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THE SIXTIETH ANNIVERSARY OF PROFESSOR MILAN KOLIBIAR

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On February 14, 1982, Professor Milan Kolibiar, one of the leading personalities of the mathematical sciences in Slovakia, reached sixty years of age.

Milan Kolibiar was born in Detvianska Huta, district Zvolen. He attended secondary schools in Zvolen and in Kláštor pod Znievom. Then he studied mathematics and physics at the Faculty of Science of the Slovak University in Bratislava, where he graduated in 1946. He became Assistant (in 1946), Associated Professor (in 1956) and Full Professor (in 1965) at the same university (holding now the name Komenský University). He reached the degree of Doctor (RNDr., in 1950) and the degree of Doctor of Science (DrSc., in 1965). In 1964 he was appointed Head of the newly established Department of Algebra and Number Theory, and at present he is still holding this office.

The scientific interests of Professor M. Kolibiar (which were influenced by his university teachers Academician O. Borůvka and Academician Š. Schwarz) concentrate on partially ordered sets, lattices and universal algebra. He examined also the border fields between algebra and topology.

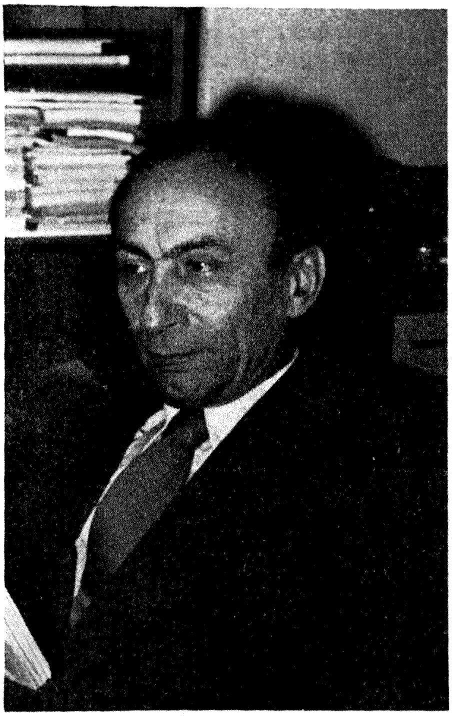
Let us give a concise survey of some results of the scientific work of M. Kolibiar.

The paper [A1] deals with the well-known problem proposed by G. Birkhoff (cf. [1], Problem 8) concerning isomorphisms of unoriented graphs of discrete lattices. The question was: *when does the isomorphism of lattices follow from the isomorphism of the corresponding unoriented graphs*? In [A1] the following theorem is proved: *The unoriented graphs of two finite distributive lattices S and S' are isomorphic if and only if there exist lattices A and B such that*

$$S = A \times B \quad \text{and} \quad S' = A \times \bar{B}, \tag{1}$$

where $\bar{B}$ is the lattice dual to B and $\times$ denotes the operation of direct product of partially ordered sets. (Cf. also the quotations of this result in [3] and [4].) M. Kolibiar returned to the relation (1) in several other connections and under different assumptions. In [A11] he proved that two metric distributive multilattices S and S' are isomorphic as metric spaces if and only if S and S' can be written in the form (1) for appropriate distributive multilattices A and B. He established a similar result for metric lattices [A6] and for semilattices [A9]. In the articles [A6] and [A26] he investigated lattices $S = (S;\ \vee,\ \wedge)$ with a third binary operation $\cap$ which is distributive with respect to both $\vee$ and $\wedge$. Far-reaching generalizations of results of B. H. Arnold are obtained there; again, the relation (1) plays an essential role in this investigation.

For a lattice S let $C\operatorname{Sub}(S)$ and $I(S)$ be the lattice consisting of all convex sublattices of S or closed intervals of S, respectively. In [A27] and [A31] M. Kolibiar studied couples of lattices S and S' such that $C\operatorname{Sub}(S)$ and $C\operatorname{Sub}(S')$ are iso-



morphic. He proved that this is the case if and only if $I(S)$ is isomorphic to $I(S')$; next he showed that this is equivalent to the condition that S and S' can be written in the form (1). In the recent paper [A32], weak homomorphisms of several types of algebraic structures are studied; nowadays these problems undergo their renaissance in view of their applications in computer science. One of the typical Kolibiar's results says: *Let $\varphi : L \to L'$ be a bijection between lattices. Then φ is a pseudo-weak isomorphism (that means, $(a_1 \vee a_2)\varphi = g(a_1\varphi, a_2\varphi)$, $(a_1 \wedge a_2)\varphi = h(a_1\varphi, a_2\varphi)$ for some binary algebraic functions g, h on L', and similarly, $(b_1 \vee b_2)\varphi^{-1} = g(b_1\varphi^{-1}, b_2\varphi^{-1})$, $(b_1 \wedge b_2)\varphi^{-1} = h(b_1\varphi^{-1}, b_2\varphi^{-1})$ for some algebraic functions g, h on L) if and only if one of the following three cases occurs:* (i) *φ is a* (*usual*) *isomorphism,* (ii) φ *is a dual isomorphism and* (iii) *both L and L' are bounded and there are lattices A and B such that L and L' can be written in the form* (1).

