Reflective and coreflective subcategories of $\mathcal{Unif}$ (and $\mathcal{Top}$)

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V. Kannan in [Ka] investigated subcategories both reflective and coreflective in categories of topological spaces. We shall look now at the same problem in the categories of uniform spaces; not because of results themselves that are not too deep (although interesting) but because of the methods which use generation of spaces in subcategories.

We shall use terms and notations as used in [Č] (thus uniform spaces need not be Hausdorff).

To prove that both reflective and coreflective subcategory of a category of continuous structures coincide with the whole category, one must prove that any object of the category can be obtained from the subcategory by combining projective and inductive generations. In the category $\mathcal{Top}$ of all topological spaces and all continuous mappings, one possible basic construction is expressed e.g. in [Č]; by means of this construction (different from that in [Ka]) we are able to prove more general results than proved in [Ka]. In $\mathcal{Unif}$, the category of all uniform spaces and all uniformly continuous mappings, one may use the idea expressed in the exercise III,3 of [I] or in 37.A.8 of [Č]:

Let $\langle X, \mathcal{U} \rangle$ be a uniform space, D and D_1 two uniform spaces with the same underlying set and such that for an $A \subset D$, the sets $A, D - A$ are proximal in D and distant in D_1. For any $U \in \mathcal{U}$ put $Q(U) = \sum_U D + \sum_{X \times X - U} D_1$. Then $\langle X, \mathcal{U} \rangle$ is a quotient of $Q_X = \inf_{\mathcal{U}} Q(U)$ under the mapping f defined as follows

$$\text{if } x, x' \in X, d \in D \text{, then } f\langle x, x', d \rangle = \begin{cases} x & \text{if } d \in A \\ x' & \text{if } d \notin A . \end{cases}$$

The preceding result can be modified in several ways. It is not necessary to take into account the points of the diagonal of $X \times X$ (we may define $Q(U) = \sum_{U - 1_X} D + \sum_{X \times X - U} D_1$ but not $Q(U) = \sum_{U - \cap \mathcal{U}} D + \sum_{X \times X - U} D_1$). The uniformity of Q_X may vary from that defined before to that projectively generated by f; e.g. we can put $Q'_X = \inf_{\mathcal{U}} Q'(U)$, where $Q'(U)$ has the same underlying set as $Q(U)$ and a base of its uniformity is composed of the equivalence

$$\bigcup_U (\langle x, x' \rangle \times D)^2 \cup \bigcup_{X \times X - U} (\langle x, x' \rangle \times A)^2 \cup \bigcup_{X \times X - U} (\langle x, x' \rangle \times (D - A))^2 .$$

Or $Q''_X = \inf_{\mathcal{U}} Q''(U)$, where $Q''(U) = (U \times D) + ((X \times X - U) \times D_1)$ and both U, $X \times X - U$ are endowed with uniformly discrete uniformities.

In all three cases described above, the infimum is in fact a projective limit of a presheaf. The approach by means of presheaves is more convenient when it is difficult to find D and D_1 belonging to the subcategory. We shall use the following modification in the case of Hausdorff $\langle X, \mathcal{U} \rangle$. For our purposes it suffices to take D_1 to be a two point discrete space with an underlying set (a, b).

Put $\widetilde{Q}(\mathfrak{U}) = \sum_{\mathfrak{U}}(a) + \sum_{X \times X - \mathfrak{U}}(a, b)$. The connecting mappings $\widetilde{\imath}_{\mathfrak{U}\mathfrak{V}} : \widetilde{Q}(\mathfrak{U}) \longrightarrow \widetilde{Q}(\mathfrak{V})$ for $\mathfrak{U}, \mathfrak{V} \in \mathcal{U}$, $\mathfrak{U} \subset \mathfrak{V}$ are

$$\widetilde{\imath}_{\mathfrak{U}\mathfrak{V}} \langle x, x', a \rangle = \langle x, x', a \rangle$$

$$\widetilde{\imath}_{\mathfrak{U}\mathfrak{V}} \langle x, x', b \rangle = \begin{cases} \langle x, x', a \rangle & \text{if } \langle x, x' \rangle \in \mathfrak{V} \\ \langle x, x', b \rangle & \text{if } \langle x, x' \rangle \in X \times X - \mathfrak{V} . \end{cases}$$

