Czechoslovak Mathematical Journal

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Book reviews. Petr Vopěnka, Petr Hájek: The theory of semisets

Czechoslovak Mathematical Journal, Vol. 23 (1973), No. 2, 347–348

Persistent URL: http://dml.cz/dmlcz/101172

WHITEMAN J. R., United Kingdom: Finite element methods for elliptic boundary value problems containing singularities.

ZLÁMAL M., Czechoslovakia: Curved elements in the finite element method.

ŽENÍŠEK A., Czechoslovakia: Hermite interpolation on simplexes in the finite element method.

The lectures included the results which had been obtained in the respective branches during the last years as well as some unpublished results suggesting the directions of further research in the corresponding fields. The complete text of these lectures will be included in the Proceedings of the conference which will appear at the beginning of the year 1973 as the first volume of the new series of publications of the University of J. E. Purkyně in Brno "Monographia".

Owing to the big number of communications, each section was further divided into two parallel subsections. In the section of ordinary differential equations, 43 communications were delivered; in the section of partial differential equations, 42 communications; in the section of numerical methods and applications, 34 communications.

The scientific contribution of the conference as well as its social program and the general organization was highly appreciated in the letters sent by Professor W. N. EVERITT to the President of the Czechoslovak Academy of Sciences, Academician J. KOŽEŠNÍK and to the President of the University of J. E. Purkyně Professor J. VAŠKŮ and by Professor E. MAGENES to the President of the Czechoslovak Academy of Sciences, Academician J. KOŽEŠNÍK.

The organization of the conference was estimated as perfect by all the participants. They appreciated particulary the punctuality in keeping the daily schedule and the instant and continuous information concerning all conference activities.

The social program for the participants was carefully prepared. The first night of the conference, almost all participants attended the opening party which took place in the Castle Wine-cellar. On Thuesday night there was a gala concert of the old Czech music at the castle Slavkov (Austerlitz) which was extraordinarily successful. On Wednesday afternoon most participants joined one of the four bus tours (Moravian Karst, Buchlov, Lednice, Telč). The farewell party took place at Queen Eliška Wine-cellar. The foreign participants were accompanied by 26 persons and children who were offered a special program.

The conference passed in a remarkably friendly atmosphere and enabled all the foreign guests to see the life and work of the people of our republic, its natural beauties and to enjoy the friendly hospitality of its people. The high scientific, social and organizational level of the conference form a good ground to believe that the conference EQUADIFF 4 which is to be held in Prague in 1977 will continue this tradition.

Miloš Ráb, Brno

BOOK REVIEWS

Petr Vopěnka, Petr Hájek: THE THEORY OF SEMISETS. Academia, Prague 1972, 400 pp., 100,— Kčs.

The reviewed book deals with the theory of semisets which is a generalization of the theory of sets. The authors set themselves two aims: on the one hand, to sum up some classical results of the set theory and, on the other hand, to establish the theory of semisets. The classical results presented here include in particular the theory of ordinal and cardinal numbers, the theory of complete Boolean algebras, Gödel's constructive model (which among other makes it possible

to prove the consistency of the axiom of choice and the continuum hypothesis), Fraenkel-Mostowski's models (which among other make it possible to prove the consistency of the negation of the axiom of choice) and the ultraproduct model. The book can very well serve as a textbook of these parts of the set theory, which may be considered the most important contributions to the set theory before the model of Cohen.

The theory of semisets enriches the set theory with new objects — semisets. To make this concept clear, let us go back to the history of the theory of sets. After Cantor had introduced the theory of sets, it turned out that the description of sets is not free from contradiction — the so-called paradoxes of the set theory were discovered. One possible way out was to introduce the concept of classes which are understood to be "big" sets (e.g. the collection of all sets is not a set but only a class). Sets are defined to be such classes which may be elements of some other class. This approach is axiomatized by the Gödel-Bernays set theory. This theory makes it possible to prove that a part of a set is again a set. On the other hand, the theory of semisets, while having the axiomatics very similar to that of the Gödel-Bernays set theory, admits the existence of classes which are parts of sets though they are not sets themselves. (And namely these objects are called semisets.) The axiomatics of the theory of semisets is chosen in such a way that the following two important assertions hold:

(1) Theory of semisets with the additional axiom "Every semiset is a set" is equivalent to the Gödel-Bernays set theory.

(2) All theorems concerning only sets can be proved in the theory of semisets if and only if they can be proved in the set theory. The above statement remains true if we add to the theory of semisets some additional axiom e.g. so called axioms of support. In particular, the theory of semisets is inconsistent if and only if the theory of sets is.

In 1963, P. Cohen solved the problem which worried mathematicians since Cantor's times — he proved the consistency of the negation of the continuum hypothesis. The theory of semisets was created originally as a mathematical theory simplifying and axiomatizing Cohen's method. However, in the course of time it has turned out that it is a theory which may prove itself useful in many other cases. As examples, let us mention at least that it is possible to build in it the non-standard analysis, to use it to establish new topological theorems etc. The theory of semisets is developed in the book gradually, new axioms being added and their consistency and independence being studied. Among the axioms, a significant part is played by the axioms of support, for it is shown that a model of the theory of semisets with the axiom of Boolean support may be extended to a model of the theory of sets. Hence a new method of construction of models of the set theory is established. Indeed, to construct a model of the set theory with required properties it is sufficient to construct a model of the theory of semisets with certain properties and to extend it into a model of the theory of sets. It may be essentially easier to construct the model of the theory of semisets than to construct directly a model of the theory of sets.

The reader is required no preliminary knowledge. Even all necessary concepts and theorems from mathematical logics are summarized in the book. The corresponding part of the book may serve an introduction into logics and the theory of syntactic models. The subject of the book is presented in a condensed form. It is the reviewer's opinion that everybody dealing with the theory of sets should get acquainted with the book.

In the end, I should like to quote from the introduction of the reviewed book: "In presenting the theory of semisets the authors hope to make some contribution to the task of breaking through the bars of the prison in which mathematicians find themselves. This prison is set theory and the authors believe that mathematicians will escape from it just as they escaped from the prison of three-dimensional space."

Antonín Sochor, Praha