Alois Švec

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SUBMANIFOLDS OF KLEIN SPACES

ALOIS ŠVEC, Praha

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It is my feeling that the theory of partial differential equations developed in the last years especially at Harvard is of great importance for the differential geometry. Because of that Chap. 1 of this paper is a report on some results due to H. GOLDSMITH and others; see [1]. In the second part I apply this theory, and I show that to solve the equivalence problem for submanifolds in Klein spaces only a finite process is needed. I restrict myself to the linear case, the non-linear case might be treated in a similar way.

1. LINEAR HOMOGENEOUS SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

1.1. Involution. Let V and W be vector spaces of finite dimension; $\dim V = n$. Denote by $S^m V^*$ the symmetric tensor product of m copies of V^*, the symmetric product of $v_1^*, v_2^* \in V^*$ denote by $v_1^* \circ v_2^*$. For $v \in V$ introduce the homomorphisms

$$\delta_v : W \otimes S^{m+1} V^* \to W \otimes S^m V^* \tag{1.1}$$

satisfying

$$\delta_v(w \otimes f) = w \otimes f_v \quad \text{for} \quad w \in W, \quad f \in S^{m+1} V^* ; \tag{1.2}$$

f_v is the derivative of f with respect to v. If $v_1, \ldots, v_n$ is a basis of V and $v^1, \ldots, v^n$ the dual basis of V^*, define the homomorphism

$$\delta \equiv \delta_{m+1,j} : W \otimes S^{m+1} V^* \otimes \Lambda^j V^* \to W \otimes S^m V^* \otimes \Lambda^{j+1} V^* \tag{1.3}$$

by means of the relation

$$\delta(\xi \otimes v^{i_1} \wedge \ldots \wedge v^{i_j}) = \sum_{i=1}^{n} \delta_{v_i} \xi \otimes v^i \wedge v^{i_1} \wedge \ldots \wedge v^{i_j} \tag{1.4}$$

for $\xi \in W \otimes S^{m+1} V^*$. This definition does not depend on the choice of the basis.

Theorem 1.1. *We have*

$$\delta^2 = 0\,, \quad \text{i.e.,} \quad \delta_{m,j+1} \delta_{m+1,j} = 0 \tag{1.5}$$

for each m and j. The sequence

$$(1.6)\qquad 0 \to W \otimes S^m V^* \xrightarrow{\delta} W \otimes S^{m-1} V^* \otimes V^* \xrightarrow{\delta} W \otimes S^{m-2} V^* \otimes \Lambda^2 V^* \xrightarrow{\delta} \dots$$
$$\dots \xrightarrow{\delta} W \otimes S^{m-n} V^* \otimes \Lambda^m V^* \to 0$$

is exact; we set $S^l V^* = 0$ *for* $l < 0$.

For each $m \geqq k$ be given a subspace $g^m \subset W \otimes S^m V^*$. The sequence $\{g^m\}$ is *consistent* if

$$(1.7)\qquad \delta_v(g^m) \subset g^{m-1}$$

for each $m > k$ and $v \in V$. If the sequence $\{g^m\}$ is consistent we have

$$(1.8)\qquad \delta(g^m \otimes \Lambda^j V^*) \subset g^{m-1} \otimes \Lambda^{j+1} V^*,$$

and we may consider the sequences

$$(1.9)\qquad 0 \to g^m \xrightarrow{\delta} g^{m-1} \otimes V^* \xrightarrow{\delta} g^{m-2} \otimes \Lambda^2 V^* \xrightarrow{\delta} \dots$$
$$\dots \xrightarrow{\delta} g^k \otimes \Lambda^{m-k} V^* \xrightarrow{\delta} W \otimes S^{k-1} V^* \otimes \Lambda^{m-k+1} V^*$$

for $m \geqq k$. Denote by $H^{m-j,j} \equiv H^{m-j,j}(g^k)$ the cohomology of the sequence (1.9) on the $(j+1)$-th place:

$$(1.10)\qquad H^{m-j,j} = \operatorname{Ker} \delta_{m-j,j} / \operatorname{Im} \delta_{m-j+1,j-1}\,.$$

This is the so-called *Spencer cohomology*. A consistent sequence $\{g^m\}$ is called *involutive* if all the sequences (1.9) are exact. $\{g^m\}$ is called *r-acyclic* if

$$(1.11)\qquad H^{m,j} = 0 \quad \text{for} \quad m \geqq k\,, \quad 0 \leqq j \leqq r\,.$$