The underlying set of the projective limit $\langle \widetilde{Q}_X, \{\widetilde{\jmath}_{\mathfrak{U}}\} \rangle$ is the set

$$\sum_{1_X}(a) + \sum_{X \times X - 1_X}(a, b)$$

and

$$\widetilde{\jmath}_{\mathfrak{U}} \langle x, x', a \rangle = \langle x, x', a \rangle$$

$$\widetilde{\jmath}_{\mathfrak{U}} \langle x, x', b \rangle = \begin{cases} \langle x, x', a \rangle & \text{if } \langle x, x' \rangle \in \mathfrak{U} \\ \langle x, x', b \rangle & \text{if } \langle x, x' \rangle \in X \times X - \mathfrak{U} . \end{cases}$$

The uniformity of $\widetilde{Q}_X$ is that projectively generated by $\{\widetilde{\jmath}_{\mathfrak{U}}\}$. It is almost clear what the quotient mapping $\widetilde{f} : \widetilde{Q}_X \longrightarrow \langle X, \mathcal{U} \rangle$ will be:

$$\widetilde{f}\langle x, x', a \rangle = x , \qquad \widetilde{f}\langle x, x', b \rangle = x' .$$

We shall prove the following theorem (all the subcategories are supposed to be full and replete):

<u>THEOREM 1.</u> Let $\mathbb{L}$ be a both reflective and coreflective subcategory of $\mathcal{U}nif$. Then $\mathbb{L} = \mathcal{U}nif$.

<u>THEOREM 2.</u> Let $\mathbb{K}$ be a reflective subcategory of $\mathcal{U}nif$ containing all uniformly discrete spaces. If $\mathbb{L}$ is both reflective and coreflective subcategory of $\mathbb{K}$, then $\mathbb{L} = \mathbb{K}$.

THEOREM 3. Let $\mathcal{K}$ be a closed-hereditary reflective subcategory of $\mathcal{U}nif$. If $\mathcal{L}$ is a both reflective and coreflective subcategory of $\mathcal{K}$, then $\mathcal{L} = \mathcal{K}$.

THEOREM 4. Let $\mathcal{K}$ be a coreflective subcategory of $\mathcal{U}nif$ generated by complete uniformly 0-dimensional Hausdorff spaces. If $\mathcal{L}$ is a both reflective and coreflective subcategory of $\mathcal{K}$, then $\mathcal{L} = \mathcal{K}$.

At first several remarks. Theorem 1 is clearly contained in all the remaining Theorems but we have stated it here because its proof is simpler than the others. We do not know whether Theorem 2 is contained in Theorem 3 (even in its weaker form - see the remark following the proof of Theorem 3). Examples 1 and 2 will show that Theorems 3 and 4 do not hold if we omit the conditions posed on the reflective (or coreflective) subcategory $\mathcal{K}$.

PROOF of THEOREMS 1 and 2. Clearly, $\mathcal{L}$ contains all uniformly discrete spaces and their products in $\mathcal{U}nif$. We may take for D a product of uniformly discrete spaces which is not discrete and for D_1 the same product endowed with uniformly discrete uniformity. Now, since sums, projective limits and quotients in $\mathcal{L}$ of objects from $\mathcal{L}$ are the same as in $\mathcal{K}$ (or $\mathcal{U}nif$ in Theorem 1), it follows that any object of $\mathcal{K}$ belongs to $\mathcal{L}$ (it is a quotient of $Q'' \in \mathcal{L}$ in Theorem 2 and of Q in Theorem 1).

Notice that we have proved more: Any uniform space is a quotient of an object of $\mathcal{K}$.

