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ON CONVERGENCE GROUPS WITH DENSE COARSE SUBGROUPS

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A systematic study of coarse convergence groups has been initiated in [3]. In this follow-up we study properties of abelian convergence groups implied by the assumption that the group in question contains a dense subgroup (i.e. each point of the group is a limit of a sequence of points of the subgroup) which is coarse. We give a necessary and sufficient condition (Criterion 2) for a dense subgroup of a coarse group to be coarse. The condition allows us to answer partially the question asked in [3] whether coarseness is preserved by products. Finally, we investigate the relationship between coarseness and completeness.

For the reader's convenience we repeat here some basic facts about coarse convergence groups. In notation and terminology we generally follow [3]. Background information on convergence groups can be found in [9], [10] and [7].

Throughout the paper a group is always an abelian group written in the additive notation. By a convergence group we understand a group G equipped with a compatible sequential convergence $\mathfrak{G} \subset G^N \times G$ which satisfies the so-called FLUSH-axioms or, equivalently, $\mathscr{L}$-axioms. Recall that H $(\equiv \mathscr{L}_0)$ stands for the uniqueness of sequential limits, S $(\equiv \mathscr{L}_1)$ means that for all $g \in G$ the constant sequence $\langle g \rangle$ converges to g, F $(\equiv \mathscr{L}_2)$ means that if a sequence converges to a point, then each subsequence of the sequence converges to this point, U $(\equiv \mathscr{L}_3)$ denotes the Urysohn axiom, and $L(\equiv \mathscr{S}\mathscr{G})$ stands for the compatibility of $\mathfrak{G}$ with the group structure of G. We say that $\mathfrak{G}$ (and also G) is coarse if there is no FLUSH-convergence for G strictly larger than $\mathfrak{G}$. The definition of a coarse convergence group resembles the definition of a minimal topological group. As pointed out in [3], these two analogous notions can have quite different properties.

By Z we denote the group of integers, by N the positive integers, by MON the set of all monotone (one-to-one) mappings of N into N and if $S = \langle x_n \rangle$ is a sequence of points, then for $s \in$ MON the subsequence of S the n-th term of which is $x_{s(n)}$ is denoted by $S \circ s$. If S and T are two sequences in a group G, then $S + T$ denotes the sequence the n-th term of which is $S(n) + T(n)$ and the sequence $-S$ is defined analogously.

We say that a sequence of points of a convergence group has the property (C) if either of the two conditions holds true:

(C_1) Some subsequence of the sequence in question converges to the zero element of the group;

(C_2'') Some finite linear combination, with coefficients from $Z \setminus \{0\}$, of subsequences of the sequence in question converges to a nonzero element of the group.

In [3] the following necessary and sufficient condition is given for a convergence group to be coarse.

Criterion 1. *A convergence group is coarse iff each sequence of points of the group has the property* (C).

As pointed out in [3], Criterion 1 resembles the characterization of maximal (with respect to inclusion) compatible sequential convergences in a linear space given in [6].

1.

In this section we give a characterization of coarse dense subgroups of a coarse convergence group. The characterization is analogous to that given for minimal topological groups in [12], [11].

Definition 1.1. Let G' be a group. A subgroup G of G' is said to be *algebraically essential* in G' if each nontrivial subgroup of G' intersects G in a nontrivial subgroup.

Proposition 1.2. *Let G' be a convergence group and let G be a dense subgroup of G'. If G is coarse, then G is algebraically essential in G'.*

Proof. Let g be a point in $G' \setminus G$. Then there is a sequence S in G converging in G' to g. Since G is coarse, the sequence S has the property (C). But each subsequence $S \circ s$ of S converges to g ($\neq 0$) and S cannot satisfy condition (C_1). Thus S satisfies (C_2''), i.e., some finite linear combination $z_1 S \circ s_1 + \ldots + z_k S \circ s_k$ (where $k \in N$, $z_i \in Z \setminus \{0\}$ and $s_i \in \mathrm{MON}$, $i = 1, \ldots, k$) converges in G to a nonzero element $h \in G$. From the fact that $z_1 S \circ s_1 + \ldots + z_k S \circ s_k$ converges to $(z_1 + \ldots + z_k)\, g$, we get $h = (z_1 + \ldots + z_k)\, g$. Thus h belongs to the cyclic subgroup of G' generated by $g \in G' \setminus G$. Consequently, G is algebraically essential in G'.

