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NECESSARY AND SUFFICIENT CONDITIONS FOR SOME CONVERGENCE METHODS

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In this note, necessary and sufficient conditions for some convergence methods [1], [2] are given (§ 2). Furthermore in § 3 there is studied the rate of convergence of these methods for the solution of the equation

$$Ax = f\,, \tag{1}$$

where $A : X \to X$ is a linear bounded operator on a Hilbert space X and $f \in X$. The methods from [1], [2] are described concisely in § 1.

1. The basic idea of the methods [1], [2] is the following: We seek the approximate solutions of (1) in the from

$$x_{n+1} = Pf + \beta_n(I - PA)\, x_n\,, \quad (n = 0, 1, 2, \ldots) \tag{2}$$

where P is a linear bounded operator having bounded inverse P^{-1} in the (real or complex) Hilbert space X. Furthermore, let P commute with A. The real numbers β_n $(n = 0, 1, 2, \ldots)$ are to be determined so as to minimize either $\|f - \beta_n A x_n\|^2$ or $\|f - Ax_{n+1}\|^2$, $(n = 0, 1, 2, \ldots)$. Hence either

$$\beta_n = \operatorname{Re}(f, Ax_n)\, \|Ax_n\|^{-2} \tag{3}$$

or

$$\beta_n = \operatorname{Re}(Lf, LAx_n)\, \|LAx_n\|^{-2}\,, \tag{4}$$

where $L = I - PA$. Thus we have two sequences $\{x_n\}$, $\{\tilde{x}_n\}$, where

$$x_{n+1} = Pf + \frac{\operatorname{Re}(f, Ax_n)}{\|Ax_n\|^2}(I - PA)\, x_n\,, \tag{5}$$

$$\tilde{x}_{n+1} = Pf + \frac{\operatorname{Re}(Lf, LA\tilde{x}_n)}{\|LA\tilde{x}_n\|^2} L\tilde{x}_n\,. \tag{6}$$

The following theorem is valid:

Theorem 1 ([1], [2]). *Let A, P be linear bounded commutative operators which map X into X, and are such that P^{-1} exists and is bounded, and $q = \|I - PA\| < 1$. Then equation (1) has a unique solution x^* in X. The sequences $\{x_n\}$, $\{\tilde{x}_n\}$ defined by (5), (6) converge in the norm topology of X to the solution x^* of (1), and their errors are bounded by*

$$\|x^* - x_n\| \leqq kq\|f - Ax_{n-1}\|\,, \quad \|x^* - \tilde{x}_n\| \leqq kq\|f - A\tilde{x}_{n-1}\|\,,$$

where $k = \|A^{-1}\| \leqq \|P\|\,(1 - q)^{-1}$.

Now set $A = I - \lambda K$, where $K : X \to X$ is a linear bounded operator from X into X, λ is a complex parameter.

Theorem 2 [3]. *Let one of the following conditions be fulfilled:*

1) $P = I$, $\|\lambda K\| < 1$.

2) $P = \vartheta I$, *A is self-adjoint in X, $mI \leqq A \leqq MI$, $0 < m \leqq M$, $\vartheta = 2(M + m)^{-1}$, where $m = \inf\limits_{\|x\|=1} (Ax, x)$, $M = \sup\limits_{\|x\|=1} (Ax, x)$.*

3) $P = \vartheta I$, $\operatorname{Re}(Ax, x) \geqq m\|x\|^2$ *for every $x \in X$, $m > 0$ and $0 < \vartheta < 2m\|A\|^{-2}$.*

4) $P = \vartheta(I - \bar{\lambda}K^*)$, *where $\bar{\lambda}$ is the complex conjugate to λ, K^* is an adjoint operator to K, K is normal, $\|Ax\| \geqq k\|x\|$ holds for every $x \in X$, $(k > 0)$ and $0 < \vartheta < k\sqrt{2}\,(1 + \|\lambda K\|)^{-1}$.*

Then the equation (1) has a unique solution x^* in X and $\|x^* - x_n\| \to 0$, $\|x^* - \tilde{x}_n\| \to 0$ whenever $n \to \infty$ in the norm topology of X, at least with the speed of a geometric sequence.