Let $V_1 \subset V$ be a subspace. Define

$$(1.12)\qquad (g^m)_{V_1} = \{\xi \in g^m \mid \delta_v \xi = 0 \quad \text{for all} \quad v \in V_1\}\,.$$

The sequence $\{g^m\}$ being consistent, we have

$$(1.13)\qquad \delta_v((g^m)_{V_1}) \subset (g^{m-1})_{V_1} \quad \text{for all} \quad v \in V_1\,.$$

Let $v_1, \dots, v_n$ be a basis of V; denote by $\{v_r, v_{r+1}, \dots, v_n\}$ the space spanned by $v_r, \dots, v_n$. The basis $v_1, \dots, v_n$ is *regular* with respect to $\{g^m\}$ if the mappings

$$(1.14)\qquad \begin{aligned} &\delta_{v_n} \;\;\;\;: g^{m+1} \to g^m\,, \\ &\delta_{v_{n-1}} : (g^{m+1})_{\{v_n\}} \to (g^m)_{\{v_n\}}\,, \\ &\delta_{v_{n-2}} : (g^{m+1})_{\{v_{n-1},v_n\}} \to (g^n)_{\{v_{n-1},v_n\}}\,, \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\delta_{v_1} \;\;\;\;: (g^{m+1})_{\{v_2,\dots,v_n\}} \to (g^m)_{\{v_2,\dots,v_n\}} \end{aligned}$$

are onto for all m's.

Theorem 1.2. *The consistent sequence* $\{g^m\}$ *is involutive if and only if there is a regular basis with respect to it.*

Be given a space $g^k \subset W \otimes S^k V^*$. Its first *prolongation* is defined as

$$pg^k = (g^k \otimes V^*) \cap (W \otimes S^{k+1} V^*)\,; \tag{1.15}$$

put $p^l g^k = p(p^{l-1} g^k)$ for $l > 1$. The sequence $\{g^m\}$ with $g^m = p^{m-k} g^k$ for $m \geqq k$ is consistent. The space g^k is called *involutive* (*r-acyclic*) if the corresponding sequence $\{g^m\} = \{p^{m-k} g^k\}$ is involutive (r-acyclic).

Theorem 1.3. *Be given a space* $g^k \subset W \otimes S^k V^*$. *Let there exist a basis* $v_1, \ldots, v_n$ *of the space* V *such that the maps* (1.14) *are onto for* $m = k$ *and* $g^{k+1} = pg^k$. *Then* g^k *is involutive and* $v_1, \ldots, v_n$ *is regular with respect to* $\{g^m\} = \{p^{m-k} g^k\}$.

Theorem 1.4. *Let, for each* $m \geqq 1$, *be given a space* $g^m \subset W \otimes S^m V^*$. *If* $pg^m \supset g^{m+1}$ *for each* $m \geqq 1$, *there is a number* m_0 *such that*

$$pg^m = g^{m+1} \quad \text{for} \quad m \geqq m_0 \tag{1.16}$$

and the space g^{m_0} *is involutive.*

1.2. Differential equations. Let X be a differentiable manifold of class C^∞, $\dim X = n$. Denote by $T = T(X)$ its tangent bundle, T^* be its cotangent bundle. E being a vector bundle over X, E_x is the fibre over $x \in X$ and $\mathscr{E}$ is the sheaf of germs of C^∞ sections of E.

Be given a C^∞ vector bundle E over X with the projection $\pi : E \to X$; let $x_0 \in X$ be a fixed point and $s_1, s_2 \in \mathscr{E}$ sections defined in a neighborhood of x_0. Let $U \subset X$ be a coordinate neighborhood of x_0 such that s_1, s_2 are defined in it and we may write $\pi^{-1}(U) = U \times V^m$, V^m being an m-dimensional vector space. Choosing a basis $v_1, \ldots, v_m$ in V^m, the section s_τ; $\tau = 1, 2$; is given (in U) by $y^\alpha = f_\tau^\alpha(x^1, \ldots, x^n)$; $\alpha = 1, \ldots, m$; $x^1, \ldots, x^n$ being the local coordinates in U. We say that s_1 and s_2 belong to the same k-jet at x_0, and we write $j_{x_0}^k(s_1) = j_{x_0}^k(s_2)$, if all the partial derivatives at x_0 of the functions f_1^α up to the order k are equal to the corresponding derivatives of f_2^α.