This proposition is also valid in the noncommutative case and the proof remains essentially the same (the noncommutative version of (C), see [3], is to be used). In this case a subgroup G of a group G' is said to be *algebraically essential* if every nontrivial subgroup of G' which is invariant under conjugations by elements of G intersects G in a nontrivial subgroup.

Corollary 1.3. *Let G be a dense subgroup of a convergence group G'. If G is coarse and torsion-free, then G' is torsion-free.*

Proposition 1.4. *Let G' be a coarse convergence group and let G be a dense subgroup of G'. If G is algebraically essential in G', then G is coarse.*

Proof. Let S be a sequence in G no subsequence of which converges in G to zero. Since G' is coarse, it follows from Criterion 1 that S satisfies condition (C_2''), i.e., some finite linear combination $z_1 S \circ s_1 + \ldots + z_k S \circ s_k$ (where $k \in N$, $z_i \in Z \setminus \{0\}$ and $s_i \in \mathrm{MON}$, $i = 1, \ldots, k$) converges in G' to a nonzero element $g \in G'$. Since G is algebraically essential in G', for some $z \in Z \setminus \{0\}$ we have $zg \in G$ and $zg \neq 0$. The sequence $z(z_1 S \circ s_1 + \ldots + z_k S \circ s_k)$ converges in G to $zg \neq 0$ and hence, by (C_2''), G is coarse.

Combining Proposition 1.2 and Proposition 1.4 we get the following.

Criterion 2. *Let G' be a coarse convergence group and let G be a dense subgroup of G'. Then G is coarse iff it is algebraically essential in G'.*

Since every sequentially compact convergence group is coarse, Criterion 2 provides examples of coarse convergence groups which fail to be sequentially compact. In fact, a dense subgroup G of a sequentially compact convergence group G' is coarse iff G is algebraically essential in G'. Clearly also, if G' is torsion-free, then G is coarse iff G'/G is torsion.

Example 1.5. Let p be a prime number and let J_p denote the group of p-adic integers equipped with the p-adic convergence. There exist proper dense coarse subgroups of J_p. Indeed, by the above remark, they are exactly the dense proper subgroups G of J_p usch that J_p/G is torsion. Every free subgroup G of J_p of maximal rank (see [5]), containing a unit of the ring J_p, has this property (J_p is not free, it is even indecomposable, see [5]).

2.

In [3] the authors asked whether the coarseness is preserved by products. Using Criterion 2, we show that for infinite products the answer is "NO". Further, using Criterion 2, we show that a product of a sequentially precompact coarse group (i.e. a convergence group having a sequentially compact completion in which the original group is dense) and a coarse convergence group is coarse. Concerning products of minimal topological groups the reader is referred to [1].

Proposition 2.1. *Let G' be a metrizable convergence group and let G be a dense subgroup of G' such that G^N is coarse. Then there exists a natural number n such that $nG' \subset G$.*

Proof. Since G^N is a dense coarse subgroup of the Fréchet convergence group $(G')^N$, by Proposition 1.2, G^N is algebraically essential in $(G')^N$; in particular, $(G')^N/G^N \cong (G'/G)^N$ is torsion. Then there exists a natural number n such that $nG' \subset G$.

Example 2.2. Let G be a proper dense coarse subgroup of J_p as in Example 1.5 (p is an arbitrary prime number). Then G is not an open subgroup of J_p, so G does not contain any subgroup nJ_p, $n \neq 0$, since they are all open. By Propositiion 2.1, G^N fails to be coarse.

Proposition 2.3. *Let G be a sequentially compact convergence group and let H be a coarse convergence group. Then $G \times H$ is coarse.*

Proof. According to Criterion 1, it suffices to show that each sequence S in $G \times H$ satisfies either (C_1) or (C_2''). Let S be a sequence in $G \times H$. Denote by S_1 and S_2 the projection of S onto G and H, respectively (i.e. $S(n) = (S_1(n), S_2(n))$, $n \in N$). Since H is coarse, S_2 satisfies either (C_1) or (C_2'').