Remark 1. The real numbers β_n $(n = 0, 1, 2, \ldots)$ can also be determined from the conditions (cf. [3]) that $\|x^* - {}^1\beta_n x_n\|^2 = \text{Min}$, $\|x^* - x_{n+1}\|^2 = \text{Min}$ $(n = 0, 1, 2, \ldots)$. Then either

$${}^1\beta_n = \frac{\operatorname{Re}(f, Px_n)}{\operatorname{Re}(x_n, PAx_n)}\,,$$

or

$${}^2\beta_n = \frac{\operatorname{Re}(Lf, PLx_n)}{\operatorname{Re}(Lx_n, PLAx_n)}\,.$$

If X is a real Hilbert space, then the parameters β_n, $\tilde{\beta}_n$, ${}^1\beta_n$, ${}^2\beta_n$ $(n = 0, 1, 2, \ldots)$ have the following form:

$$\beta_n = (f, Ax_n)\,\|Ax_n\|^{-2}\,, \tag{7}$$

$$\tilde{\beta}_n = (Lf, LA\tilde{x}_n)\,\|LA\tilde{x}_n\|^{-2}\,, \tag{8}$$

$${}^1\beta_n = (f, Px_n)\,(x_n, PAx_n)^{-1}\,, \tag{9}$$

$${}^2\beta_n = (Lf, PLx_n)\,(Lx_n, PLAx_n)^{-1}\,. \tag{10}$$

If we choose $P = \vartheta I$, where ϑ is a positive number such that the norm $\|I - \vartheta A\|$ assumes its minimal value, then the methods (2), (7); (2), (9) (in (2) for β_n set ${}^1\beta_n$) are simple and convenient for the solution of linear algebraic and integral equations of the second order.

2. Let X be a real Hilbert space. Let $A : X \to X$, $P : X \to X$ be linear bounded commutative operators such that P^{-1} exists and is a bounded operator and $q = \|I - PA\| \leqq 1$. In a real space X the formulae (5), (6) have the form:

$$(5') \qquad x_{n+1} = Pf + (f, Ax_n) \|Ax_n\|^{-2} (I - PAx_n),$$

$$(6') \qquad \tilde{x}_{n+1} = Pf + (Lf, LA\tilde{x}_n) \|LA\tilde{x}_n\|^{-2} L\tilde{x}_n, \quad L = I - PA.$$

Set $h_n = f - Ax_n$, $\tilde{h}_n = f - A\tilde{x}_n$, $(n = 0, 1, 2, \ldots)$. Then

$$\|h_{n-1}\|^2 - \|h_n\|^2 = \|f - Ax_{n-1}\|^2 - \|f - Ax_n\|^2 = \|f - Ax_{n-1}\|^2 - \|f - APf - \beta_{n-1}A(I - PA)\, x_{n-1}\|^2 = \|f - Ax_{n-1}\|^2 - \|(I - PA)\,[f - \beta_{n-1}Ax_{n-1}]\|^2 .$$

Because $\|f - \beta_n Ax_n\|^2 = \text{Min}$, $(n = 0, 1, 2, \ldots)$, there is

$$\|f - \beta_n Ax_n\| \leqq \|f - Ax_n\|$$

for every n $(n = 0, 1, 2, \ldots)$. Hence

$$\|h_{n-1}\|^2 - \|h_n\|^2 \geqq \|f - Ax_{n-1}\|^2 - q^2\|f - \beta_{n-1}Ax_{n-1}\|^2 \geqq \geqq (1 - q^2)\, \|f - Ax_{n-1}\|^2 \geqq 0 .$$

Thus $\|h_{n-1}\| \geqq \|h_n\|$ for every n $(n = 1, 2, \ldots)$. Similarly

$$\|\tilde{h}_{n+1}\|^2 = \|L(f - \tilde{\beta}_n A\tilde{x}_n)\|^2 \leqq \|L(f - A\tilde{x}_n)\|^2 \leqq \leqq q^2\|f - A\tilde{x}_n\|^2 \leqq \|f - A\tilde{x}_n\|^2 = \|\tilde{h}_n\|^2 .$$

