays a fundamental role: starting with a sequential convergence $\mathfrak{L}$ on X, a filter $\mathscr{F}$ on X converges to a point x whenever there is a finer filter $\mathscr{G}$ with a countable basis such that every sequence $\mathfrak{L}$-converges to x whenever $\mathscr{F}(S) \supset \mathscr{G}$; denote by $\gamma(\mathfrak{L})$ the resulting filter convergence.

The importance of γ follows, for instance, from the fact that the Novák completion of an abelian sequential convergence (the convergence is maximal, i.e., satisfies the Urysohn axiom) group G (cf. [11], [8]) can be constructed via the completion of the filter convergence group γG (see Corollary 3.16 in [1]) and, for every sequentially determined filter convergence group H (i.e. $H = \gamma\mathscr{L}H$) with a maximal sequential convergence, the completion of H can be constructed via the Novák completion of $\mathscr{L}H$ (see Corollary 3.18 in [1], cf. Theorem 8 in [3]). This is partly due to the fact that in case of a maximal sequential convergence a sequence S is Cauchy in G iff $\mathscr{F}(S)$ is a Cauchy filter in γG. In view of Proposition 3.11 in [1], if the sequential convergence in G is not maximal then this might be not true any more. Indeed, answering a question by R. Beattie and H.-P. Butzmann, we construct an $\mathscr{L}$-group G and a Cauchy sequence S such that $\mathscr{F}(S)$ fails to be a Cauchy filter in γG.

Our construction is based on the fact that in a group G every compatible sequential convergence on G can be identified with a certain subgroup of G^N. The straightforward proofs of the next two propositions are omitted. Similar propositions (with different axioms of convergence) can be found in [9] and [12].

Proposition 1. *Let $(G, \mathfrak{G})$ be an $\mathscr{L}$-group and let e be the neutral element of G. Then $\mathfrak{G}^{\leftarrow}(e)$ has the following properties:*

(i) *$\mathfrak{G}^{\leftarrow}(e)$ is a subgroup of G^N;*

(ii) *$\mathfrak{G}^{\leftarrow}(x) = \langle x\rangle\, \mathfrak{G}^{\leftarrow}(e) = \mathfrak{G}^{\leftarrow}(e)\, \langle x\rangle$ for all $x \in G$;*

(iii) *if $S \in \mathfrak{G}^{\leftarrow}(e)$ and $s \in MON$, then $S \circ s \in \mathfrak{G}^{\leftarrow}(e)$;*

(iv) *$\mathfrak{G}$ has unique limits iff $\langle e\rangle$ is the only constant sequence in $\mathfrak{G}^{\leftarrow}(e)$;*

(v) *$\mathfrak{G}$ satisfies axiom $(\mathscr{M}\mathscr{L})$ iff the following implication holds: if $S \in \mathfrak{G}^{\leftarrow}(e)$, then $S \wedge \langle e\rangle \in \mathfrak{G}^{\leftarrow}(e)$;*

(vi) *$\mathfrak{G}$ satisfies axiom $(\mathscr{F}\mathscr{L})$ iff the following implication holds: if $S \in \mathfrak{G}^{\leftarrow}(e)$, $T \in G^N$ and $\mathscr{F}(S) = \mathscr{F}(T)$, then $T \in \mathfrak{G}^{\leftarrow}(e)$.*

Let G be a group. Identifying $x \in G$ with $\langle x\rangle \in G^N$, we can consider G to be a subgroup of G^N. A subgroup H of G^N is said to be *normal with respect to* G if $gSg^{-1} = \langle g\, S(n)\, g^{-1}\rangle \in H$ whenever $g \in G$ and $S \in H$. Let $\mathscr{A}$ be a subset of G^N. Let $\mu\mathscr{A}$ be the set of all sequences $S \wedge \langle e\rangle$ such that $S \in \mathscr{A}$, let $\delta\mathscr{A}$ be the set of all sequences

$S \circ s$ such that $S \in \mathscr{A}$ and $s \in MON$, and let $\varphi\mathscr{A}$ be the set of all sequences $T \in G^N$ such that $\mathscr{F}(T) = \mathscr{F}(S)$ for some $S \in \mathscr{A}$. Consider the set of all subgroups of G^N containing $\mathscr{A}$ and normal with respect to G. Denote by $[\mathscr{A}]_G$ the intersection of all such subgroups. Then G^N is the largest and $[\mathscr{A}]_G$ the smallest element of the set.