1. Assume that S_2 satisfies (C_1), i.e., for some $s \in \mathrm{MON}$ the subsequence $S_2 \circ s$ converges in H to 0. Since for some $t \in \mathrm{MON}$ the sequence $S_1 \circ s \circ t$ converges in G to some point $g \in G$, the sequence $S \circ s \circ t$ converges in $G \times H$ to $(g, 0)$. Thus S satisfies either (C_1) (if $g = 0$) ro (C_2'') (if $g \neq 0$).

2. Assume that S_2 satisfies (C_2''), i.e., for some $k \in N$ there are $s_i \in \mathrm{MON}$ and $z_i \in Z \setminus \{0\}$, $i = 1, \dots, k$, such that $z_1 S_2 \circ s_1 + \dots + z_k S_2 \circ s_k$ converges in H to some nonzero element h. Observe that for each $t \in \mathrm{MON}$ the sequence $\langle z_1 S_2(s_1(t(n))) + \dots + z_k S_2(s_k(t(n)))\rangle$ converges in H to h. We claim that for some $t \in \mathrm{MON}$ each sequence $S_1 \circ s_i \circ t$ converges to some point $g_i \in G$, $i = 1, \dots, k$. Indeed, since G is sequentially compact, for some $t_1 \in \mathrm{MON}$ the sequence $S_1 \circ s_1 \circ t_1$ converges in G to a point $g_1 \in G$, for some $t_2 \in \mathrm{MON}$ the sequence $S_1 \circ s_2 \circ t_1 \circ t_2$ converges in G to a point $g_2 \in G, \dots$, for some $t_k \in \mathrm{MON}$ the sequence $S_1 \circ s_k \circ \circ t_1 \circ \dots \circ t_k$ converges in G to a point g_k in G. Put $t = t_1 \circ \dots \circ t_k$. Since $z_1 S_1 \circ \circ s_1 \circ t + \dots + z_k S_1 \circ s_k \circ t$ converges in G to $z_1 g_1 + \dots + z_k g_k$, the sequence $\langle z_1 S(s_1(t(n))) + \dots + z_k S(s_k(t(n)))\rangle$ converges in $G \times H$ to $(z_1 g_1 + \dots + z_k g_k, h)$ and hence the sequence S satisfies (C_2''). This completes the proof.

Proposition 2.4. *Let G and H be coarse convergence groups. If G is sequentially precompact, then $G \times H$ is coarse.*

Proof. Let G' be a sequentially compact (hence coarse) convergence group such that G is a dense subgroup of G'. By Proposition 2.3, $G' \times H$ is coarse. According to Criterion 2, G is algebraically essential in G'. Thus $G \times H$ is algebraically essential in $G' \times H$. Clearly, $G \times H$ is a dense subgroup of $G' \times H$. Thus, by Criterion 2, $G \times H$ is coarse.

3.

In this section we deal with the relationship between coarseness and completeness.

Let G be a convergence group. Recall that a sequence S in G is said to be *Cauchy* if for each $s \in \mathrm{MON}$ the sequence $S \circ s - S$ converges in G to the zero element of G. If each Cauchy sequence in G converges, then G is said to be *complete*. A thorough discussion of the notion of completeness in various types of continuous sequential groups can be found in [7].

Theorem 5 in [3] states that a torsion-free divisible coarse convergence group is complete. Our next proposition shows that the assumption that the group is torsion-free is superfluous.

Proposition 3.1. *Let G be a divisible coarse convergence group. Then G is complete.*

Proof. Let G' be the Novák completion ([10]) of G. Choose $g \in G' \setminus G$. By Proposition 1.2, G is algebraically essential in G', i.e. there exists $k \in N$ such that $kg \neq 0$ and $kg \in G$. Let $\langle g_n \rangle$ be a sequence of points of G converging in G' to g. Then the sequence $\langle kg_n \rangle$ of points of kG converges in G to kg. Since G is divisible, we have $kG = G$. Consequently, for some $h \in G$ we have $kh = kg$. Without loss of generality, we may assume that k is prime. In fact, if $k = p_1 p_2 \dots p_s$, where each p_i is prime ($i = 1, \dots, s$), take the least $t \in \{1, \dots, s\}$ such that $p_1 p_2 \dots p_t g \in G$. Then $g_1 = p_1 p_2 \dots p_{t-1} g \notin G$ and $p_t g_1 \in G$ (for $t = 1$ set $p_0 = 1$), so we can take g_1 instead of g. Now, consider the cyclic subgroup C of G' generated by $h - g$. Since $k(h - g) = 0$ and k is prime, C is simple. As G is algebraically essential in G', we have $C \cap G \neq \{0\}$ and hence $C \subset G$. Consequently, $h - g \in G$ and also $g \in G$. Thus $G' = G$ and G is complete.