Denote by $J^k(E)$ the set of all k-jets of sections of E. This set has a natural structure of a vector bundle over X. The bundle $J^0(E)$ is obviously equal to E. Let $\pi : J^k(E) \to J^l(E)$, $k > l$, be the natural projection. Denote by $\mathscr{J}^k(E)$ the sheaf of the germs of the sections of $J^k(E)$. The mapping $j^k : \mathscr{E} \to \mathscr{J}^k(E)$ be defined as follows: $s \in \mathscr{E}$ being defined on $U \subset X$, set $[j^k(s)](x) = j_x^k(s)$ for $x \in U$.

There is just one bundle morphism $\varepsilon : S^k T^* \otimes E \to J^k(E)$ characterized by the following property. Let $x \in X$; $t_1, \ldots, t_k \in T_x^*$; $e \in E_x$. Choose functions $f_1, \ldots, f_k$ on a neighborhood of x such that $(df_\sigma)_x = t_\sigma$ and $f_\sigma(x) = 0$ for $\sigma = 1, \ldots, k$; further, choose $s \in \mathscr{E}$ such that $s(x) = e$. Now, we have

$$\varepsilon(t_1 \circ \ldots \circ t_k \otimes e) = j_x^k(f_1 \ldots f_k s)\,.$$

Theorem 1.5. *The sequence*

$$0 \to S^kT^* \otimes E \xrightarrow{\varepsilon} J^k(E) \xrightarrow{\pi} J^{k-1}(E) \to 0 \tag{1.17}$$

is exact.

Let E, F, G be vector bundles over X; $f : E \to F$ be a bundle morphism and $\mathfrak{f} : \mathscr{E} \to \mathscr{F}$ the corresponding sheaf morphism.

Theorem 1.6. *To a given bundle morphism $f : E \to F$ there is a unique bundle morphism $j^l(f) : J^l(E) \to J^l(F)$ such that the diagram*

$$\begin{array}{ccc} \mathscr{E} & \xrightarrow{\mathfrak{f}} & \mathscr{F} \\ {\scriptstyle j^l}\downarrow & & \downarrow{\scriptstyle j^l} \\ \mathscr{J}^l(E) & \xrightarrow[\mathfrak{j}^l(f)]{} & \mathscr{J}^l(F) \end{array} \tag{1.18}$$

is commutative.

Theorem 1.7. *There is a unique bundle monomorphism*

$$p^l(\mathrm{id}^k) : J^{k+l}(E) \to J^l(J^k(E))$$

such that the diagram

$$\begin{array}{ccc} \mathscr{E} & \xrightarrow{j^k} & \mathscr{J}^k(E) \\ {\scriptstyle j^{k+l}}\downarrow & & \downarrow{\scriptstyle j^l} \\ \mathscr{J}^{k+l}(E) & \xrightarrow[\mathfrak{p}^l(\mathrm{id}^k)]{} & \mathscr{J}^l(J^k(E)) \end{array} \tag{1.19}$$

is commutative.

Considering (1.19) and (1.18) with $G = J^k(E)$, we get the following corrolary: Given a bundle morphism $f : J^k(E) \to F$ there is a unique bundle morphism $p^l(f) :$ $: J^{k+l}(E) \to J^l(F)$ such that the diagram

$$\begin{array}{ccc} \mathscr{E} & \xrightarrow{\mathfrak{f}j^k} & \mathscr{F} \\ {\scriptstyle j^{k+l}}\downarrow & & \downarrow{\scriptstyle j^l} \\ J^{k+l}(E) & \xrightarrow[\mathfrak{p}^l(f)]{} & \mathscr{J}^l(F) \end{array} \tag{1.20}$$

is commutative; here, $p^l(f) = j^l(f)\, p^l(\mathrm{id}^k)$.

Considering $F = J^k(E)$ and $f = \mathrm{id}$, we get $p^l(\mathrm{id}) = p^l(\mathrm{id}^k)$, thus explaining why we have denoted by $p^l(\mathrm{id}^k)$ the morphism of Theorem 1.7. The morphism $p^l(f)$ is the l-th *prolongation* of $f : J^k(E) \to F$.