Therefore $\|\tilde{h}_{n+1}\| \leqq \|\tilde{h}_n\|$ for every n $(n = 0, 1, 2, \ldots)$. In the sequel we shall assume that $h_n \neq 0$, $\tilde{h}_n \neq 0$, $(n = 0, 1, 2, \ldots)$ i.e. that $\|h_n\| > 0$ $\|\tilde{h}_n\| > 0$ $(n = 0, 1, 2, \ldots)$. Set (cf. [4]) $x_{n+1} = x_n + \alpha_n y_n$, $\tilde{x}_{n+1} = \tilde{x}_n + \tilde{\alpha}_n \tilde{y}_n$, where $\alpha_n = \|x_{n+1} - x_n\|$, $\tilde{\alpha}_n = \|\tilde{x}_{n+1} - \tilde{x}_n\|$, $\|y_n\| = 1$, $\|\tilde{y}_n\| = 1$. Then

$$\Delta(\|h_n\|^2) = \|h_{n+1}\|^2 - \|h_n\|^2 = -2\alpha_n(Ay_n, h_n) + \alpha_n^2\|Ay_n\|^2 ,$$

and hence

$$(11) \qquad \alpha_n^2\|Ay_n\|^2 - 2\alpha_n(Ay_n, h_n) \leqq 0 .$$

Similarly

$$\Delta(\|\tilde{h}_n\|^2) = \|\tilde{h}_{n+1}\|^2 - \|\tilde{h}_n\|^2 = \tilde{\alpha}_n^2\|A\tilde{y}_n\|^2 - 2\tilde{\alpha}_n(A\tilde{y}_n, \tilde{h}_n) .$$

Because $\Delta(\|\tilde{h}_n\|^2) \leqq 0$,

$$(12) \qquad \tilde{\alpha}_n^2\|A y_n\|^2 - 2\tilde{\alpha}_n(A\tilde{y}_n, \tilde{h}_n) \leqq 0 .$$

(11), (12) are equivalent to

$$(13) \qquad \alpha_n = q_n(Ay_n, h_n)\, \|Ay_n\|^{-2} ,$$

$$(14) \qquad \tilde{\alpha}_n = \tilde{q}_n(A\tilde{y}_n, \tilde{h}_n)\, \|A\tilde{y}_n\|^{-2} ,$$

respectively, where $0 \leqq q_n \leqq 2$, $0 \leqq \tilde{q}_n \leqq 2$. (If $(Ay_n, h_n) = 0$ or $(A\tilde{y}_n, \tilde{h}_n) = 0$. then $q_n = 1$, or $\tilde{q}_n = 1$). Now introduce angles φ_n, $\tilde{\varphi}_n$ by $(Ay_n, h_n) = \|Ay_n\| \cdot \|h_n\| \cos \varphi_n$, $(A\tilde{y}_n, \tilde{h}_n) = \|A\tilde{y}_n\|\, \|\tilde{h}_n\| \cos \tilde{\varphi}_n\, (n = 0, 1,.2, \ldots)$. According to (13), (14),

$$(15) \qquad \Delta(\|h_n\|^2) = -q_n(2 - q_n)\, \|h_n\|^2 \cos^2 \varphi_n$$

and

$$\Delta(\|\tilde{h}_n\|^2) = -\tilde{q}_n(2 - \tilde{q}_n)\, \|\tilde{h}_n\|^2 \cos^2 \tilde{\varphi}_n .$$

The sequence $\{\|h_n\|\}_{n=0}^{n=\infty}$, $\{\|\tilde{h}_n\|\}_{n=0}^{n=\infty}$ are bounded and monotone decreasing. Therefore there exist

$$h_\infty = \lim_{n\to\infty} \|h_n\|^2 , \quad \tilde{h}_\infty = \lim_{n\to\infty} \|\tilde{h}_n\|^2$$

and

$$h_\infty = \|h_0\|^2 + \sum_{n=0}^{\infty} \Delta(\|h_n\|^2) , \quad \tilde{h}_\infty = \|\tilde{h}_0\|^2 + \sum_{n=0}^{\infty} \Delta(\|\tilde{h}_n\|^2) .$$

Thus we obtain

Theorem 3. *Let X be a real Hilbert space. Let $A : X \to X$, $P : X \to X$ be linear bounded commutative operators such that P^{-1} exists and is a bounded operator, and $q = \|I - PA\| \leqq 1$. Then the series*

$$\sum_{n=0}^{\infty} q_n(2 - q_n)\, \|h_n\|^2 \cos^2 \varphi_n , \quad \sum_{n=0}^{\infty} \tilde{q}_n(2 - \tilde{q}_n)\, \|\tilde{h}_n\|^2 \cos^2 \tilde{\varphi}_n$$

converge.

