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CORRECTION TO MY PAPER ON STRUCTURE AND IDEAL THEORY OF COMMUTATIVE SEMIGROUPS

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Professor B. PONDĚLÍČEK has kindly drawn my attention to some errors in my paper [1]. These errors have crept in because of the neglect of verifying the theorems in trivial cases. The following are the corrected versions of Theorems 1.6 and 1.7 in [1]. Here we number them in the same way for ready reference.

Theorem 1.6. *Let S be a U-semigroup which is not a group. If P is the union of all its proper prime ideals, then $P \neq \square$ and $S = x \cup xs$ for every $x \in S \setminus P$. The converse holds if $P \neq S$.*

Proof. If S does not contain any proper prime ideals, then for every a in S, $\sqrt{(a \cup aS)} = S$, which implies $S = a \cup aS$ by U-semigroup property. Then S becomes a group, which is not true by an assumption. Thus P is non-empty. Now, if $x \in S \setminus P$, $S = \sqrt{(x \cup xS)}$, which implies $S = x \cup xS$. Conversely let S be not a group and let A be an ideal different from S. If $x \in A \setminus P$, then $S = x \cup xS$ by assumption and so $A = S$, which is a contradiction. Therefore $A \subseteqq P$ we claim now that $\sqrt{(A)} \neq S$. If possible, let $\sqrt{(A)} = S$. Then if $x \in s$, $x^n \in A$ for some natural number n and so $x^n \in P$, which is a prime ideal and thus $x \in P$. Therefore $S = P$, which is a contradiction. Thus S is a U-semigroup.

Theorem 1.7. *Let S be a semigroup which is not the union of all its proper prime ideals but contains maximal ideals. Then the following are equivalent:*

i) $S = S^2$,

ii) *S contains a unique maximal ideal which is prime.*

Proof. (i) $\Rightarrow$ (*ii*). Let $T = \{a : \sqrt{(aS^1)} \neq S\}$. If $T = \square$, then for every $a \in S$, $\sqrt{(aS^1)} = S$ and so S contains no proper prime ideals. But maximal ideals are prime by [2]. Hence this case is inadmissible. If $T \neq S$, then T is the unique maximal ideal. For, let M be any maximal ideal. Since $S = S^2$, M is a prime ideal and so $\sqrt{(M)} = M$.

Now if $a \in M \setminus T$, then $S = \sqrt{(a \cup aS)} \subseteq \sqrt{(M)} = M$. Thus $M \subseteq T$ and so $M = T$. The only other possibility is $T = S$. Since S is not the union P of its prime ideals, we have then for $x \in S \setminus P$, $\sqrt{(x \cup xS)} = S$, which is not true since $T = S$.

(ii) $\Rightarrow$ (i) follows by Schwartz's result [2].

References

[1] *Satyanarayana, M.:* Structure and ideal theory of commutative semigroups, Czech. Math. Jour., *28 (103)* (1978), 171–180.

[2] *Schwarz, Štefan:* Prime ideals and maximal ideals in semigroups, Czech. Math. Jour., *19 (94)* (1969), 72–79.

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