PROOF of THEOREM 3. If K contains only indiscrete spaces then the assertion is obvious. Let X be a non-discrete space belonging to K and denote by D a two-point set. If X is Hausdorff, then D with discrete uniformity $\mathcal{D}_1$ belongs to K as a uniform homeomorph of a closed subspace of X. If X is not Hausdorff then it is not T_0, and, hence, D with indiscrete uniformity $\mathcal{D}$ belongs to K as a retract of X; consequently, $\langle D, \mathcal{D}_1 \rangle \in K$ because otherwise $\langle D, \mathcal{D} \rangle$ would be a reflection of $\langle D, \mathcal{D}_1 \rangle$ in K, which is impossible since K contains a non-indiscrete space.

Fix now a two-point discrete space D_1 with points a, b (clearly, D_1 belongs also to L since L is coreflective in K). Let a Hausdorff $\langle X, \mathcal{U} \rangle \in K$ be given and denote by r a reflector $\mathit{Unif} \longrightarrow K$ with reflections $Y \xrightarrow{i_Y} rY$. Define

$$\langle Q, \{p_U \mid U \in \mathcal{U}\} \rangle = \underleftarrow{\lim} \{r\tilde{Q}(U), \{r\tilde{i}_{UV} \mid U \subset V\} \mid U \in \mathcal{U}\};$$

since L is reflective and coreflective in K we have $r\tilde{Q}(U) \in L$ and $Q \in L$. We shall prove that X is a quotient in Unif of Q under a mapping g and, thus, that X belongs to L. The mapping g will be in a sense a limit of mappings from $r\tilde{Q}(U)$ into X. First we shall define mappings $g_U : \tilde{Q}(U) \longrightarrow X$ like this

$$g_U \langle x, x', d \rangle = \begin{cases} x & \text{if } d = a \\ x' & \text{if } d = b. \end{cases}$$

For $z \in Q$ we define $gz = (rg_U) p_U z$, where $U \in \mathcal{U}$

is selected in such a way that $p_U z \in \varkappa \sum_{X \times X - U} D_1$ if such a U exists (if $Y = Y_1 + Y_2$ we may take $\varkappa Y = \varkappa Y_1 + \varkappa Y_2$), otherwise U is an arbitrary member of $\mathcal{U}$. The definition of g is correct because if $gz = (\varkappa g_U) p_U z$ as defined, then $gz = (\varkappa g_V) p_V z$ for any $V \in \mathcal{U}$, $V \subset U$, which follows from the fact that

$$\varkappa g_U / \varkappa \sum_V (a) + \varkappa \sum_{X \times X - U} D_1 = \varkappa g_V / \varkappa \sum_V (a) + \varkappa \sum_{X \times X - U} D_1 .$$

It remains to show that g is uniformly continuous because it is quotient then (its composition with $j : \tilde{Q}_X \longrightarrow Q$, the limit of $\{ i_{\tilde{Q}(U)} \circ j_U : \tilde{Q}_X \longrightarrow \tilde{Q}(U) \longrightarrow \varkappa \tilde{Q}(U) \}$, equals to the quotient $\tilde{f}$). Let $U \in \mathcal{U}$, $V \in \mathcal{U}$, $V = V^{-1}$, $V \circ V \circ V \subset U$, V is closed in $X \times X$ (hence $V \in K$). We shall prove that $(g \times g)^{-1}[U] \supset ((\varkappa g_V) \circ p_V \times (\varkappa g_V) \circ p_V)^{-1}[V]$. Let $z_1, z_2 \in Q$, $\langle (\varkappa g_V) p_V z_1, (\varkappa g_V) p_V z_2 \rangle \in V$; it is easily seen that it suffices to prove $\langle gz, (\varkappa g_V) p_V z \rangle \in V$ for any $z \in Q$. Suppose $gz = (\varkappa g_W) p_W z$ for a $W \subset V$, $W \in \mathcal{U}$; we may suppose that $p_W z \in \varkappa \sum_{X \times X - W} D_1$, $p_V z \in \varkappa \sum_{X \times X - V} D_1$ because otherwise we could put $W = V$; moreover we may suppose $p_W z \in \varkappa \sum_{V - W} (b)$ because that is the only set where $\varkappa g_V$ and $\varkappa g_W$ may differ. Now, $\varkappa g_V / \varkappa \sum_{V-W} (a) = \varkappa \sum_{V-W} (a) \xrightarrow{\varkappa h} V \xrightarrow{pr_1} X$ and $\varkappa g_W / \varkappa \sum_{V-W} (b) = \varkappa \sum_{V-W} (b) \xrightarrow{\varkappa h'} V \xrightarrow{pr_2} X$, where

