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ON CONGRUENCE LATTICES IN A CATEGORY*)

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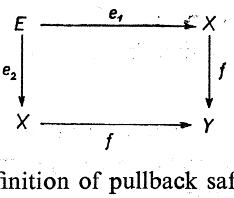
A congruence on an object X of a category $\mathbf{C}$ is a subobject of $X \times X$ which is a kernel pair of some morphism with domain X. All congruences on X form a lattice (under weak assumptions on $\mathbf{C}$, discussed below) and they can be composed via pullbacks. The aim of the present paper is to exhibit a categorial proof of the fact (well-known in universal algebra) that if congruences are permutable then the congruence lattice is modular. Congruences on algebraic structures are mentioned.

1. Congruences on an object

1.1 a) In this part we make several easy (and probably well-known) observations about congruences in a category $\mathbf{C}$. We assume throughout the paper that $\mathbf{C}$ has finite limits.

b) A *relation* E on an object X is a subobject of $X \times X$, i.e., a monomorphism $E \to X \times X$, up to isomorphism. Composing this with projections we obtain a pair $e_1, e_2 : E \to X$ and we shall identify the relation with (e_1, e_2). (A pair of morphisms $f_1, f_2 : E \to X$ represents a relation iff it is collectively monomorphic, i.e., given distinct $p, q : Y \to F$ then either $f_1 . p \neq f_1 . q$ or $f_2 . p \neq f_2 . q$.)

Let $f : X \to Y$ be a morphism in $\mathbf{C}$. The *kernel pair* of f is the relation (e_1, e_2), defined by the pullback



(This is correct because the definition of pullback safeguards (e_1, e_2) as a collective mono.) We denote $\ker f = (e_1, e_2)$. A relation on X is a *congruence* if it is the kernel pair of some morphism with domain X.

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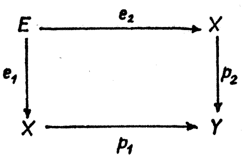
c) Relations on X are naturally ordered, as subobjects of $X \times X$. We denote by Con (X) the ordered class of congruences on X. This is a poset, provided that $\boldsymbol{C}$ is well-powered, of course. It is worth noticing that, under side conditions, it is a poset also if $\boldsymbol{C}$ is regularly cowell-powered (i.e., if every object has only a set of coequalizer-quotients). Recall that a multiple pushout of regular epis is called regular cointersection.

1.2. Proposition. *Let* $\boldsymbol{C}$ *have finite limits. A necessary and sufficient condition for each* Con (X), $X \in \boldsymbol{C}^o$, *to be a (small) complete lattice is:*

$\boldsymbol{C}$ be regularly cowell-powered and have regular cointersections.

Proof. I) Sufficiency.

a) Every congruence $e_1, e_2 : E \to X$ has a coequalizer. Proof: let $(e_1, e_2) = \ker f$, then there is $d : X \to E$ with $e_1 . d = e_2 . d = 1$ (because $f . 1 = f . 1$). Hence, e_1 and e_2 are regular (even split) epis and, by hypothesis they have a pushout



Then p_1 is the coequalizer of e_1, e_2 because $p_1 = p_1 . e_1 . d = p_1 . e_2 . d = p_2$.

b) Con (X) is a set for every X. This follows from the fact that $\boldsymbol{C}$ is regularly cowell-powered, and that two distinct congruences have clearly coequalizers, distinct as quotient-objects.

c) Con (X) is a complete lattice: for each non-void set M of congruences, let c be the cointersection of the coequalizers c_m $(m \in M)$ of the pairs m. The ker c is the join of M. And the least element of Con (X) is $\Delta_X = \ker \mathrm{id}_X$.

II) Necessity is easy; it suffices to recall that for a regular epi f, $\ker f$ has always a coequalizer, viz., f.

1.3. a) Definition [2]. A category is *regular* if it has finite limits and if pullbacks carry regular epis (i.e., given a pullback $\bar{p} . p = \bar{q} . q$ such that p is a regular epi, then also $\bar{q}$ is a regular epi).