Under the assumptions of theorem 3 also suppose that A has a bounded inverse A^{-1} and $(A^{-1}x, x) \geqq m\|x\|^2$, $(m > 0)$ holds for every $x \in X$. Set $c_n = (A^{-1}h_n, h_n)$, $\tilde{c}_n = (A^{-1}\tilde{h}_n, \tilde{h}_n)$. Then the series

$$(16) \qquad \sum_{n=0}^{\infty} c_n q_n(2 - q_n) \cos^2 \varphi_n , \quad \sum_{n=0}^{\infty} \tilde{c}_n \tilde{q}_n(2 - \tilde{q}_n) \cos^2 \tilde{\varphi}_n$$

also converge. Set $r_n = \|h_n\|^2\, c_n^{-1}$, $\tilde{r}_n = \|\tilde{h}_n\|^2\, \tilde{c}_n^{-1}$. Then

$$(17) \qquad \|A^{-1}\|^{-1} \leqq r_n \leqq M , \quad \|A^{-1}\|^{-1} \leqq \tilde{r}_n \leqq M ,$$

where $M = m^{-1}$. According to (15), (16), (17),

$$(18)\qquad (\|h_k\|^2 - h_\infty)(\|A^{-1}\|)^{-1} \leqq \sum_{n=k}^{\infty} c_n q_n (2 - q_n) \cos^2 \varphi_n \leqq$$
$$\leqq M(\|h_k\|^2 - h_\infty),$$

$$(\|\tilde{h}_k\| - \tilde{h}_\infty) \|A^{-1}\|^{-1} \leqq \sum_{n=k}^{\infty'} \tilde{c}_n \tilde{q}_n (2 - \tilde{q}_n) \cos^2 \tilde{\varphi}_n \leqq$$
$$\leqq M(\|\tilde{h}_k\|^2 - \tilde{h}_\infty).$$

Because $\|h_n\|^2 = c_n r_n$, $\|\tilde{h}_n\|^2 = \tilde{r}_n \tilde{c}_n$,

$$(19)\qquad (r_k c_k - \lim_{k\to\infty} r_k c_k) \|A^{-1}\|^{-1} \leqq \sum_{n=k}^{\infty} c_n q_n (2 - q_n) \cos^2 \varphi_n \leqq$$
$$\leqq M(r_k c_k - \lim_{k\to\infty} r_k c_k),$$

$$(\tilde{r}_k \tilde{c}_k - \lim_{k\to\infty} \tilde{r}_k \tilde{c}_k) \|A^{-1}\|^{-1} \leqq \sum_{n=k}^{\infty} \tilde{c}_n \tilde{q}_n (2 - \tilde{q}_n) \cos^2 \tilde{\varphi}_n \leqq M(\tilde{r}_k \tilde{c}_k - \lim_{k\to\infty} \tilde{r}_k \tilde{c}_k).$$

If x^* denotes the unique solution of (1), then $x_n \to x^*$ (or $\tilde{x}_n \to x^*$) if and only if $h_n \to 0$ (or $\tilde{h}_n \to 0$) as $n \to \infty$. Hence if $x_n \to x^*$ and $\tilde{x}_n \to x^*$, then

$$(20)\qquad \frac{1}{\|A^{-1}\|} r_k c_k \leqq \sum_{n=k}^{\infty} c_n q_n (2 - q_n) \cos^2 \varphi_n \leqq M r_k c_k,$$

$$\frac{1}{\|A^{-1}\|} \tilde{r}_k \tilde{c}_k \leqq \sum_{n=k}^{\infty} \tilde{c}_n \tilde{q}_n (2 - \tilde{q}_n) \cos^2 \tilde{\varphi}_n \leqq M \tilde{r}_k \tilde{c}_k.$$

Using (17),

$$\frac{1}{\|A^{-1}\|^2} c_k \leqq \sum_{n=k}^{\infty} c_n q_n (2 - q_n) \cos^2 \varphi_n \leqq M^2 c_k$$

and similarly for the second inequalities in (20). According to (20) and from

$$c_n r_k^{-1} c_k^{-1} = (A^{-1} h_n, h_n) \|h_k\|^{-2} \leqq \|A^{-1}\| \|h_n\|^2 \|h_k\|^{-2} \leqq \|A^{-1}\|$$

