Jan Pelant

One folkloristic lemma on cardinal reflections in Unif

In: Zdeněk Frolík (ed.): Seminar Uniform Spaces. , 1975. pp. 145–147.

Persistent URL: http://dml.cz/dmlcz/703124

Terms of use:

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project DML-CZ: The Czech Digital Mathematics Library http://dml.cz

ONE FOLKLORISTIC LEMMA ON CARDINAL REFLECTIONS IN UNIF

Let (X, \mathcal{U}) be a uniform space. Let X be a cardinal number. Let $\{U_a\}_{a \in A}$ could A = X, be a \mathcal{U} -cover of X. Denote by $Y: \{V_a\}_{a \in B}$ some \mathcal{U} -cover which star-refines $\{U_a\}$. For $x \in X$, denote $S(x) = \{a \mid a \in A\}$. Let $\{x, Y\} \in U_a\}$. For $\{x \in X\}$, denote $\{(x)\} = \{a \mid a \in A\}$. Let $\{(x, Y)\} \in U_a\}$. For $\{(x, Y)\} \in \{(x, Y)\} = \{a \mid a \in A\}$. Let $\{(x, Y)\} \in \{(x, Y)\}$ and $\{(x, Y)\} \in \{(x, Y)\}$ and $\{(x, Y)\} \in \{(x, Y)\}$. A mapping $\{(x, Y)\} \in \{(x, Y)\}$ by each $\{(x, Y)\} \in \{(x, Y)\}$. Let $\{(x, Y)\} \in \{(x, Y)\}$ and $\{(x, Y)\} \in \{(x, Y)\}$. Let $\{(x, Y)\} \in \{(x, Y)\}$ be a uniform space. Let $\{(x, Y)\} \in \{(x, Y)\}$ be a cardinal number. Let $\{(x, Y)\} \in \{(x, Y)\}$ be a uniform space. Let $\{(x, Y)\} \in \{(x, Y)\}$ be a cardinal number. Let $\{(x, Y)\} \in \{(x, Y)\}$ be a uniform space.

Lemma: There is a \mathcal{U} -cover $\{C_a\}_{a\in A}$ which starrefines $\{U_a\}_{a\in A}$ and is refined by V if and only if the following assertions hold:

there exist a choice mapping κ and a partition $\{B_a\}_{a\in A}$ of B such that

$$R(\bigcup_{b \in B_a} V_b) = \bigcup_{b \in B_a} R(V_b) \subset \bigcap_{b \in B_a} I(V_b) = I(\bigcup_{b \in B_a} V_b).$$

Proof: The necessity of the condition is obvious. Conversely, if a choice mapping x and a partition $\{B_a\}_{a\in A}$ of B from Lemma exist, then we define $\{C_a\}_{a\in A}$ by $C_a = \bigcup_{k\in B_a} V_k$. Clearly, $V \prec \{C_a\}_{a\in A}$. Let $x\in X$.

$$I(\operatorname{st}(x, \{C_a\}_{a\in A})) = \bigcap \{I(C_a) \mid x \in C_a\} \supset \bigcap \pi(C_a) \supset \pi(x).$$

Application:

1) Assume that $2^{\alpha} < K$ for any $\alpha < K$. Each M-cover ver $\{U_{\alpha}\}_{\alpha \in A}$, cand A < K, is star-refined by a M-cover

-146-(Ca 3aeA (See [1].)

Proof: Consider 4 Va 3 . Define $x: X \longrightarrow A$ by $r(x) = min \ (a \ | st(x, \{V_k\}_{k \in B})) \subset \coprod_{a} 3$; we may assume that A is an ordinal. For a e A and J = = a, define $B(a, J) = \{b \mid \text{ is the minimal number of } A$ such that $SI(V_g, iV_g)_{g \in B}$ c $La & I(V_g) \cap a = J$. The assumption implies that cord $f(a, J) \mid J \in a \in A$ and A. It follows from the definition of κ and B(a, J) that $\mu(V_{g_r}) \subset J \cup \{a\}$ for any $b \in B(a, J)$, hence $\bigcap_{k \in B(a, J)} I(V_k) \supset I(V_k) \cap I(V_k)$ ⊃ Jula 3 ⊃ U אוע,) and Lemma applies. לבנת, סי

2) (X, \mathcal{U}) is a uniform space, { \mathbb{U}_a } and A < Kis \mathcal{U} -cover. Let $\{W_{\mathbf{c}}\}_{\mathbf{ceC}} = \mathbf{W}$ be a 6-pointwise finite $\mathcal U$ -cover, that refines $\{U_a\}_{a\in A}$. Then there is s \mathcal{U} -cover $\{V_a\}_{a \in A}$ which refines starwise $\{U_a\}_{a \in A}$ See [2].

Proof: Choose any mapping q: C --- A such that $W_c = U_{q(c)}$. By assumptions, $C = \bigcup_{m=0}^{\infty} C_m$ and each collection $\{W_c \mid c \in C_m\}$ is pointwise finite. Consider V == { V } & & W . Assume that C is a well-ordered set, take C_m with the induced ordering. Define $x:X\longrightarrow A$ by n(x) = q(c) where $c \in C$ is the minimal element of $C_{\underline{a}}$ such that at $(x, \{V_{\underline{\mu}}, \}) \subset W_{\underline{c}}$ and there is not eny point with this property being an element of Cae for $k < \beta$. As W is pointwise finite and $V \stackrel{*}{\rightleftharpoons} W$, $\kappa(V_{A})$

-147is finite for any $l \in B$. For $J \in \mathcal{P}_{lin}(A)$ define $B_J =$ = {beB|r(Vg)=J]. As cord Pcin(A) = cord A $V_{\mathcal{E}}$ $V_{\mathcal{E}}$ = $J \subset \bigcap_{\mathcal{E}} I(V_{\mathcal{E}})$, Lemma applies.

References:

- [1] Anna Kucia: On coverings of a uniformity, Coll. Math. 27(1973),73-74.
- [2] Giovanni Vidosaich: A note on cardinal reflections in the category of uniform spaces, Proc. A.M.S. 23(1969),55-58.

Added in proof: In Application 1, if $K = \alpha^+$, then it is enough to suppose 2^{β} a for all $\beta < \infty$.