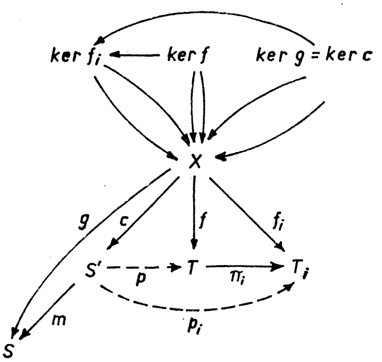
b) Proposition. *Let* $\boldsymbol{C}$ *be a regular category in which each* Con (X) *is a complete lattice. Then* $\boldsymbol{C}$ *has regular factorizations, i.e., every morphism can be factorized as a regular epi followed by a mono.*

Proof. As noted in the previous proof, each kernel pair has a coequilizer. It is proved in [2] (see 1.6 on page 130) that every regular category with this property has regular factorizations (a result of Tierney).

c) The following proposition shows that every well-behaved category has complete lattices of congruences. (If well-behaved means cocomplete, finitely complete and regularly co-well powered, then this follows from the above, of course.) Hoehnke [4] proves this proposition more generally for factorization systems $(\mathscr{E}, \mathscr{M})$; yet we exhibit a proof here since it is a short one and we need it below.

d) Proposition. *Let* C *be a complete, well-powered category with regular factorizations. Then each* Con(X) *is a complete lattice: Given* $\ker f_i$ *in* Con(X) $(i \in I)$, *with* $f_i : X \to T_i$, *let* $f : X \to \prod_{i \in I} T_i$ *be the canonical morphism, then* $\ker f = \bigwedge_{i \in I} \ker f_i$ *in* Con(X).

Proof. Denote by $\pi_{i_0} : T \to T_{i_0}$ the projections from $T = \prod T_i$, then $f_i = \pi_i . f$, hence $\ker f_i \geqq \ker f$, for each $i \in I$. Let $g : X \to S$ be given with $\ker f_i \geqq \ker g$ for each i:



If $g = m . c$, m a mono and $c : X \to S'$ a coequalizer, then $\ker c = (e_1, e_2)$ implies $c = \mathrm{coeq}\,(e_1, e_2)$ and $\ker g = \ker c$. Now, $\ker c \leqq \ker f_i$, hence $f_i . e_1 = f_i . e_2$ for each i and we get $p_i : S' \to T_i$ with $f_i = p_i . c$ for each $i \in I$. This yields a canonical $p : S' \to T$ with $f = p . c$. The last implies $\ker f \geqq \ker c = \ker g$. Hence, $\ker f = \bigwedge \ker f_i$.

e) Corollary. *Let* C, D *be complete, well-powered categories with regular factorizations. Given a functor* $F : C \to D$ *we obtain, for each object* X *in* C, *an induced (compatible) mapping*

$$F_{(X)} : \mathrm{Con}_C(X) \to \mathrm{Con}_D(FX) : \ker f \mapsto \ker Ff .$$

If F *preserves products then each* $F_{(X)}$ *preserves meets.*

1.4. a) The above Corollary shows why concrete categories usually have the property that congruences on X form a closure system in the lattice of equivalences on X (i.e., congruences are closed to meets). The reason is that the forgetful functor $U : C \to$ Set often as not preserves limits; and the induced mappings $U_{(X)}$ are embeddings $\mathrm{Con}_{\boldsymbol{C}}(X) \to \mathrm{Eq}(X) =^{\mathrm{def}} \mathrm{Con}_{\mathrm{set}}(UX)$. For finitary concrete categories, i.e. such that U preserves filtered colimits, $\mathrm{Con}(X)$ is an algebraic closure system in $\mathrm{Eq}(X)$ (i.e., congruences are closed to directed joins):

b) Proposition. *Let $\boldsymbol{C}, \boldsymbol{D}$ be finitely complete well-powered categories with regular factorizations and directed colimits. Let $F : \boldsymbol{C} \to \boldsymbol{D}$ be a functor preserving directed colimits. Then the induced mappings $F_{(X)} : \mathrm{Con}_{\boldsymbol{C}}(X) \to \mathrm{Con}_{\boldsymbol{D}}(FX)$ all preserve directed joins.*

Proof is easy.

c) To generalize the well-known fact that for varieties of finitary algebras $\mathrm{Con}(X)$ is a complete sublattice of $\mathrm{Eq}(X)$, we must consider functors $F : \boldsymbol{C} \to \boldsymbol{D}$ which preserve some pushouts. (This guarantees that then all $F_{(X)}$ preserve finite joins.)