The *differential equation* (of order k) is a C^∞ sub-bundle $R^k \subset J^k(E)$. The l-th ($l \geqq 0$) *prolongation* of R^k is the subset

$$p^l R^k = R^{k+l} = J^l(R^k) \cap J^{k+l}(E)\,. \tag{1.21}$$

Obviously, $J^l(R^k) \subset J^l(J^k(E))$ and $J^{k+l}(E)$ is identified with its image in the monomorphism $p^l(\mathrm{id}^k)$ of Theorem 1.7. Define $R^{k-l} = J^{k-l}(E)$ for $1 \leqq l \leqq k$. R^{k+l} need not to be a sub-bundle of $J^{k+l}(E)$ as $\dim R_x^{k+l}$ is not constant in general.

Theorem 1.8. *To each differential equation R^k on E there is a bundle F and a bundle morphism $f : J^k(E) \to F$ such that $R^k = \mathrm{Ker} f$. Then $R^{k+l} = \mathrm{Ker}\, p^l(f)$.*

1.3. The formal complete integrability. The equation $R^k \subset J^k(E)$ is *formally completely integrable* if, for each $l \geqq 0$, R^{k+l} is a vector bundle and

$$R^{k+l+1} \to R^{k+l} \to 0 \tag{1.22}$$

is exact.

One of the main results is to obtain criteria for the formal complete integrability of a given differential equation.

Let $R^k \subset J^k(E)$ be an equation. For each $l \geqq 0$, define $g^{k+l} \subset S^{k+l}T^* \otimes E$ by the exact sequence

$$0 \to g^{k+l} \xrightarrow{\varepsilon} R^{k+l} \xrightarrow{\pi} R^{k+l+1}\,; \tag{1.23}$$

further, set $g^{k-l} = S^{k-l}T^* \otimes E$ for $1 \leqq l \leqq k$. g^k is the *symbol* of R^k. In general, g^{k+l} is not a bundle.

Theorem 1.9. *Let R^k be an equation of order k and g^{k+l} the corresponding system of spaces. Then $g_x^{k+l} = p^l g_x^k$ for each $l \geqq 0$ and $x \in X$, and the sequence*

$$0 \to g^{k+l+1} \xrightarrow{\delta} T^* \otimes g^{k+l} \xrightarrow{\delta} \Lambda^2 T^* \otimes g^{k+l-1} \tag{1.24}$$

is exact for $l \geqq 0$; i.e., $\{g^{k+l}\}$ is 1-acyclic.

Theorem 1.10. *The manifold X being connected, there is an integer $k_0 > k$* (depending only on k, $n = \dim X$ and $\dim E$) *such that $H^{k_0+m,j} = 0$* for each $m \geqq 0$, $j \geqq 0$.

Theorem 1.11. *If R^{k+l} is a vector bundle* (for an $l \geqq 0$) and $\pi : R^{k+l} \to R^{k+l-1}$ *is an epimorphism, there is a bundle morphism* (the so-called *curvature* of R^{k+l})

$$\varkappa = \varkappa(R^{k+l}) : R^{k+l} \to H^{k+l-1,2} \tag{1.25}$$

such that the sequence

(1.26) $$R^{k+l+1} \xrightarrow{\pi} R^{k+l} \xrightarrow{\varkappa} H^{k+l-1,2}$$

is exact.

Theorem 1.12. *If g^k and g^{k+1} are vector bundles and g^k is 2-acyclic, g^{k+l} is a vector bundle for any $l \geqq 0$.*

Combining the preceding two theorems, we get

Theorem 1.13. *Let $R^k \subset J^k(E)$ be a differential equation and suppose: (1) R^{k+1} is a vector bundle, (2) $\pi : R^{k+1} \to R^k$ is an epimorphism, (3) g^k is a 2-acyclic system, (4) g^k and/or g^{k+2} is a vector bundle. Then R^k is formally completely integrable.*

Finally, combining this theorem with Theorem 1.10, we get

Theorem 1.14. *If X is connected and $R^k \subset J^k(E)$ a differential equation, there is a number $k_0 > k$* (depending only on n, k and dim E) *such that if R^{k+l+1} is a vector bundle and $\pi : R^{k+l+1} \to R^{k+l}$ is an epimorphism for $0 \leqq l \leqq k_0 - k$, then R^k is formally completely integrable.*

A (local) section $s \in \mathscr{E}$ is called a *solution* of R^k if $j^k s \in \mathscr{J}^k(E)$ is contained in R^k.

