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PROFESSOR MICHAL GREGUŠ SEXAGENARIAN

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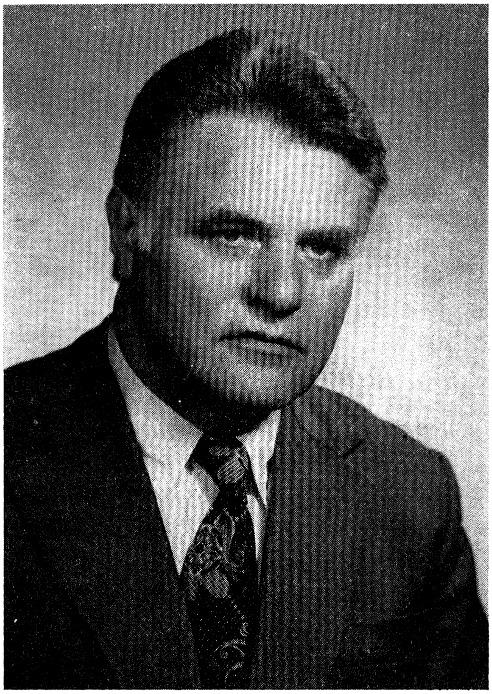
A distinguished Slovak mathematician, Professor Michal Greguš, DrSc., corresponding member of the Czechoslovak and Slovak Academies of Sciences and the Dean of the Faculty of Mathematics and Physics, Comenius University, Bratislava, has reached sixty years of age on 22 December 1986. His work has significantly influenced the scientific, educational and cultural life in Slovakia.



Professor M. Greguš was born on 22 December 1926 in Zbehy (Nitra district), in a family of a railwayman. He studied in 1946–1950 at Faculty of Science of Bratislava University. After a short period of work at the Slovak Technical University and the Military Technical Academy he joined in 1953 the Faculty of Science of Comenius University, Bratislava. In 1957 he received his CSc. (Candidate of Science) degree from the Faculty of Science of Purkyně University, Brno, and, at the same Faculty, he defended his DrSc. thesis at the age of 39 years. In 1959 he was appointed Associated Professor, in 1965 Full Professor of Mathematics. As the successor of Academician Jur. Hronec he was appointed Head of the Department of Mathematics in 1959.

Professor Greguš has always played an important part in the organization of science and education. He was Vice-Dean in the years 1959–1962, then Dean of the Faculty of Science, and from 1965 till 1968 he held the office of Vice-Chancellor of Comenius University. In these offices he considerably contributed to the development of the Faculty of Science, especially to the construction of its new quarters in Mlynská dolina. In 1968 he was appointed Deputy Minister of Education of Slovak Socialist Republic. In this office he remained till 1973, successfully working for the development of higher education. During the period 1973–1978 he was head of the Czechoslovak Permanent Mission at UNESCO. With remarkable success he took part in the implementation of the cultural programme of socialist countries, which was directed at strengthening the friendship and understanding among all peoples. After coming back to the University he took part in preparing the establishment of the Faculty of Mathematics and Physics, and after its foundation he became its first Dean. The merits for the extraordinary increase of the activity of the Faculty goes to a great extent to Professor Greguš. This concerns not only education and research, but also the cooperation of the Faculty with selected entreprises and research institutes. This cooperation was intended to help, on the one hand, to the cooperating institutions, and on the other, to the Faculty in improving the training of the students for their future tasks. Professor Greguš is member of Scientific Boards for Mathematics of the Czechoslovak and Slovak Academies of Sciences, of Editorial Boards of Mathematica Slovaca, Acta Mathematica Universitatis Comenianae, and of many other boards and committees.

The experience of many years of teaching Mathematics is reflected in the textbook Ordinary Differential Equations [B3], of which Professor Greguš is a co-author. He also prepared lecture notes Partial Differential Equations [B2]. Both texts can be characterized by the author's effort not only to present the basic facts but also to introduce to the reader the main trends of the present research. The stress is laid on applicability and on actual applications. As a teacher, Professor Greguš has very close and friendly relation to his students. In his Seminar on the Theory of Differential Equations he has educated a great number of excellent students. Last but not least, he has taken part in popularization of Mathematics.

Professor Greguš has been member of the Union of Czechoslovak Mathematicians and Physicists for many years. Among other, he was Vice-President of the Union of Slovak Mathematicians and Physicists. For his merits he was awarded a number of distinctions, among them the medal of the Union in 1962 and the Golden Medal of the Comenius University. For his contributions to international cooperation he was granted medals of the Universities in Leningrad, Gent and Sofia.

However, the foundation stone of Professor Greguš' activity has always been his research in Mathematics. In early fifties he started to study linear differential equations of the third order. It was the famous Italian mathematician G. Sansone who drew the attention to this field, and Academician O. Borůvka who stimulated intense research in this direction in Czechoslovakia. M. Greguš joined these efforts with all his vigour. Within twenty years he developed the theory of the third order differential equation, and educated a number of students, most of them now already well-known mathematicians and his close colaborators. The fruit of his assiduous, systematical, and creative work is the monograph *Linear differential equation of the third order* [B1], whose English version is now being prepared. Since the book presents the main results of Greguš' research, let us give here a brief survey of its contents.