Generally, preservation of pushouts is a much less natural condition than preservation of limits and directed colimits (when forgetful functors are studied). But an interesting result of Barr [3] is helpful here. Barr proves that, given EX5 categories (see 2,1c below) $\boldsymbol{C}, \boldsymbol{D}$, every functor $F : \boldsymbol{C} \to \boldsymbol{D}$ which preserves finite limits and exact coequalizers (i.e., coequalizers of kernel pairs), preserves finite regular cointersections. And preservation of exact coequalizers is a rather natural condition (e.g., every set-functor preserves exact coequalizers).

d) Proposition. *Let $\boldsymbol{C}, \boldsymbol{D}$ be complete, well-powered categories with directed colimits and regular factorizations. If a functor $F : \boldsymbol{C} \to \boldsymbol{D}$ preserves products, directed colimits and pushouts of regular epis, then each $F_{(X)} : \mathrm{Con}_{\boldsymbol{C}}(X) \to$ $\to \mathrm{Con}_{\boldsymbol{D}}(FX)$ is a complete lattice homomorphism.*

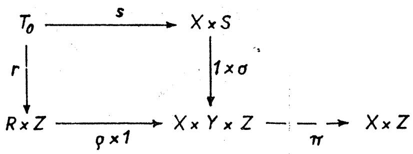
Proof is easy.

Corollary. *Let $\boldsymbol{C}, \boldsymbol{D}$ be complete, well-powered* EX5 *categories. Then every finitary functor $F : \boldsymbol{C} \to \boldsymbol{D}$, preserving limits and exact coequalizers, has the property that each $F_{(X)}$ is a complete homomorphism.*

Example. Let $\boldsymbol{C}$ be a complete, well-powered, finitary concrete EX5 category with a continuous forgetful functor $U : \boldsymbol{C} \to \mathrm{Set}$. (This is a very natural set of conditions, satisfied e.g. by every variety of finitary algebras.) If U preserves exact coequalizers, then each $\mathrm{Con}(X)$ is isomorphic to a complete sublattice of the lattice of equivalences on UX.

2. Permutable congruences

2.1. a) Relations on an object X (more generally, relations from X to Y, i.e. subobjects of $X \times Y$) are not only ordered but they also have a natural algebraic structure of composition. Following Barr and Grillet [2] we define the *composition* *of relations* $\varrho : R \to X \times Y$ and $\sigma : S \to Y \times Z$ (ϱ, σ are monos) in a regular category as follows. Consider the pullback of $\varrho \times 1$ and $1 \times \sigma$



and the projection $\pi : X \times Y \times Z \to X \times Z$. Then $\pi . \varrho \times 1 . r = \pi . 1 \times \sigma . s :$ $: T_0 \to X \times Z$ can be factorized as a regular epi, followed by a mono, giving rise to a subobject $\pi : T \to X \times Z$. The composition of (R, ϱ) and (S, σ) is defined as the relation (T, τ). Concisely

$$T = S \circ R .$$

Proposition [2]. *In a regular category composition of relations is well-defined and associative.*

b) A relation R on an object X, i.e. a collective mono $e_1, e_2 : R \to X$, is said to be an *equivalence* if it is

(i) reflexive, i.e. $\Delta_X \subset R$ (or, equivalently, there exists $d : X \to R$ with $e_1 . d =$ $= 1 = e_2 . d$);

(ii) symmetric, i.e. $e_2, e_1 : R \to X$ is the same relation (the same subobject of $X \times X$); and

(iii) transitive, i.e. $R \circ R \subset R$.

Every congruence is easily seen to be an equivalence. The converse does not hold in general; it is called in [2]

(Lawvere Condition): *Every equivalence is a congruence.*

c) Barr calls a regular category $\boldsymbol{C}$ an *EX5-category* if it has filtered colimits and exact coequalizers, it satisfies Lawvere Condition and finite limits commute with filtered colimits.

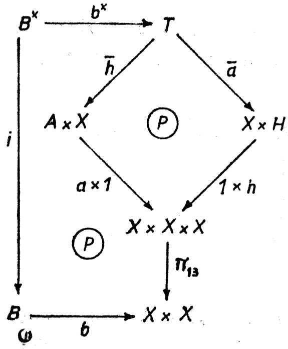
2.2. Lemma. *Given relations* A, B, H *on an object* X, *we have*

$$(A \circ H) \cap B \subset A \circ (H \cap (A^{-1} \circ B)) .$$

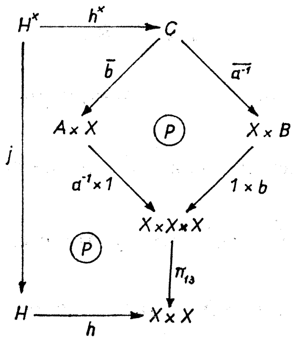
Proof. We have monos $a : A \to X \times X$; $b : B \to X \times X$ and $h : H \to X \times X$.