Proposition 2. *Let G be a group and let $\mathscr{A}$ be a subset of G^N.*

(i) *$[\mathscr{A}]_G$ consists precisely of the finite products of sequences of the form $gS^\varepsilon g^{-1} = \langle g\, S(n)^\varepsilon\, g^{-1}\rangle$, where $g \in G$, $S \in \mathscr{A}$ and $\varepsilon = \pm 1$.*

(ii) *$[\varphi\delta\mathscr{A}]_G$ is the smallest subgroup of G^N containing $\mathscr{A}$, normal with respect to G and closed with respect to δ and φ.*

(iii) *There is a sequential convergence $\mathfrak{H}_{\mathscr{A}}$ on G satisfying axiom $(\mathscr{F}\mathscr{L})$ such that $(G, \mathfrak{H}_{\mathscr{A}})$ is an $\mathscr{L}$-group and $\mathscr{A} \subset [\varphi\delta\mathscr{A}]_G = \mathfrak{H}^{\leftarrow}_{\mathscr{A}}(e)$.*

(iv) *If $(G, \mathfrak{H})$ is an $\mathscr{L}$-group such that $\mathfrak{H}$ satisfies axiom $(\mathscr{F}\mathscr{L})$ and $\mathscr{A} \subset \mathfrak{H}^{\leftarrow}(e)$, then $\mathfrak{H}_{\mathscr{A}} \subset \mathfrak{H}$.*

(v) *$\mathfrak{H}_{\mathscr{A}}$ has unique limits iff $[\varphi\delta\mathscr{A}]_G$ contains no constant sequence except $\langle e\rangle$.*

(vi) *$[\varphi\delta\mu\mathscr{A}]_G$ is the smallest subgroup of G^N containing $\mathscr{A}$, normal with respect to G and closed with respect to μ, δ and φ.*

(vii) *There is a sequential convergence $\mathfrak{G}_{\mathscr{A}}$ on G satisfying axioms $(\mathscr{F}\mathscr{L})$ and $(\mathscr{M}\mathscr{L})$ such that $(G, \mathfrak{G}_{\mathscr{A}})$ is an $\mathscr{L}$-group and $\mathscr{A} \subset [\varphi\delta\mu\mathscr{A}]_G = \mathfrak{G}^{\leftarrow}_{\mathscr{A}}(e)$.*

(viii) *If $(G, \mathfrak{G})$ is an $\mathscr{L}$-group such that $\mathfrak{G}$ satisfies axioms $(\mathscr{F}\mathscr{L})$ and $(\mathscr{M}\mathscr{L})$ and $\mathscr{A} \subset \mathfrak{G}^{\leftarrow}(e)$, then $\mathfrak{G}_{\mathscr{A}} \subset \mathfrak{G}$.*

(ix) *$\mathfrak{G}_{\mathscr{A}}$ has unique limits iff $[\varphi\delta\mu\mathscr{A}]_G$ contains no constant sequence except $\langle e\rangle$.*

2.

Cauchy sequences in $\mathscr{L}$-groups have been studied, e.g., in [5] and [10]. Recall that a sequence S in an $\mathscr{L}$-group is Cauchy if $S \circ s - S \circ t$ converges to 0 for all $s, t \in MON$.

Definition. Let G be an $\mathscr{L}$-group. A sequence S of points of G is said to be *FTON-Cauchy* if $S \circ s - S \circ t$ converges to 0 for all $s, t \in$ FTON.

By Proposition 3.11 in [1], in an $\mathscr{L}$-group G a sequence S is FTON-Cauchy iff $\mathscr{F}(S)$ is a Cauchy filter in γG. Further (cf. Corollary 3.12 in [1]), if the sequential convergence in G is maximal, then each Cauchy sequence in G is FTON-Cauchy. In this section we construct (Example 1) an $\mathscr{L}$-group G satisfying axiom $(\mathscr{F}\mathscr{L})$ in which a Cauchy sequence need not be FTON-Cauchy. The construction is then modified (Example 2) so that G satisfies axioms $(\mathscr{F}\mathscr{L})$ and $(\mathscr{M}\mathscr{L})$.

Example 1. Let X be a countably infinite set arranged into a one-to-one sequence $S = \langle S(n)\rangle$. Let G be the free abelian group generated by X. Denote by $\mathscr{A}$ the set of all sequences of the form $S \circ s - S \circ t$, where $s, t \in MON$. Observe that for each $T \in \mathscr{A}$ and each $s \in MON$ we have $-T \in \mathscr{A}$ and $T \circ s \in \mathscr{A}$. We shall define a sequential convergence $\mathfrak{H}$ on X satisfying axiom $(\mathscr{F}\mathscr{L})$ in such a way that, first, each sequence in $\mathscr{A}$ will $\mathfrak{H}$-converge to 0 (hence S will be a Cauchy sequence), and, secondly, for $u \in FTON$ defined by $u(1) = 1$, $u(2) = u(3) = 2$, $u(4) = u(5) = u(6) = 3, \ldots$, the sequence $S \circ u$ will not $\mathfrak{H}$-converge to 0 (hence S will not be be a FTON-Cauchy sequence).

