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THE 60TH ANNIVERSARY OF PROFESSOR FRANTIŠEK ŠIK

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On September 29, 1981 RNDr. František Šik, DrSc., Professor of the Faculty of Science, J. E. Purkyně University of Brno, celebrated the 60th anniversary of his birthday.

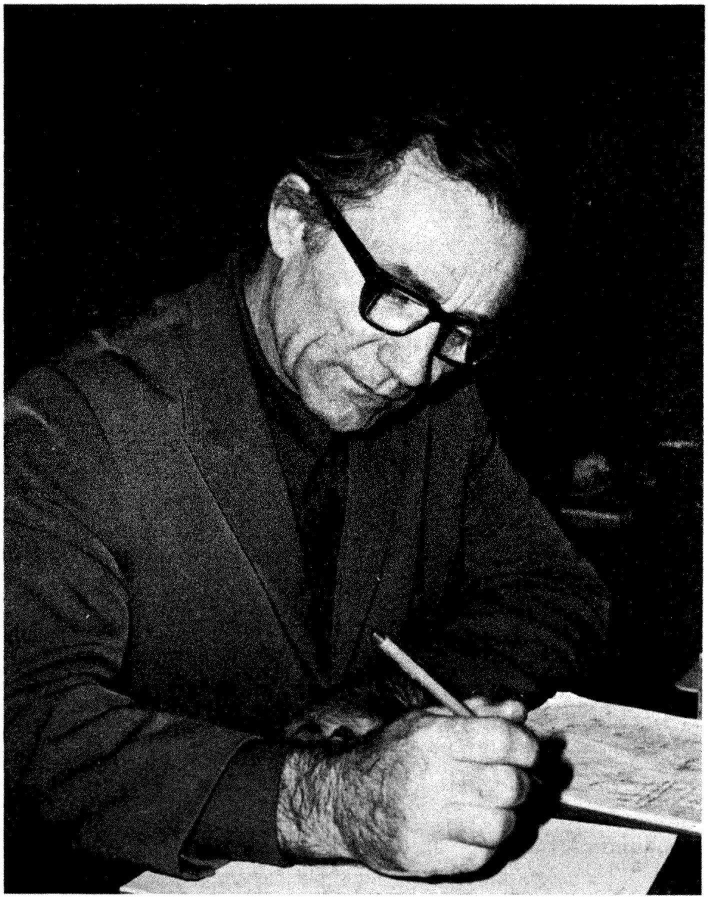
He was born in Brno in the family of a post-office employee. In 1948 he graduated from the Faculty of Science in Brno. During his studies he was much influenced by his professor Academician O. Borůvka. In the years 1948–50 he worked as lecturer at the Technical University in Brno. He completed his postgraduate studies at the Institute of Mathematics of the Cezchoslovak Academy of Sciences in Prague. His supervisors were Academians E. Čech and V. Kořínek. In 1958 he became Associated Professor and in 1963 Full Professor of the Faculty of Science, J. E. Purkyně University of Brno, where he has been working up to now. He was Dean of the Faculty for the period 1965–69. His courses for students have been devoted mainly to algebra and topology. In 1962–64 he was Visiting Professor at the University of Havana.

His first scientific publication appeared in 1951. He concentrated his scientific interest upon algebra and partly upon topology, first of all upon the border fields between algebra and topology, and in the last years upon the optimalization methods and their applications. The essential part of his work is concerned with the study of ordered groups. This theory is also the topic of his research seminar, where he has educated a great number of young mathematicians. Professor F. Šik is the auhor of more than 40 scientific papers. Because of the extent of his work it is not possible to mention all the results he has achieved. We can only give a short characterization of the main trends of his research.

The characteristic feature of Professor Šik's scientific work is the fact that he does not only try to derive "isolated theorems" but investigates the studied field in detail, step by step, to comprehend fully the mutual relations of the mathematical objects in question. To discover the inner structure of these objects, he combines in some cases several methods (e.g. algebraic and topological ones.)

F. Šik introduced the notion of orthogonality on a quasiordered set and used it to define the notion of polar. Then he applied these notions to deducing new results on direct product decompositions of a group. Further, he investigated the orthogonality on a lattice ordered group G which is defined by the relation $x \,\delta\, y \Leftrightarrow |x| \wedge |y| = 0$. For $A \subseteq G$ let us denote $A' = \{x \in A : x \,\delta\, a \text{ for each } a \in A\}$; the set A' is called a polar in G. If $a \in G$, then $\{\{a\}'\}'$ and $\{a\}'$ is called a principal polar and a dual principal polar, respectively. The systems of all polars, all principal polars and all dual principal polars of G are denoted by $\Gamma(G)$, $\Pi(G)$ and $\Pi'(G)$, respectively.