a) The left-hand relation $(A \circ H) \cap B$ is the image of $i . b$ in the following diagram *) (where **P** denotes pullbacks):

*) In all diagrams index $^{\times}$ substitute by *.



b) The relation $H \cap (A^{-1} \circ B)$ is the image of $h \,.\, j$ in the following diagram:



We denote the respective projections by

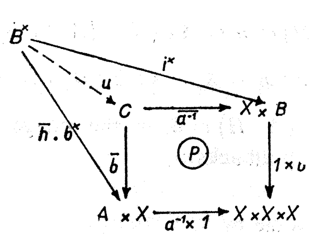
$$\pi_i : X \times X \times X \to X \text{ and } \pi_{i,j} : X \times X \times X \to X \times X \quad (i, j = 1, 2, 3 \text{ and } i \neq j) \text{ and}$$
$$\pi^k : X \times X \to X \quad (k = 1, 2)\,.$$

c) Now we define $i^* : B^* \to X \times B$ (= product with projections p_X, p_B) by

(1) $$p_B \,.\, i^* = i \quad \text{and} \quad p_X \,.\, i^* = \pi_1 \,.\, (a^{-1} \times 1) \,.\, \bar{h} \,.\, b^*\,.$$

Let us verify that $(1 \times b) \,.\, i^* = (a^{-1} \times 1) \,.\, (\bar{h} \,.\, b^*)$ and that, therefore, $u : B^* \to C$ exists with

(2) $$\overline{a^{-1}} \,.\, u = i^* \quad \text{and} \quad \bar{b} \,.\, u = \bar{h} \,.\, b^*:$$



We have: $\pi_1 . [(1 \times b) . i^*] = p_X . i^* = \pi_1 . [(a^{-1} \times 1) . (\bar{h}, b^*)]$, by definition of i^*. Furthermore, since $\pi_{2,3} . (1 \times b) = b . p_B$:

$$\pi_{2,3} . [(1 \times b) . i^*] = b . i$$

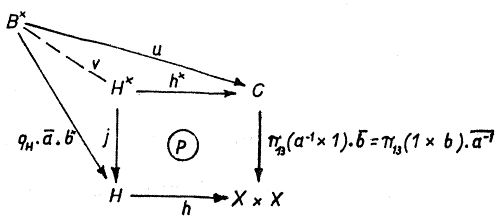
while $\pi_{2,3} . (a^{-1} \times 1) = \pi_{1,3} . (a \times 1)$ yields

$$\pi_{2,3} . [(a^{-1} \times 1) . (\bar{h} . b^*)] = \pi_{1,3} . (a \times 1) . \bar{h} . b^* = b . i .$$

This proves that $(1 \times b) . i^* = (a^{-1} \times 1) . (\bar{h} . b^*)$.

d) Let us verify that (denoting by q_X, q_H the projections of $X \times H$) we have $h . (q_H . \bar{a} . b^*) = [(a^{-1} \times 1) . \bar{b}] . u$ and that, therefore, there exists $v : B^* \to H^*$ with

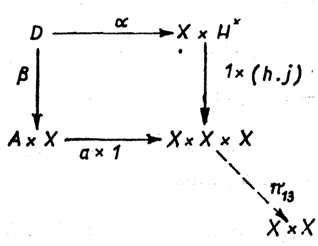
(3) $$h^* . v = u \quad \text{and} \quad j . v = q_H . \bar{a} . b^* :$$



Indeed, clearly $h . q_H = \pi_{2,3} . (1 \times h)$ and so

$$\begin{aligned} h . (q_H . \bar{a} . b^*) &= \pi_{2,3} . [(1 \times h) . \bar{a}] . b^* \\ &= \pi_{2,3} . (a \times 1) . \bar{h} . b^* \\ &= \pi_{1,3} . (a^{-1} \times 1) . \bar{h} . b^* \\ &= \pi_{1,3} . (a^{-1} \times 1) . \bar{b} . u . \end{aligned}$$

e) Finally, the right-hand relation $A \circ (H \cap (A^{-1} \circ B))$ is the image of $\pi_{1,3} . (a \times 1) . \beta$ in the pullback