Corollary 3.2. *Every divisible abelian group admits a nondiscrete complete convergence structure.*

Proof. By Theorem 1 in [3], every abelian group admits a nondiscrete coarse convergence structure. Now apply Proposition 3.1.

According to Theorem 1 in [3], each FLUSH-convergence for a group can be enlarged to a coarse convergence. Consider the group Q of rational numbers equipped with a coarse convergence coarser than the usual metric convergence for Q; denote by Q_c the resulting coarse convergence group. Since Q is divisible, it follows from Proposition 3.1 that Q_c is complete (cf. Corollary 2 in [3]). Similarly, consider the rational torus $G = Q/Z$ equipped with a coarse convergence coarser than the usual metric convergence for G; denote it by G_c. Since G is divisible, it again follows from Propositiin 3.1 that G_c is complete. We are now going to examine the group Q_c more closely.

Answering a problem posed by J. Novák at the Kanpur Topological Conference in 1968 (cf. Problem 12 in [8]), F. Zanolin constructed in 1977 a convergence group (having unique sequential limits) in which there are two distinct points which cannot be separated by disjoint neighbourhoods (cf. [14]). Later on, P. Kratochvíl and independently K. Wichterle claimed that they can construct a convergence group no two points of which can be separated by disjoint neighbourhoods (unpublished). We show that Q_c has the the same property. It is a consequence of the completeness of Q_c.

Proposition 3.3. *No two points of Q_c can be separated by disjoint neighbourhoods.*

Proof. It suffices to show that Q_c satisfies the following condition:

(c) For each $x \in Q$ and for each positive real number ε there is in Q a sequence $S = \langle x_n \rangle$ such that: (i) S converges in Q_c to 0; (ii) $|x - x_n| < \varepsilon$ for each $n \in N$.

Indeed, (c) implies that no $x \in Q$ can be separated in Q_c from 0 by disjoint neighbourhoods. Since the convergence in Q_c satisfies the Urysohn axiom U, it is homogeneous (i.e. a sequence $\langle x_n \rangle$ converges to a point x iff the sequence $\langle x_n - x \rangle$ converges to 0) and hence no two points in Q_c can be separated by disjoint neighbourhoods. The proof of (c) is done in three steps.

1. As stated earlier, Q_c is complete.

2. Let $\langle y_n \rangle$ be a sequence of rational numbers converging in the real line to $\sqrt{2}$. It is a Cauchy sequence in Q_c and hence it converges in Q_c to a rational number y. The sequence $\langle y_n - y \rangle$ converges in Q_c to 0 but in the real line it converges to the irrational number $\sqrt{2} - y$.

3. Let ε be a positive real number and x a rational number. Then there is an integer p and a natural number q such that for all but finitely many $n \in N$ we have $|x - p(y_n - y)/q| < \varepsilon$. Clearly, the sequence $\langle p(y_n - y)/q \rangle$ converges in Q_c to 0. Let $S = \langle x_n \rangle$ be a sequence obtained from $\langle p(y_n - y)/q \rangle$ by leaving out sufficiently long initial segment. Then S has the properties required by (c).

Remark 3.4. The convergence in Q_c has the following interesting "antidiagonal property":

(AD) For each $v \in Q$, $v \neq 0$, there is a p-system $(\langle S_m \rangle, 0)$ (i.e. for each $m \in N$, $S_m = \langle x_{mn} \rangle$ is a sequence converging in Q_c to 0, cf. [4]) such that each diagonal sequence S_f, $f \in N^N$ (defined by $S_f(n) = S_n(f(n)) = x_{nf(n)}$), converges in Q_c to v.