The notion of polar turned out to have fundamental importance for studying several questions that concern the structure of lattice ordered groups. (The shorter name l-group is often preferred to lattice ordered group.) F. Šik [7] proved that the



system $\Gamma(G)$ partially ordered by inclusion is a complete Boolean algebra and that $\bigwedge_{i\in I} X_i = \bigcap_{i\in I} X_i$ for each $\emptyset \neq \{X_i\}_{i\in I} \subseteq \Gamma(G)$. This Šik's theorem is quoted in monographs on lattice ordered groups (L. Fuchs; P. Conrad; A. Bigard, K. Keimel, S. Wolfenstein; V. M. Kopytov) and was applied in many papers. F. Šik also investigated thoroughly analogous questions for directed groups [18]. The conditions for $\Pi(G)$ or $\Pi'(G)$ to be Boolean algebras can be found in [21]; one of equivalent conditions is $\Pi(G) = \Pi'(G)$. The properties of the lattices $\Pi(G)$ and $\Pi'(G)$ are used in [24] for determining conditions under which certain factor l-groups of G are linearly ordered.

For the case of complete vector lattices the notion of polar was studied by F. Riesz (1940); the orthogonality in complete lattice ordered groups has been investigated by L. V. Kantorovič and his school in Leningrad.

Each direct factor of a lattice ordered group G is a polar of G. Direct and subdirect product decompositions of l-groups and of directed groups are dealt with in papers [9], [10], [13] and [19]. The major part of the results concerns the case when the direct or the subdirect factors are linearly ordered. In the literature dealing with l-groups the following criterion on the representability of an l-group G due to Šik is well-known and often applied: G is representable if and only if each polar of G is a normal subgroup. (The representability of G means that G is a subdirect product of linearly ordered groups.) The notion of completely subdirect product introduced by F. Šik has become a standard tool in studying the structure of l-groups. Further notions introduced by Šik for classifying the subdirect product decompositions (e.g., the notion of reduced subdirect product) were used for investigating the structure of l-groups by topological methods (cf. below).

A lattice ordered group G is said to be compactly generated if for each $a \in G$ and each system $\{a_i\}_{i \in I} \subseteq G$ such that $a = \bigvee_{i \in I} a_i$ there exists a finite subset $J \subseteq I$ with $a = \bigvee_{j \in J} a_j$. Compatcly generated l-groups are investigated in [20] and [27]. We quote a typical result: G is compactly generated if and only if it fulfils the conditions a) minimal prime subgroups of G are maximal polars, and b) minimal polars of G are discretely ordered sets. Inspired by these Šik's papers several authors (A. Bigard, P. Conrad, S. Wolfenstein, and others) continued the exploration of compactly generated l-groups.

The paper [8] (published in 1958) belongs to the first studies on ordered permutation groups anticipating the deep theory which was developed years later (cf. the monograph "Ordered permutation groups" 1976 written by A. M. W. Glass).

In the papers [14], [15] and [16] F. Šik dealt with additive and isotone mappings of a partially ordered group G into the additive group R of all reals (with the natural linear order). Conditions are given under which a mapping with the mentioned properties defined on a subgroup H of G can be extended to the whole group G, as well as conditions for the uniqueness of such an extension. In the proofs, the results of the paper [11] concerning extensions of partially ordered groups are essentially applied.

Another domain of algebra systematically studied by F. Šik are the questions on partitions and congruence relations treated in [1], [2], [3], [31], [32], [33], [42] and [43]. His theorem on permutable partitions is quoted in the book of A. G. Kuroš "Lekcii po obščej algebre". The theorem states that two partitions P and Q on a set G are permutable if and only if PQ is a partition on G; in such a case the relation $PQ = P \vee Q$ is valid. From this series of papers the fundafental importance of [31] and [42] deserves to be emphasized because of the deep insight and the ingenious constructions leading to a theorem on the existence of isomorphic refinements for two chains of congruence relations of a universal algebra (a theorem of Schreier-Zassenhaus type), and to a common generalization of a theorem of O. Borůvka (1941) and a theorem of A. Chatelet (1944) (both these authors were dealing with situations analogous to the group-theoretical study of invariant chains).