(since for $n \geqq k \|h_k\| \geqq \|h_n\|$) we conclude that

$$\frac{1}{\|A^{-1}\|} \leqq \sum_{n=k}^{\infty} q_n (2 - q_n) \cos^2 \varphi_n c_n (r_k c_k)^{-1} \leqq \|A^{-1}\| \sum_{n=k}^{\infty} q_n (2 - q_n) \cos^2 \varphi_n.$$

From these inequalities it follows that

$$\sum_{n=k}^{\infty} q_n (2 - q_n) \cos^2 \varphi_n \geqq \frac{1}{\|A^{-1}\|^2}.$$

Analogously one may prove that

$$\sum_{n=k}^{\infty} \tilde{q}_n(2 - \tilde{q}_n) \cos^2 \tilde{\varphi}_n \geqq \frac{1}{\|A^{-1}\|^2} .$$

Thus we have proved the first part of the following

Theorem 4. *Let X be a real Hilbert space. Let $A : X \to X$, $P : X \to X$ be linear bounded commutative operators with bounded inverses; let $(A^{-1}x, x) \geqq m\|x\|^2$ hold for every $x \in X$, $(m > 0)$ and $q = \|I - PA\| \leqq 1$. Then the sequence $\{x_n\}$ or $\{\tilde{x}_n\}$ defined by (5′) or (6′) converge in the norm topology of X to the unique solution x^* of (1) if and only if the series $\sum_{n=0}^{\infty} q_n(2 - q_n) \cos^2 \varphi_n$ or $\sum_{n=0}^{\infty} \tilde{q}_n(2 - \tilde{q}_n) \cos^2 \tilde{\varphi}_n$ respectively, is divergent.*

Second part. Let the series $\sum_{n=0}^{\infty} q_n(2 - q_n) \cos^2 \varphi_n$, $\sum_{n=0}^{\infty} \tilde{q}_n(2 - \tilde{q}_n) \cos^2 \tilde{\varphi}_n$ diverge. According to theorem 3, the series $\sum_{n=0}^{\infty} q_n(2 - q_n) \|h_n\|^2 \cos^2 \varphi_n$, $\sum_{n=0}^{\infty} \tilde{q}_n(2 - \tilde{q}_n) \cdot \|\tilde{h}_n\|^2 \cos^2 \tilde{\varphi}_n$ are convergent. Since $\{\|h_n\|\}$, $\{\|\tilde{h}_n\|\}$ are bounded and monotone, there is $\lim_{n\to\infty} \|h_n\| = 0$, $\lim_{n\to\infty} \|\tilde{h}_n\| = 0$. From the boundedness of A^{-1} we conclude that $x_n \to x^*$, $\tilde{x}_n \to x^*$. This completes the proof.

Remark 2. Theorems 3,4 remain valid for the sequence $\{x_n\}$, where $x_{n+1} = Pf + (I - PA)\, x_n$ (i.e. if $\beta_n = 1$, $\tilde{\beta}_n = 1$, $(n = 0, 1, 2, \ldots)$).

3. Set

$$a^2 = \overline{\lim_{n\to\infty}}\, n^{-1} \sum_{k=0}^{n-1} \|h_{k+1}\|^2 \|h_k\|^{-2} ,$$

$$\tilde{a}^2 = \overline{\lim_{n\to\infty}}\, n^{-1} \sum_{k=0}^{n-1} \|\tilde{h}_{k+1}\|^2 \|\tilde{h}_k\|^{-2} .$$

Under the assumptions of theorem 3 we have that

$$\|h_{k+1}\| \leqq \|h_k\|, \ \|\tilde{h}_{k+1}\| \leqq \|\tilde{h}_k\| , \quad (k = 0, 1, 2, \ldots) .$$