In view of Proposition 2 it suffices to construct $\mathscr{N} \subset G^N$ such that:

(i) $\mathscr{N}$ is a subgroup of G^N;
(ii) $\mathscr{A} \subset \mathscr{N}$;
(iii) $T \circ s \in \mathscr{N}$ whenever $T \in \mathscr{N}$ and $s \in MON$;
(iv) if $T \in \mathscr{N}$, $U \in G^N$ and $\mathscr{F}(T) = \mathscr{F}(U)$, then $U \in \mathscr{N}$;
(v) $\langle x \rangle \notin \mathscr{N}$ whenever $x \neq 0$;
(vi) $S - S \circ u \notin \mathscr{N}$;

and then put $(T, x) \in \mathfrak{H}$ iff $T - \langle x \rangle \in \mathscr{N}$. Observe that $\mathscr{N} = \mathfrak{H}^{\leftarrow}(0)$.

Define $\mathscr{N}$ as follows: $T \in \mathscr{N}$ iff there are $k \in N$, $T_i \in \mathscr{A}$, $s_i \in FTON$, $i = 1, \ldots, k$, such that $T(n) = (T_1 \circ s_1 + \ldots + T_k \circ s_k)(n)$ for all but finitely many $n \in N$.

Claim. *$\mathscr{N}$ satisfies all conditions* (i)–(vi).

Proof. Clearly, $\mathscr{N}$ satisfies conditions (i), (ii) and (iii). Condition (iv) follows immediately from Proposition 2 in [2] which asserts that (in sequential convergence spaces in which the convergence of a sequence does not depend on finitely many terms of the sequence) axiom $(\mathscr{F}\mathscr{L})$ is equivalent to the fact that a sequence $T \circ t$ converges to x whenever T converges to x and $t \in FTON$. Since G is a free group over the set $\{S(n); n \in N\}$, $\langle x \rangle \notin \mathscr{N}$ for all $x \in G$, $x \neq 0$, and hence $\mathscr{N}$ satisfies condition (v). Finally, given $k \in N$ and $s_i, t_i \in MON$, $u_i \in FTON$, $i = 1, \ldots, k$, consider for each $n \in N$ the following proposition:

$$(S - S \circ u)(n) = ((S \circ s_1 - S \circ t_1) \circ u_1)(n) + \ldots + ((S \circ s_k - S \circ t_k) \circ u_k)(n)\,;$$

denote it by $P(n, (s_1, \ldots, s_k), (t_1, \ldots, t_k), (u_1, \ldots, u_k))$ or, simply by $P(n)$. To prove condition (vi) it suffices to prove that for each $p \in N$ there exists $q \in N$, $q > p$, such that proposition $P(q)$ is false. The proof is based on the so called box principle (if we place more than n objects into n boxes, then one of the boxes contains at least two objects) and the following observations.

(O_1) For each $j \in N$ there exists $m \in N$ such that $j < |\{p \in N;\ u(p) = m\}|$ and $j < m < \min\{p \in N;\ u(P) = m\}$; hence the sequence $S \circ u$ has arbitrarily long (finite) constant segments, while $\langle (S - S \circ u)(n + 2) \rangle$ is a one-to-one sequence.

(O_2) For each $i \in \{1, \ldots, k\}$ we have $(S \circ s_i - S \circ t_i) \circ u_i = S \circ s_i \circ u_i - S \circ t_i \circ u_i$ and $(S \circ s_i \circ u_i)(n) = (S \circ s_i \circ u_i)(m)$ iff $(S \circ t_i \circ u_i)(n) = (S \circ t_i \circ u_i)(m)$, i.e., the sequences $S \circ s_i \circ u_i$ and $S \circ t_i \circ u_i$ are constant on the same segments of N.

Now assume that, on the contrary, for some $k \in N$ and for some $s_i, t_i \in MON$, $u_i \in FTON$, $i = 1, \ldots, k$, proposition $P(n)$ holds for all but finitely many $n \in N$. We claim that then for each $p \in N$ there exist $j_1, j_2 \in N$ such that $p < j_1 < j_2$ and proposition (Pj_1) is of the form

$$x_1 - x = (y_1 - z_1) + \cdots + (y_k - z_k),$$

and at the same time proposition $P(j_2)$ is of the form

$$x_2 - x = (y_1 - z_1) + \cdots + (y_k - z_k),$$

where x, x_1, x_2 and also y_i, z_i, $i = 1, \ldots, k$, are generators of the free group G and $x_1 \neq x_2$. Since this is clearly impossible, either $P(j_1)$ or $P(j_2)$ is a false proposition. However, the claim is a straightforward consequence of (O_1) and (O_2) and the box principle. Indeed, using (O_1), start with a sufficiently large set $M \subset N$ such that propositions $P(j)$, $j \in M$, have the form

$$x_j - x = (y_{j1} - z_{j1}) + \ldots + (y_{jk} - z_{jk}),$$

where x and $x_j, y_{j1}, \ldots, y_{jk}, z_{j1}, \ldots, z_{jk}$, $j \in M$, are generators of the free group G and $x_j \neq x_m$ whenever $j, m \in M$ and $j \neq m$; using repeatedly (O_2) and the box principle, we find a subset $\{j_1, j_2\}$ of M such that for each $i \in \{1, \ldots, k\}$ we have $y_{j_1 i} = y_{j_2 i}$ and $z_{j_1} = z_{j_2 i}$. This complete the proof.