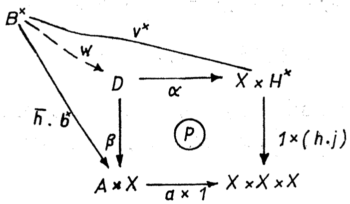


Denoting by r_X, r_{H^*} the projections of $X \times H^*$, we define $v^* : B^* \to X \times H^*$ by

(4) $$r_{H^*} \, . \, v^* = v \quad \text{and} \quad r_X \, . \, v^* = \pi_1 \, . \, (a \times 1) \, . \, \bar{h} \, . \, b^* \, .$$

Let us verify that $(a \times 1) \, . \, (\bar{h} \, . \, b^*) = (1 \times (h \, . \, j)) \, . \, v^*$, and that, therefore, there exists $w : B^* \to D$ with

(5) $$\alpha \, . \, w = v^* \quad \text{and} \quad \beta \, . \, w = \bar{h} \, . \, b^* :$$



We have, by definition of v^*,

$$\pi_1 \, . \, [(a \times 1) \, . \, \bar{h} \, . \, b^*] = r_X \, . \, v^*$$
$$= \pi_1 \, . \, [(1 \times (h \, . \, j)) \, . \, v^*] \, .$$

Furthermore, $(a \times 1) \, . \, \bar{h} = (1 \times h) \, . \, \bar{a}$ and $\pi_{2,3} \, . \, (1 \times h) = h \, . \, q_H$, thus

$$\pi_{2,3} \, . \, [(a \times 1) \, . \, \bar{h} \, . \, b^*] = h \, . \, q_H \, . \, \bar{a} \, . \, b^*$$
$$= h \, . \, j \, . \, v \quad \text{by (3)}$$
$$= h \, . \, j \, . \, r_{H^*} \, . \, v^* \quad \text{by (4)}$$
$$= \pi_{2,3} \, . \, [(1 \times (h \, . \, j) \, . \, v^*] \, .$$

f) To conclude the proof we only recall that

$$(A \circ H) \cap B = \text{im} \, (b \, . \, i) \quad \text{and} \quad A \circ (H \cap (A^{-1} \circ B)) = \text{im} \, (\pi_{1,3} \, . \, (a \times 1) \, . \, \beta)$$

and that

$$b \, . \, i = b \, . \, p_B \, . \, i^* \quad \text{by (1)}$$
$$= \pi_{2,3} \, . \, (1 \times b) \, . \, i^* =$$
$$= \pi_{2,3} \, . \, (a^{-1} \times 1) \, . \, \bar{b} \, . \, u \quad \text{by (2)}$$
$$= \pi_{1,3} \, . \, (a \times 1) \, . \, \bar{h} \, . \, b^*$$
$$= (\pi_{1,3} \, . \, (a \times 1) \, . \, \beta) \, . \, w \quad \text{by (5)} \, .$$

Since $b \, . \, i$ factorizes through $\pi_{1,3} \, . \, (a \times 1) \, . \, \beta$, its image is clearly smaller or equal to the image of the latter morphism, thus

$$(A \circ H) \cap B \subset A \circ (H \cap (A^{-1} \, . \, B)) \, .$$

2.3. a) Theorem. *Let* C *be a regular category with Lawvere Condition. Let* X *be its object with permutable congruences, i.e. such that* $R \circ S = S \circ R$ *for* $R, S \in \in \mathrm{Con}(X)$. *Then* $\mathrm{Con}(X)$ *is a modular lattice with joins defined by*

$$R \vee S = R \circ S \quad for \quad R, S \in \mathrm{Con}(X).$$

Proof. All we have to prove is that $R \circ S$ is a congruence, for $R, S \in \mathrm{Con}(X)$. Then

1. $R \circ S$ contains R and S (for $\Delta \subset R$ implies $S = \Delta \circ S \subset R \circ S$, analogously $\Delta \subset S$ implies $R \subset R \circ S$);
2. $R \circ S = R \vee S$ because, given a congruence T containing R and S we have $R \circ S \subset T \circ T \subset T$;
3. $\mathrm{Con}(X)$ is a modular lattice because, given R, S, T in $\mathrm{Con}(X)$ with $R \subset S$, we use the above lemma:

$$(R \circ T) \cap S \subset R \circ (T \cap (R^{-1} \circ S))$$

and, since $\Delta \subset R^{-1} \subset S$ implies $R^{-1} \circ S = S$, we get

$$(R \circ T) \cap S \subset R \circ (T \cap S).$$

By the Lawvere condition it suffices to verify that $R \circ S$ is an equivalence.