Theorem 1.15. *Let X be an analytic manifold, E an analytic vector bundle and $R^k \subset J^k(E)$ an analytic sub-bundle. Let R^k be a formally completely integrable equation. Be given a point $u \in R^{k+l}$, let $\pi(u) = x \in X$. Then there is a neighborhood $U \subset X$ of x and an analytic section $s : U \to E$ which is a solution of R^k and $j_x^{k+l}(s) = u$.*

2. EQUIVALENCE OF SUBMANIFOLDS

2.1. Initial conditions of differential equations. Let X be a C^∞ differentiable manifold and E a vector bundle over it. Suppose that all the manifolds and maps considered are of the class C^∞. Let X_1 be a submanifold of X. Let E_1 be the restriction of E to X_1, i.e., $E_1 = E \mid X_1$. Define the bundle morphisms

(2.1) $$\eta^k : J^k(E) \mid X_1 \to J^k(E_1)$$

as follows. Let $u \in J^k(E) \mid X_1$, i.e., $x = \pi(u) \in X_1$. Then there is a neighborhood $U \subset X$ of x and a section $s : U \to E$ such that $u = j_x^k(s)$. We set $\eta^k(u) = j_x^k(s \mid X_1 \cap U)$; $\eta^k(u)$ obviously does not depend on s.

Be given a formally completely integrable equation $R^k \subset J^k(E)$. For each $m \geqq k$ define a system of spaces $S_{(m)}^m \subset J^m(E_1)$ as

(2.2) $$S_{(m)}^m = \eta^m(R^m \mid X_1) ;$$

of course, R^m is the $(m - k)$-th prolongation of R^k. Although $S^m_{(m)}$ is not always a differential equation, we may define its prolongation $S^{m+p}_{(m)}$ in a natural way. The diagram

$$(2.3) \qquad \begin{array}{ccc} J^{m+1}(E) \,|\, X_1 & \xrightarrow{\eta^{m+1}} & J^{m+1}(E_1) \\ \Big\downarrow \pi & & \Big\downarrow \pi \\ J^m(E) \,|\, X_1 & \xrightarrow[\eta^m]{} & J^m(E_1) \end{array}$$

being commutative, we get

$$(2.4) \qquad \pi(S^{m+1}_{(m+1)}) = S^m_{(m)}$$

for each $m \geqq k$ because of the formal complete integrability of R^k. Indeed, let $u \in S^m_{(m)}$. Then there exists a $v \in R^m$ such that $\eta^m(v) = u$. Because of the complete integrability there is a $w \in R^{m+1}$ such that $\pi(w) = v$, from the commutativity of the diagram (2.3) it follows $\pi(\eta^{m+1}(w)) = u$, and the mapping $\pi : S^{m+1}_{(m+1)} \to S^m_{(m)}$ is onto. Further, to each $t \in S^{m+1}_{(m+1)}$ there is a $t_1 \in R^{m+1}$ such that $\eta^{m+1}(t_1) = t$, hence $\pi(t) = \eta^m(\pi(t_1)) \in$ $\in S^m_{(m)}$ because of $\pi(t_1) \in R^m$. Thus we get (2.4). It is easy to see that

$$(2.5) \qquad S^{m+1}_{(m+1)} \subset S^{m+1}_{(m)} = pS^m_{(m)}$$

for each $m \geqq k$. The system of subspaces $g^{m+p+1}_{(m)}$ be defined as in (1.23) by the exact sequence

$$(2.6) \qquad 0 \to g^{m+p+1}_{(m)} \xrightarrow{\varepsilon} S^{m+p+1}_{(m)} \xrightarrow{\pi} S^{m+p}_{(m)} .$$

Let us choose a fixed point $y \in X$. For each $m > k$ consider the space

$$(2.7) \qquad h^m = g^m_{(m)}(y) ;$$

from (2.5) we get $h^{m+1} \subset ph^m$ for each $m \geqq k$. Applying Theorem 1.4, we get the existence of an integer $k_0 \geqq k$ such that

$$(2.8) \qquad h^{m+1} = ph^m \quad \text{for each} \quad m \geqq k_0 .$$

Thus we obtain

Theorem 2.1. *Let $R^k \subset J^k(E)$ be a formally completely integrable equation. Let $X_1 \subset X$ be a submanifold, $y \in X_1$ a fixed point and $S^m_{(m)}$ be defined by (2.2). Then there is a $k_0 \geqq k$ such that*

$$(2.9) \qquad S^{m+1}_{(m+1)}(y) = S^{m+1}_{(m)}(y) \quad \text{for each} \quad m \geqq k_0 .$$