Example 2. Let X, G, S and $\mathscr{A}$ be the same as in Example 1. Let $\mu\mathscr{A}$ be the set of all sequences T in G such that $T = U \wedge \langle 0 \rangle$ for some $U \in \mathscr{A}$. Define $\mathfrak{G}^{\leftarrow}(0) \subset G^N$ as follows: T belongs to $\mathfrak{G}^{\leftarrow}(0)$ iff there are $k \in N$, $T_i \in \mu\mathscr{A}$, $s_i \in FTON$, $i = 1, \ldots, k$, such that $T(n) = (T \circ s_1 + \ldots + T \circ s_k)(n)$ for all but finitely many $n \in N$. Finally, define $\mathfrak{G} \subset G^N \times G$ by putting $(T, x) \in \mathfrak{G}$ iff $(T - \langle x \rangle) \in \mathfrak{G}^{\leftarrow}(0)$. In a similar way as in Example 1 it can be proved that G equipped with $\mathfrak{G}$ is an $\mathscr{L}$-group satisfying axioms $(\mathscr{F}\mathscr{L})$ and $(\mathscr{M}\mathscr{L})$ in which S is a Cauchy sequence but fails to be FTON-Cauchy.

Corollary 1. *There exists an $\mathscr{L}$-group G satisfying axioms $(\mathscr{F}\mathscr{L})$ and $(\mathscr{M}\mathscr{L})$, and a Cauchy sequence S in G such that $\mathscr{F}(S)$ fails to be a Cauchy filter in γG.*

The following result has been announced in [3].

Corollary 2. *There exists an incomplete $\mathscr{L}$-group H satisfying axioms $(\mathscr{F}\mathscr{L})$ and $(\mathscr{M}\mathscr{L})$ such that γH is complete.*

Proof. Consider the $\mathscr{L}$-group $(G, \mathfrak{G})$ from Example 2. Then G equipped with $\lambda = \gamma(\mathfrak{G})$ is a sequentially determined convergence group. Let $(\hat{G}, \hat{\lambda}, e_G)$ be the categorical completion of (G, λ). By Theorem 3.9 in [1], $(\hat{G}, \hat{\lambda})$ is sequentially determined. Put $H = (\hat{G}, \mathscr{L}(\hat{\lambda}))$. Then $\gamma H = (\hat{G}, \hat{\lambda})$. Clearly, $\mathscr{L}(\hat{\lambda})$ satisfies axioms $(\mathscr{F}\mathscr{L})$ and $(\mathscr{M}\mathscr{L})$, and $\mathscr{L}(\hat{\lambda})$ restricted to G equals $\mathfrak{G}$. Then S is a Cauchy sequence in H but fails to converge. Otherwise, $\hat{\lambda}$ being sequentially determined, $\mathscr{F}(S)$ would be $\hat{\lambda}$-convergent and hence $\hat{\lambda}$-Cauchy. But Proposition 3.11 in [1] would imply that S is FTON-Cauchy in $(G, \mathfrak{G})$, a contradiction.

3.

Motivated by Example 2 let us consider the following problem. Let $(G, \mathfrak{G})$ be an $\mathcal{L}$-group, let $\mathcal{C}$ be the set of all Cauchy sequences in G and let $\sim$ be the usual equivalence for $\mathcal{C}$, i.e., $S \sim T$ iff $S - T$ converges to 0. Let f be a mapping of $\mathcal{C}$ into the set $\mathcal{P}(G^N)$ of all subsets of G^N. Under what conditions is $f(S)$ a set of Cauchy sequences each of which is equivalent to S, $S \in \mathcal{C}$? For instance, if $f(S) = \{T \in G^N; \mathcal{F}(S) = \mathcal{F}(T)\}$ and $\mathfrak{G}$ is a maximal sequential convergence, then each $T \in f(S)$ is a Cauchy sequence equivalent to S. Is this true if $\mathfrak{G}$ satisfies axioms $(\mathcal{F}\mathcal{L})$ and $(\mathcal{M}\mathcal{L})$ but fails to be maximal?

A similar question can be asked for general Cauchy structures, namely, given a Cauchy structure and an equivalence relation, under what conditions what operations on Cauchy objects preserve the equivalence classes?

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