When studying the properties of the linear differential equation of the third order it is of advantage to write it in the normal form

$$y''' + 2\,A(x)\,y' + [A'(x) + b(x)]\,y = 0 \tag{a}$$

where the functions A', b are continuous in a certain interval i. This form plays an important role in the transformation theory and, moreover, makes it possible to write the adjoint equation to (a) in the form

$$z''' + 2\,A(x)\,z' + [A'(x) - b(x)]\,z = 0\,. \tag{b}$$

The function b is called the Laguerre invariant. The principal notion for the theory of the equation (a) is the notion of a pencil of solutions. A two-dimensional subspace of solutions y of equation (a) satisfying $y(x_1) = 0$ is called a pencil of solutions of the first kind of equation (a) at the point x_1. Similarly pencils of solutions of the second ($y'(x_1) = 0$) and third ($y''(x_1) = 0$) kinds are introduced. Each pencil satisfies a 2nd order equation

$$w(x)\,y'' - w'(x)\,y' + [w''(x) + 2\,a(x)\,w(x)]\,y = 0 \tag{c}$$

where w is a certain solution of equation (b). Under certain conditions we have $w(x) \neq 0$ either to the right or to the left from x_1, and thus the theorem on separation of zeros and eventually, the whole theory of 2nd order equations holds for a pencil of solutions. These considerations helped to establish the properties of the zero points of solutions of equation (a). For example, a necessary and sufficient condition was proved for every solution of (a) with at least one zero to have infinitely many zeros. Further, a sufficient condition was found for equation (a) to have at least one solution without zero points. The crucial role in this condition is played by the pencil of solutions at the endpoint of the definition interval i of equation (a). We can similarly introduce the pencil of solutions of equation (b) at the point x_1, and to every assertion concerning the pencils of solutions of equation (a) there is a dual assertion on the pencils of solutions of equation (b).

When studying the differential equation (a), Professor Greguš has focused his attention mainly on the oscillatory properties of solutions, that is, on the position of their zero points. He also discussed the disconjugate equation (a). Such an equation is, in a certain sense, a generalization of the equation $y''' = 0$, and has many interesting properties, which is why the disconjugate equations and their nonlinear perturbations attract the attention of many mathematicians. M. Greguš found sufficient conditions for the disconjugacy of equation (a). By comparing two equations of the form (a) he derived sufficient conditions for equation (a) to have at least one oscillatory solution, in which case equation (a) is said to be oscillatory. An interesting case occurs when each solution of equation (a) with at least one zero point has infinitely many of them, and at the same time there exists a solution without zero points. In this case M. Greguš discusses further properties of the solution without zero points, in particular, the problem when each of its derivatives of order 0, 1, 2 has a constant sign.

Another fundamental concept of the theory of oscillation of solutions of equation (a) is that of a conjugate point. It is introduced in a way analogous to the case of the 2nd order equation, but more complicated. As concerns the relations between conjugate points of two equations, comparison theorems hold, on the basis of which further criteria of oscillatoricity of equation (a) were derived. From the other problems solved in the monograph we should mention the study of equation (a) with an oscillating Laguerre's invariant, and the construction of an equation (a) whose all solutions oscillate. This situation cannot occur for equations (a) with constant coefficients, and for a long time the (incorrect) opinion prevailed that each equation of the form (a) has a solution without zero points.

The discussion of oscillatory properties of solutions of equation (a) is amended by the investigation of the asymptotic behaviour of solutions of this equation, in particular, by a study of such properties of solutions in an interval $[c, \infty)$ as boundedness, existence of a limit at infinity, construction of asymptotic formulae, and the problem whether the solution belongs to the space L^1 or L^2. The simplest case is that of solutions without zero points, while the situation for oscillatory solutions is more complicated.

An important part of Professor Greguš' research consists in studying boundary value problems. First of all, he constructed Green's function for the multipoint boundary value problem (the three-point problem in the case of equation (a)), which makes it possible to transform nonlinear boundary value problems to nonlinear integral equations, which can then be solved by various methods. Further, he extended the Sturm oscillation theorem to equation (a) and solved the three-point boundary value problem for equation (a) whose coefficients depend on two parameters.

Many results obtained for equation (a) are transferred to the 3rd order equation in the general form

$$\text{(A)} \qquad y''' + p_1(x)\, y'' + p_2(x)\, y' + p_3(x)\, y = 0\,.$$

Again an important role is played by the notion of a pencil of solutions at a given point. Among the results for equation (A), the criteria of disconjugacy of equation (A), conditions for existence of oscillatory solutions and comparison theorems are of interest.

Recently, Professor Greguš has intensively worked in the applications of the theory of differential equations in physics, especially in the physics of plasma. His liking for physics goes back to his University years, and his results demonstrate that every good mathematician looks for impulses for his research not only within the theory but also in considering mathematical models in other fields of science.

For his excellent results in research, Professor M. Greguš was deservedly elected the corresponding member of both the Czechoslovak and the Slovak Academy of Sciences. He has published about fifty papers which are frequently referred to in works of Czechoslovak as well as foreign mathematicians, among other in Hartman's and Swanson's monographs. Many mathematicians in Czechoslovakia and abroad have found stimuli for their work in Greguš' results.

It is difficult to briefly describe the many-sided and useful activities of Professor Greguš. It is to be pointed out that he is always doing what is most important, and is devoting himself to his work with all his well-known vigor and enthusiasm. Therefore we wish him for the future all the energy necessary to bring his plans into reality and to continue his work in good health and with his usual youthful zest, and all the opportunities to enjoy the results of his works.

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