The *formal solution* of the equation R^k at the point $y \in X$ is a sequence $\{s^q\}$ with

$s^q \in R^q(y)$ and $\pi(s^{q+1}) = s^q$ for $q = 1, 2, \ldots$ The *formal initial condition* at $y \in X_1 \subset \subset X$ with respect to the submanifold X_1 is a sequence $\{r^q\}$ with $r^q \in J^q(E_1)_y$ and $\pi(r^{q+1}) = r^q$ for $q = 1, 2, \ldots$ The formal solution $\{s^q\}$ *goes* through the formal initial condition $\{r^q\}$ if $\eta^q(s^q) = r^q$ for each q. The formal initial condition $\{r^q\}$ is *m-admissible* if $r^{m+p} \in S^{m+p}_{(m)}(y)$ for each $p \geqq 0$.

It is easy to prove the following

Theorem 2.2. *Be given the situation described in Theorem* 2.1. *There is a* $k_0 \geqq k$ *such that through each m-admissible,* $m \geqq k_0$, *formal initial condition at* y *with respect to* X_1 *there goes a formal solution of* R^k.

2.2. Equivalence of submanifolds. Let X be a vector space. Consider the trivial vector bundle $E = X \times X$ over it with the projection $\pi = \mathrm{pr}_1 : X \times X \to X$, pr_1 being the projection on the first factor. Be given a formally completely integrable equation $R^k \subset J^k(E)$ and a submanifold $X_1 \subset X$. Let $s \in \mathscr{E}$ be a local section of E over $U \subset X$, i.e., a mapping $s(x) = (x, \hat{s}(x))$ for $x \in U$ determining thus the mapping $\hat{s} : U \to X$. Denote by $\mathscr{S}$ the set of all local mappings $\hat{s} : U \to X$ such that the corresponding section $s(x) = (x, \hat{s}(x))$, $x \in U$, is a solution of R^k. Further, be given a mapping $\tilde{f} : X_1 \to X$. $\tilde{f}$ is called the *deformation* of order m of X_1 with respect to $\mathscr{S}$ if, for each $x_1 \in X_1$, there is a $\hat{s}_{x_1} \in \mathscr{S}$ (defined in the neighborhood of x_1) such that $j^m_{x_1}(\tilde{f}) = j^m_{x_1}(s_{x_1} \mid X_1)$. The mapping $\tilde{f} : X_1 \to X$ determines the section $f : X_1 \to E_1 \equiv \equiv E \mid X_1$ given by $f(x_1) = (x_1, \tilde{f}(x_1))$. It is easy to see that f is the deformation of order m if and only if $\tilde{f}$ is a solution of the (generalized) equation $S^m_{(m)}$ constructed above.

The mapping $\tilde{f}$ is called the *formal equivalence* at $y \in X_1$ with respect to $\mathscr{S}$ if there is a sequence of local maps $\{\hat{s}^q : U^q \to X\}$ of the neighbourhoods U^q of y such that, for $q = 1, 2, \ldots$, $j^q_y(\hat{s}^{q+1}) = j^q_y(\hat{s}^q)$, $\hat{s}^q \in \mathscr{S}$, $j^q_y(\tilde{f}) = j^q_y(\hat{s}^q \mid X_1)$. We get from Theorem 2.2

Theorem 2.3. *Be given a vector space* X *and a set* $\mathscr{S}$ *of local maps on it which are the solutions of an equation* R^k (in the way described above). *Be given a submanifold* $X_1 \subset X$ *and* $y \in X_1$. *Then there is an integer* $k_0 \geqq k$ *with the following property*: $\tilde{f} : X_1 \to X$ *being the deformation of order* $m \geqq k_0$, $\tilde{f}$ *is the formal equivalence at* y.

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Author's address: Sokolovská 83, Praha 8 - Karlín, ČSSR (Matematicko-fyzikální fakulta UK).