Indeed, let v be a rational number, $v \neq 0$. According to (c), for each $m \in N$ there is a sequence $S_m = \langle x_{mn} \rangle$ of rational numbers converging in Q_c to 0 such that for all $n \in N$ we have $|(v + 1/m) - x_{mn}| < 2^{-m}$. Since the convergence in Q_c is coarser than the usual metric convergence in Q, it is easy to see that each diagonal sequence S_f, $f \in N^N$, converges in Q_c to v.

Observe that from (AD) it follows immediately that the closure operator in Q_c fails to be idempotent. It would be nice to find out more about the so-called sequential order of the closure in Q_c, i.e., the least ordinal α $(1 < \alpha \leqq \omega_1)$ such that the α-th iteration of the closure in Q_c is idempotent (cf. [8]).

As shown in [12], the rational torus Q/Z is a minimal topological group. It is known that the completion of a minimal topological abelian group is minimal again (cf. [11]). As we shall see, for coarse convergence groups the situation is different.

J. Novák has shown in [10] that each FLUSH-convergence abelian group has a completion (referred to as the Novák completion) and it can have several non-homeomorphic completions (see also [7]). We are going to construct a coarse convergence group the Novák completion of which fails to be coarse.

Example 3.5. Put $X = \{x_n;\ n \in N\} \cup \{x_{mn};\ m, n \in N\}$. Let G be the free abelian group with X as the set of its generators. We are going to equip G with a coarse

FLUSH-convergence $\mathfrak{G}_c$ in such a way that for each $m \in N$ the sequence $S_m = \langle x_{mn} \rangle$ is a totally divergent Cauchy sequence, the sequences S_k and S_l are not equivalent whenever $k \neq l$ (i.e. the sequence $S_k - S_l$ does not converge to zero) and, if p_m denotes the ideal point in the Novák completion G' of G to which the sequence S_m converges, then the sequence $\langle p_m \rangle$ does not have the property (C). The convergence $\mathfrak{G}_c$ is constructed in two steps.

1. Define a set $\mathscr{A}$ of sequences of points of G as follows: $\mathscr{A} = \{\langle x_{ms(n)} - x_{mt(n)} \rangle; m \in N \text{ and } s, t \in \text{MON}\} \cup \{\langle 2x_{mn} - x_m \rangle; m \in N\} \cup \{\langle x_n \rangle\}$. According to Theorem 0 in [2], there is a FLUS-convergence $\mathfrak{G}_{\mathscr{A}}$ for G such that $\mathscr{A} \subset \mathfrak{G}_{\mathscr{A}}^{\leftarrow}(0)$ (i.e. each sequence in $\mathscr{A}$ $\mathfrak{G}_{\mathscr{A}}$ – converges to 0) and $\mathfrak{G}_{\mathscr{A}}$ is the smallest FLUS-convergence for G with respect to this property. To prove that $\mathfrak{G}_{\mathscr{A}}$ satisfies axiom H (the uniqueness of sequential limits) it suffices to verify that no constant sequence $\langle x \rangle$ $\mathfrak{G}_{\mathscr{A}}$ – converges to 0 except the case when $x = 0$. So, assume that for some $x \in G$ the constant sequence $\langle x \rangle$ $\mathfrak{G}_{\mathscr{A}}$ – converges to 0. Then x is a finite linear combination, with coefficients from $Z \setminus \{0\}$, of subsequences of sequences from $\mathscr{A}$ (cf. [13]). This yields an infinite system of equations in the free group G with x on the lefthand side in each of the equations. A straightforward calculation shows that this is possible only for $x = 0$. Thus $\mathfrak{G}_{\mathscr{A}}$ is a FLUSH-convergence.

2. Next, according to Theorem 1 in [3], there is a coarse FLUSH-convergence for G coarser than $\mathfrak{G}_{\mathscr{A}}$; denote it by $\mathfrak{G}_c$.