One of the most important parts of Šik's scientific activity concerns the topological aspects of representation of l-groups as subdirect sums of linearly ordered groups. The basic notion for this research proved to be a regulator of an l-group. A regulator of an l-group G is a pair (R, U), where R is a non-empty set and U is a map of the set R in to the system of all simple subgroups of G with the property $\bigcap\{Ux : x \in R\} = \{0\}$ (0 being the neutral element of G). A regulator is called standard, if $Ux \neq G$ for any $x \in R$. (R, U) is Hausdorff if $x \neq y$ implies the existence of $f, g \in G$ for which $f \,\delta\, g$, $f \bar{\in} Ux$, $g \bar{\in} Uy$. (R, U) is completely regular if to each $f \in Ux$ there exists $g \in G$ such that $x \in R - Zg$. Here $Zg = \{x : x \in R, g \in Ux\}$. A regulator (R, U) is reduced if $Ux \parallel Uy$ for $x \neq y$. The sets Zf, $f \in G$ form a base of closed sets for the topological space (R, U) on the set R. An important special case occurs if all the subgroups Ux are ideals. Then (R, U) is called a realizator and the l-group G is the subdirect sum of linearly ordered groups $\{G_x : x \in R\}$, where $G_x \cong G/Ux$. Under these conditions the topological space is an analogon of the weak topology on a set X defined by a system C of real functions on X, in which all maps from C are continuous. A topological inspiration for such a study came from the book Gillamn, Jerison: Rings of continuous functions. The papers [28]–[26] from the years 1961–1968 are mainly devoted to realizators, the papers [36]–[41] deal with regulators. Let us introduce several results as examples of the detailed analysis in this area.

If (R, U) is a standard regulator of an l-group G, the following assertions are equivalent: 1. (R, G) is a T_1 – space. 2. (R, G) is a T_2 – space. 3. (R, U) is reduced. 4. (R, U) is Hausdorff ([38]).

A standard regulator is completely regular iff Ux is a minimal simple subgroup for all $x \in G$ [27]. Therefore, the space (R, G) is completely regular for a standard, completely regular regulator (R, U).

It should be mentioned that the essential part of Šik's results on this subject are included in the third chapter of the book A. Biggard, K. Keimel, S. Wolfenstein, Groupes et anneaux reticulés, Springer, Berlin 1977.

Clopen subsets of the space (R, G) for a realizator (R, U) are studied in [29]. Γ – regulator formed by set-union of ultrafilters in $\Pi(G)$ and Π' – regulator constructed analogously from $\Pi'(G)$ are examples of standard regulators. A comparison of Π – regulators with Π' – regulators was done in [40]. A special type of a regulator is obtained if one supposes that $\bigcap\{Uy : y \in R,\ y \neq x\} \neq \{0\}$ for all $x \in R$. Existence of such a regulator in an l-group is equivalent to the assertion that every polar is an intersection of maximal ones [41].

Topological papers [5], [6], [36] deal with rather general problems connecting algebra to the topology in the framework of ideas introduced by E. Čech in his topological seminar in Brno in 1937 and continued by the seminar of Professor K. Koutský in the fifties. The papers [34], [35[start with problems of the numerical praxis. A procedure of solving systems of linear algebraic equations is described, in which every coefficient runs through a certain interval of the real numbers (results of some measurements).

Šik's stimulating ideas and the open problems formulated by him have found many continuators among mathematicians in Czechoslovakia and abroad. Finally, the extensive work of his in reviewing papers for Czechoslovak Mathematical Journals, for Mathematical Reviews and Zentralblatt für Mathematik should be highly appreciated.

Prof. František Šik celebrated the 60th anniversary of his birthday full of optimism and scientific plans for the future. On behalf of his friends, colleagues and students we wish him personal well-being and further success in his research.

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SIXTIETH ANNIVERSARY OF BIRTHDAY OF PROF. RNDR. VÁCLAV METELKA, CSC.

RNDr. Václav Metelka, CSc., Associated Professor of Mathematics at the Technical University in Liberec (Northern Bohemia), reached sixty years of age on June 2, 1981.

The main field of scientific interest of Prof. Metelka is algebraic geometry, in particular the theory of configurations.

PROFESSOR RNDR. LADISLAV SEDLÁČEK, CSC., SEXAGENARIAN

RNDr. Ladislav Sedláček, CSc., Professor of Mathematics and Head of Department of Algebra and Geometry of the Faculty of Science at Palacký University in Olomouc (Northern Moravia), reached sixty years of age on May 2, 1981.

The main domain of scientific activity of Prof. Sedláček is algebra, particularly the theory of general algebraic structures, special groupoids and universal algebras, and the theory of education in Mathematics.

Prof. Sedláček is an outstanding officer of the Society of Czechoslovak Mathematicians and Physicists. In 1979 he was awarded the J. A. Komenský medal.

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