$h \langle x, x', a \rangle = \langle x, x' \rangle$, $h' \langle x, x', b \rangle = \langle x, x' \rangle$ and pr_i is the i-th projection. Since $\varkappa h \circ \varkappa \tilde{i}_{WV} / \varkappa \sum_{V-W} (b) = \varkappa h'$ we have $(\varkappa h') p_W z = (\varkappa h)(\varkappa \tilde{i}_{WV}) p_W z = (\varkappa h) p_V z \in V$ and, hence, $\langle (\varkappa g_W) p_W z, (\varkappa g_V) p_V z \rangle \in V$.

If an object X of K is not Hausdorff then K is bireflective in $\mathcal{Unif}$ and we can use the spaces $Q(\mathfrak{U})$ instead of spaces $\widetilde{Q}(\mathfrak{U})$ and $Q = \inf_{\mathcal{U}} \{\varkappa Q(\mathfrak{U})\}$ then. It remains to prove that the mapping f (see the introduction) is uniformly continuous on Q into X and this can be done by the same method as in the preceding proof of uniform continuity of g but the procedure is much easier.

All we needed in the foregoing proof for the reflective subcategory K was that if K is composed of Hausdorff spaces only, then a discrete two-point space belongs to K (or equivalently, all compact 0-dimensional T_2-spaces belong to K) and that any object of K has a base of its uniformity composed of objects of K. I do not know whether these conditions imply that K is closed-hereditary (probably not).

PROOF of THEOREM 4. It suffices to prove that any complete uniformly 0-dimensional Hausdorff object $\langle X, \mathcal{U} \rangle$ of K belongs to L. Let $\mathcal{U}'$ be the base of $\mathcal{U}$ composed of equivalences and for each $\mathfrak{U} \in \mathcal{U}'$, $f_{\mathfrak{U}}$ be the canonical mapping on X onto the quotient $X/\mathfrak{U}$ (a uniformly discrete space). Let $X \xrightarrow{h} \varkappa X$ be a reflection of X in L ; since $X/\mathfrak{U}$ belongs to L, any $f_{\mathfrak{U}}$ has the unique factorization $\widetilde{f}_{\mathfrak{U}}$ via h. For $x \in \varkappa X - h[X]$, $\mathfrak{U} \in \mathcal{U}'$, denote $V_{\mathfrak{U},x} = f_{\mathfrak{U}}^{-1}[\widetilde{f}_{\mathfrak{U}} x]$. For a fixed x, the collection $\{V_{\mathfrak{U},x} \mid \mathfrak{U} \in \mathcal{U}\}$ is a base for a Cauchy filter F in X and, thus, has a limit gx. If we put $ghx = x$

for $x \in X$, then $g : \kappa X \longrightarrow X$ is uniformly continuous, $f_{U} \circ g = \tilde{f}_{U}$ for all $U \in \mathcal{U}'$ and $\{f_{U}\}$ projectively generates X . Consequently, g is a retraction and X belongs to L .

Theorem 4 is the easiest result that can be proved by the same method. It would be sufficient to suppose that K is coreflective in $\mathcal{U}nif$, K is generated by a class of complete spaces such that any its member is projectively generated by Hausdorff members of L and the reflections κ are dense one-to-one mappings.