4. Reflexivity. Since $\Delta \subset R$ and $\Delta \subset S$, we have

$$\Delta = \Delta \circ \Delta \subset R \circ S.$$

5. Symmetry. Since $R^{-1} \subset R$ and $S^{-1} \subset S$, we get

$$(R \circ S)^{-1} = S^{-1} \circ R^{-1} \subset S \circ R = R \circ S.$$

6. Transitivity. Since $R \circ R \subset R$ and $S \circ S \subset S$, we get

$$R \circ S \circ R \circ S = R \circ (R \circ S) \circ S \subset R \circ S.$$

This concludes the proof.

b) Corollary. *Congruence lattices in an abelian category are modular.*

A non-abelian application: *congruence lattices in the category of groups are modular.*

3. Congruences on algebraic structures

3.1. a) We shall investigate algebraic structures of a given type $S = (S_F, S_P, \mathrm{ar})$: S_F and S_P are disjoint classes of functions symbols, respectively predicate symbols, and ar maps $S_F \cup S_P$ to the class of all cardinals. An S-structure is a set A together with operations $f_A : A^{\text{ar}(f)} \to A$ $(f \in S_F)$ and relations $R_A \subset A^{\text{ar}(R)}$ $(R \in S_P)$. S-structures form an obvious category $\mathfrak{A}(S)$ the morphisms of which (S-homomorphisms) are mappings which respect the operations and relations (according to their symbols).

b) Let us mention, as a matter of interest, that these categories $\mathfrak{A}(S)$ are very general. Kučera and Pultr [5] prove that every reasonable concrete category (e.g. such that morphisms factorize as onto, followed by one-to-one, and that fibres are small sets) has a full embedding, respecting underlying sets, into some $S(F)$. Here F : sets $\to$ sets is a functor; objects of $S(F)$ are pairs (A, α) with $\alpha \subset FA$ and morphisms $h : (A, \alpha) \to (B, \beta)$ are mappings with $F f(\alpha) \subset \beta$.

Following Yoneda lemma, $S(F)$ can be clearly viewed as a full subcategory of some $\mathfrak{A}(S)$ where $S = (\emptyset, S_P, \text{ar})$, provided that, for each cardinal $\mathfrak{m}$, S_P contains at least card $(F\mathfrak{m})$ symbols R with ar $(R) = \mathfrak{m}$.

Thus, we see that every reasonable concrete category has a concrete embedding into some $\mathfrak{A}(S)$. Since these embeddings are not expected to preserve anything, this result does not inform us about congruence lattices and so it is not quite relevant for our purposes.

c) Congruences in $\mathfrak{A}(S)$ are independent of the predicate symbols, i.e. they are just the algebraic congruences in $\mathfrak{A}(S^*)$ where $S = (S_F, S_P, \text{ar})$ implies $S^* = (S_F, \emptyset, \text{ar}/S_F)$. The following is well-known:

Proposition. *The forgetful functor $U : \mathfrak{A}(S) \to$ Set is continuous. It is finitary iff S_F is finitary (i.r.* ar $(f) < \aleph_0$ *for each $f \in S_F$). If S_F is finitary, U preserves exact coequalizers.*

Corollary. *$\mathfrak{A}(S)$-congruences form closure systems in equivalence lattices. If S_F is finitary then $\mathfrak{A}(S)$-congruences form complete sublattices of equivalence lattices.*

This corollary can be reversed for varieties of algebras, i.e. full subcategories of $(S_F, \emptyset, \text{ar})$-structures, closed to products, subobjects and quotient objects:

Proposition [1]. *If **C** is a variety such that congruences form complete sublattices of equivalence lattices then there exists a variety **C**′ of finitary algebras, isomorphic to **C** as a concrete category.*

d) For subcategories ***C*** of $\mathfrak{A}(S)$, the predicate part of S makes a substantial role concerning ***C***-congruences. E.g. let $S_F = \emptyset$, $S_P = \{\leqq\}$ and ar $(\leqq) = 2$. Then $\mathfrak{A}(S)$ is the category of graphs and compatible mappings. Let Pos denote its subcategory of posets. Pos-congruences are studied in a number of papers of the second author see [6–9] and references there.

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