Proposition 3.5.1. *The group G equipped with $\mathfrak{G}_c$ has the following properties:*

(i) *For each $m \in N$, $S_m = \langle x_{mn} \rangle$ is a totally divergent Cauchy sequence;*

(ii) *The sequences $S_k = \langle x_{kn} \rangle$ and $S_l = \langle x_{ln} \rangle$ are not equivalent whenever $k \neq l$.*

Proof. (i) Since $\mathscr{A} \subset \mathfrak{G}_{\mathscr{A}}^{\leftarrow}(0) \subset \mathfrak{G}_c^{\leftarrow}(0)$, each S_m is a Cauchy sequence. Further, $2S_m = \langle 2x_{mn} \rangle$ converges to x_m. Assume, on the contrary, that a subsequence $S_m \circ s$ of S_m converges to some $y \in G$. Then $2S_m \circ s$ converges to $2y$ and, according to the uniqueness of limits, we have $x_m = 2y$. But this contradicts the fact that x_m is a generator of the free group G.

(ii) For $k \neq l$ the sequence $2(S_k - S_l) = \langle 2x_{kn} - 2x_{ln} \rangle$ converges to $x_k - x_l$. Thus $S_k - S_l = \langle x_{kn} - x_{ln} \rangle$ cannot converge to 0 and the Cauchy sequences S_k and S_l are not equivalent. This completes the proof.

Denote by G' the Novák completion of the group G equipped with $\mathfrak{G}_c$. For each $m \in N$, denote by p_m the point in $G' \setminus G$ to which the Cauchy sequence $S_m = \langle x_{mn} \rangle$ converges in G'. Recall that, by Lemma 11 in [10], if a sequence S converges in G' to 0, then there is a subsequence $S \circ s$ of S such tht for all $m, n \in N$ we have $(S(s(m)) - - S(s(n))) \in G$.

Proposition 3.5.2. (i) *For each $m \in N$ we have $2p_m = x_m$.*

(ii) *No subsequence of the sequence $\langle p_n \rangle$ converges in G'.*

Proof. (i) For each $m \in N$, the sequence $\langle x_{mn} \rangle$ converges to p_m and the sequence

$\langle 2x_{mn} \rangle$ converges to x_m. By the uniqueness of sequential limits we have $2p_m = x_m$.

(ii) Contrariwise, assume that for some $s \in \text{MON}$ the subsequence $\langle p_{s(n)} \rangle$ of $\langle p_n \rangle$ converges in G' to some $x \in G'$. Then $\langle p_{s(n)} - x \rangle$ converges in G' to 0 and hence there is $t \in \text{MON}$ such that $((p_{t(m)} - x) - (p_{t(n)} - x)) = (p_{t(m)} - p_{t(n)}) \in G$ for all $m, n \in N$. Now, if $k \neq l$ and $p_k - p_l \in G$, then for some $y \in G$ we have $p_k - p_l = y$. Then $\langle x_{kn} - x_{ln} \rangle$ converges to y. Consequently, $\langle 2x_{kn} - 2x_{ln} \rangle$ converges to $2y$ and at the same time to $x_k - x_l$. This implies $x_k - x_l = 2y$, a contradiction with the fact that $k \neq l$ and x_k and x_l are generators of the free group G.

Proposition 3.5.3. *The group G' fails to be coarse.*

Proof. According to Criterion 1, it suffices to prove that:

(i) No subsequence of $\langle p_n \rangle$ converges to 0;

(ii) No finite linear combination, with coefficients from $Z \setminus \{0\}$, of subsequences of $\langle p_n \rangle$ converges in G' to a nonzero element.

Condition (i) follows directly from Proposition 3.5.2. To prove (ii), assume on the contrary that for some $m \in N$ there are subsequences P_i of $\langle p_n \rangle$ and integers $z_i \neq 0$, $i = 1, \ldots, m$, such that the sequence $z_1 P_1 + \ldots + z_m P_m$ converges to some $p \neq 0$. By Proposition 3.5.2, we have $2p_n = x_n$, $n \in N$, and hence $2(z_1 P_1 + \ldots + z_m P_m) = z_1 Q_1 + \ldots + z_m Q_m$, where each Q_i is a subsequence of the sequence $\langle x_n \rangle$. Since each Q_i converges to 0, we have $2p = 0$. Hence G' is not a torsion-free group. By Corollary 1.3, we have a contradiction with the fact that G si torsion-free.

Added in proof. Recently J. Gerlits proved that the sequential order of the closure in Q_c is ω_1 (see page 476).

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