The next example shows a reflective subcategory K of $\mathcal{U}nif$ that contains a non-trivial both reflective and coreflective subcategory L . In spite of this example we know that the conditions on K in Theorem 3 are too strong (e.g. any reflective subcategory of $\mathcal{U}nif$ composed of compact Hausdorff spaces has no non-trivial coreflective subcategory).

EXAMPLE 1. Let C be the de Groot's strongly rigid metrizable non-compact space, [G], and let u be a collection of uniformities on C that is meet-stable in the collection of all uniformities on C . The subcategory $C(u)$ of $\mathcal{U}nif$ composed of all products of uniformities from u form reflective subcategory of $\mathcal{U}nif$. The proof is almost the same as the H. Herrlich's proof that the subcategory of $\mathcal{T}op$ composed of all powers of C is reflective in $\mathcal{T}op$, $[H_1]$, $[H_2]$. We shall mention here only necessary changes. For a uniform space X let

$c_X = \{\langle f \langle C, \mathcal{U} \rangle\rangle \mid f: X \longrightarrow \langle C, \mathcal{U}\rangle$ is inductively generating with respect to $\mathcal{u}$, f is not constant $\}$,

$\varkappa X = \prod \{\langle C, \mathcal{U}_{\langle f, \langle C, \mathcal{U}\rangle\rangle} \rangle \mid \langle f, \langle C, \mathcal{U}\rangle\rangle \in c_X, \mathcal{U}_{\langle f, \langle C, \mathcal{U}\rangle\rangle} = \mathcal{U} \}$,

$h: X \longrightarrow \varkappa X$ be the reduced product of all $f: X \longrightarrow$ $\longrightarrow \langle C, \mathcal{U}_{\langle f, \langle C, \mathcal{U}\rangle\rangle} \rangle$. Then $X \xrightarrow{h} \varkappa X$ is the reflection of X in $C(\mathcal{u})$.

Now, the category C(fine uniformity on C) is both reflective and coreflective in C(all uniformities on C) which is reflective in $\mathcal{Unif}$. Notice that if $\mathcal{V}$ is the fine uniformity on C then $\langle C, \mathcal{V}\rangle^I$ is the coreflection of $\prod_I \langle C, \mathcal{U}_i \rangle$.

EXAMPLE 2. Let K be the subcategory of $\mathcal{Unif}$ composed of uniform spaces $\langle X, \mathcal{U}\rangle$ such that

$$U_i \in \mathcal{U}, \ U_i \text{ equivalences} \Longrightarrow \bigcap U_i \in \mathcal{U} .$$

Let L be the subcategory of $\mathcal{Unif}$ composed of uniform spaces $\langle X, \mathcal{U}\rangle$ such that $\bigcap \mathcal{U} \in \mathcal{U}$. It is not difficult to prove that L and K are coreflective in $\mathcal{Unif}$ and L is bireflective in K. If we put L' to be all uniformly discrete spaces, then L' is coreflective in $\mathcal{Unif}$ and bireflective in L - this shows that we cannot omit the condition on completeness in Theorem 4.

The method used to prove Theorem 3 can be also used to improve results from [Ka]. V. Kannan proved there that if K is either a bireflective subcategory of $\mathcal{Top}$ or an open-hereditary coreflective subcategory of $\mathcal{Top}$ generated by 0-dimensional spaces, then any both reflective and coreflective subcategory of K coincides with K . There is also an example in [Ka] showing that there exists a subcategory K of $\mathcal{Top}$ containing a nontrivial both reflective and coreflective subcategory; the subcategory K is neither reflective nor coreflective in $\mathcal{Top}$ - our Examples 3, 4 will show that such a K can be found coreflective or epireflective in Hausdorff spaces.

THEOREM 5. Let K be a reflective subcategory of $\mathcal{Top}$ containing a two-point space and such that every $X \in K$ is locally K . Then any both reflective and coreflective subcategory L of K coincides with K .