Already in the paper [A1] the importance of studying ternary operations and ternary relations on lattices was mentioned, e.g. the median operation

$$(2) \qquad (a, b, c) = (a \wedge b) \vee (a \wedge c) \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \wedge (b \vee c)$$

and the ternary relation "between" axb defined by

$$(3) \qquad (a \wedge x) \vee (x \wedge b) = x = (a \vee x) \wedge (x \vee b).$$

The papers [A4]–[A6], [A9], [A15], [A18], [A20], [A23] and [A32] are related to these problems. There are many reasons why ternary relations and operations on lattices are studied. The first result in this direction was the well-known theorem of V. Glivenko, who the problem restricted to metric lattices (recall that every metric lattice is a modular one) and proved that the metric relation "between" defined by the equality

$$\varrho(a, x) + \varrho(x, b) = \varrho(a, b)$$

is identical with the relation "between" determined by relation (3). Relation (3) turned out to be more suitable as a starting point because it can be introduced on arbitrary lattices. The problem consisted in finding conditions for a ternary relation $a\,x\,b$ defined on a set L under which there exists a lattice $(L; \vee, \wedge)$ having the property that (3) holds identically on L.

This problem was settled for modular lattices by L. M. Kelly and by M. F. Smiley with W. R. Transue for bounded lattices. M. Kolibiar [A9] solved the problem for general lattices (cf. also [7]). In the paper [A6] he proved that if two lattices S and S' are constructed by the method of [A9] from a set L with a ternary relation, then S and S' need not be isomorphic but must have the form (1). A further impulse to study ternary operations on lattices was given by an article of G. Birkhoff and S. Kiss and by Problem 66 of Birkhoff's monograph [1] concerning the characterization of a lattice by means of a ternary operation. M. Kolibiar [A5] (cf. also [7]) partially solved this problem (namely, he found a solution for bounded lattices, using a partial ternary operation of the form (2)).

M. Altweg (see also [7]) investigated a system of axioms for abstract description of the ternary relation

$$a \leqq x \leqq b \quad \text{or} \quad b \leqq x \leqq a$$

in partially ordered sets. M. Kolibiar in [A11] and [A15] examined some modifications of Altweg's conditions and applied them for generalizing some results of [A9] to the case of directed multilattices.

New methods of using the ternary relation "between" on partially ordered sets to the study of classical notions are developed in the papers [A10], [A11], [A15] and [A18]. In terms of the "betweenness" relation M. Kolibiar defined the notion "line" (a particular case of line being a chain). By means of the notion of a line he was able to formulate and prove several far-reaching generalizations of theorems of Jordan-Hölder type.

The very important result of the paper [A7] (cf. also the monographs [2]–[4] and [7]) consists in finding two identities expressed in terms of the operations $\vee$ and $\wedge$ which characterize modular lattices. According to a recent result of R. McKenzie and R. Padmanabhan (cf. [4]) modular lattices cannot be characterized by means of a single identity involving the binary operations $\vee$ and $\wedge$.

In the paper [A8] M. Kolibiar succeeded in describing relatively complemented distributive lattices in five different ways (extending several previous results); L. A. Skornjakov [8] calls this result Kolibiar-Hashimoto-Grätzer-Schmidt's Theorem. Paper [A22] also deals with distributive lattices; it essentially extends a result of B. Jónsson.

The papers [A13], [A17], [A28] and [A30] have two common aims: to establish an algebraic description of partially ordered sets P such that the interval topology on P is Hausdorff, and to apply this description to proving fixed point theorems for isotonic mappings.

The papers [A3], [A14], [A25] (and also part of [A32]) are of purely universal algebraic character. Already in Birkhoff's book [1] it was shown that the direct product decompositions $A = A_1 \times A_2 \times \ldots \times A_n$ of an algebra A are in a one-to-one correspondence with systems of permutable congruence relations $\theta_1, \theta_2, \ldots, \theta_n$ on A fulfilling certain conditions. In 1957 J. Hashimoto investigated direct product decompositions of algebras of the form $A = \prod(A_i : i \in I)$, where the set I can be infinite. M. Kolibiar [A19] did the same under more general assumptions (namely, for relational structures); cf. also [A14] and [5].

The results of scientific papers written by M. Kolibiar were applied by many mathematicians both in Czechoslovakia and abroad and they were quoted in monographs on lattice theory, universal algebra and graph theory (cf. e.g. G. Birkhoff [2], G. Grätzer [4], [5] and O. Ore [6]).

Professor M. Kolibiar has devoted a good deal of his time and energy to the education of young mathematicians dealing with algebra. His scientific and pedagogical work has essentially influenced two generations of Slovak mathematicians. He has been unceasingly interested in problems of teaching mathematics at secondary schools, universities and technical universities. Professor Kolibiar was one of the enthusiastic founders of the secondary-school students competition "Mathematical Olympiad" in the fifties.

The University has confered several duties upon Professor M. Kolibiar. Besides, for many years he has been member of the Scientific Board for Mathematics at the Czechoslovak Academy of Sciences and of the Commitee for Mathematics at the Slovak Academy of Sciences, member or chairman of several committees for doctoral and post-doctoral dissertations, member of editorial boards of the journals Acta Mathematica Univ. Comenianae and Mathematica Slovaka, etc. His participation in the organization and administration of the State Research Program (in the field of algebra and mathematical logic) is also of great importance. He is one of the founders of the traditional summer schools on partially ordered sets and universal algebra in Czechoslovakia.

The Czechoslovak mathematical community wishes Professor Milan Kolibiar good health and many further successes in both his scientific and educational activities.

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