Then $0 \leqq a \leqq 1$, $0 \leqq \tilde{a} \leqq 1$. Moreover, if $a < 1$ (or $\tilde{a} < 1$), then $h_n \to 0$ (or $\tilde{h}_n \to 0$). For instance, we shall prove this for $\{h_n\}$. Suppose $a < 1$. If $\|h_n\| \to b \neq 0$, then $\|h_{n+1}\| \|h_n\|^{-1} \to 1$ and $\lim_{n\to\infty} n^{-1} \sum_{k=0}^{n-1} \|h_{k+1}\|^2 \|h_k\|^{-2} = 1$. Hence $h_n \to 0$. Set $\alpha^2 = \overline{\lim_{n\to\infty}} \sqrt[n]{\|h_n\|^2}$, $\tilde{\alpha}^2 = \overline{\lim_{n\to\infty}} \sqrt[n]{\|\tilde{h}_n\|^2}$. Then $\alpha \leqq a$, $\tilde{\alpha} \leqq \tilde{a}$. Indeed from

$$\|h_n\|^{2/n} \leqq \|h_0\|^{2/n}\, n^{-1} \sum_{k=0}^{n-1} \|h_{k+1}\|^2 \|h_k\|^{-2} ,$$

and $\lim_{n\to\infty} \|h_0\|^{2/n} = 1$ it follows immediately that $\alpha \leqq a$. Analogously for the second inequality. If the mapping $A : X \to X$ is such that A^{-1} exists and is continuous, then the sequences $\{x_n\}$, $\{\tilde{x}_n\}$ defined by (5′), (6′) converge to the solution x^* of (1) at least with the rate of geometric sequences with quotients a, $\tilde{a}$. Thus we have proved the following theorem:

Theorem 5. *Let X be a real Hilbert space, $A : X \to X$, $P : X \to X$ linear bounded commutative operators with bounded inverses such that $q = \|I - PA\| \leqq 1$. If $a < 1$, $\tilde{a} < 1$, then the sequences $\{x_n\}$, $\{\tilde{x}_n\}$ defined by (5′), (6′) converge in the norm topology of X to the solution x^* of (1) at least with the rate of geometric sequences with quotients a, $\tilde{a}$.*

Theorem 6. *Under the assumptions of theorem 3, let A have a bounded inverse A^{-1}. If $\sigma = \underline{\lim}_{n\to\infty} n^{-1} \sum_{k=0}^{n-1} q_k(2 - q_k) \cos^2 \varphi_k > 0$ (or $\tilde{\sigma} = \underline{\lim}_{n\to\infty} n^{-1} \sum_{k=0}^{n-1} \tilde{q}_k(2 - \tilde{q}_k) \cos^2 \tilde{\varphi}_k > 0$) then the sequence $\{x_n\}$ (or $\{\tilde{x}_n\}$) converges in the norm topology of X to the solution x^* of (1) at least with the rate of geometric sequences with the quotients $1 - \sigma$ (or $1 - \tilde{\sigma}$, respectively).*

Proof. Because

$$\|h_k\|^2 - \|h_{k+1}\|^2 = q_k(2 - q_k) \|h_k\|^2 \cos^2 \varphi_k ,$$

there is

$$1 - \|h_{k+1}\|^2 \|h_k\|^{-2} = q_k(2 - q_k) \cos^2 \varphi_k ,$$

and

$$n^{-1} \sum_{k=0}^{n-1} \|h_{k+1}\|^2 \|h_k\|^{-2} = 1 - n^{-1} \sum_{k=0}^{n-1} q_k(2 - q_k) \cos^2 \varphi_k .$$

Since $\sigma > 0$, one has $a < 1$ and therefore $h_n \to 0$. From the existence of bounded A^{-1} one obtains that $x_n \to x^*$. The assertation on the rate of convergence of $\{x_n\}$ obviously holds. This completes the proof.

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Резюме

НЕОБХОДИМЫЕ И ДОСТАТОЧНЫЕ УСЛОВИЯ ДЛЯ НЕКОТОРЫХ СХОДЯЩИХСЯ МЕТОДОВ

ЙОСЕФ КОЛОМЫ (Josef Kolomý), Прага

Пусть дано уравнение $Ax = f$, где $A : X \to X$ – линейный ограниченный оператор в гильбертовом пространстве X, $f \in X$. Последовательные приближения вычисляются по формуле (2), где P – линейный ограниченный оператор в X такой, что P перестановочен с A и существует ограниченный P^{-1}. Действительные коэффициенты β_n ($n = 0, 1, 2, \ldots$) определяются так, чтобы выполнялось одно из условий: $\|f - \beta_n A x_n\|^2 = \text{Min}$, $\|f - A x_{n+1}\|^2 = \text{Min}$. В работе изучены необходимые и достаточные условия для сходимости и быстрота этих методов