Proof. If all the two-point spaces from K are indiscrete then K consists of indiscrete spaces only and the assertion is trivial. If a two-point connected T_0-space belongs to K , then K is closed- and open-hereditary and contains all 0-dimensional T_0-spaces hence also T_2 spaces with at most one accumulation point. But these spaces will belong also to L and inductively generate all $\mathcal{Top}$ - our assertion follows. It remains to prove Theorem 5 in the case that all objects of K are T_1-spaces.

Let $X \in K$, $A \subset X$, $x_0 \in \bar{A} - A$, $\mathcal{U}_{x_0}$ be the neighborhood base of x_0 in X members of which belong to K . For $U \in \mathcal{U}_{x_0}$ put $S_U = (X - U) \cup (x_0)$ with the discrete topology. If $U, V \in \mathcal{U}_{x_0}$, $V \subset U$, then we define $i_{VU}:$ $: S_V \longrightarrow S_U$

$$= \left\langle \begin{array}{l} x \text{ if } x \in X - U \\ x_0 \text{ if } x \in (U - V) \cup (x_0) . \end{array} \right.$$

Put $\langle S, \{j_U\} \rangle = \varprojlim \{ S_U, \{ i_{VU} \mid V \subset U \} \mid V \in \mathcal{U}_{x_0} \}$; clearly, $S = A \cup (x_0)$, A is open discrete in S and $\mathcal{U}^S_{x_0} = [\mathcal{U}_{x_0}] \cap A$. All the spaces S obtained in just described way for various A, x_0, together with identity mappings into X inductively generate X. Now define $\langle S', \{p_U\} \rangle = \varprojlim \{ \varkappa S_U, \{ \varkappa i_{VU} \} \}$, where $\varkappa$ is the reflector $Top \to K$. The space S' belongs to L because $\varkappa S_U$ belong to L for all $U \in \mathcal{U}_{x_0}$. Let $q_U : S_U \to X$ be the identity mappings and for a $z \in S'$ let $qz = (\varkappa q_U) p_U z$, where U is selected in such a way that $p_U z \in \varkappa (A - U)$ if such a $U \in \mathcal{U}_{x_0}$ exists and U is arbitrary otherwise (then $qz = x_0$). The correctness of this definition of q can be shown in the same way as in the proof of Theorem 3. It remains to prove that q is continuous (all such mappings q then inductively generate X). Let $z \in S'$; if $p_U z \in \varkappa (A - U)$ for a $U \in \mathcal{U}_{x_0}$ then $G = p_U^{-1} [\varkappa (A - U)]$ is an open neighborhood of z and $q/G = (\varkappa q_U) \circ p_U / G$ is continuous. If $qz = x_0$ and $U \in \mathcal{U}_{x_0}$, $G = p_U^{-1} [x_0]$, then $q[G] \subset U$ - indeed, if $z' \in G$, $V \subset U$, $V \in \mathcal{U}_{x_0}$ and $p_V z' \in \varkappa (A - V)$ then $(\varkappa q_V) p_V z' \in \varkappa (U - V)$.

It should be noted that the first part of the preceding proof (till the definition of S') gives another proof of Theorem 2. in [Ka] because a bireflective subcategory K of $\mathcal{Top}$ is hereditary (thus, if nontrivial, it contains a two-point space and any its object is locally K). The condition that a two-point space belongs to K is essential only if $K \subset \mathcal{Top}_{T_1}$ (otherwise it obviously follows from the reflectivity of K in $\mathcal{Top}$.

<u>EXAMPLE 3.</u> Let K be the category of locally connected spaces and L be composed of all spaces the collections of open or closed sets of which coincide. Then both L and K are coreflective in $\mathcal{Top}$, [H_3], and L is bireflective in K . If L'={discrete spaces} then L' is coreflective in $\mathcal{Top}$ and bireflective in L .

If $K \subset \mathcal{Top}_{Reg}$ and K is closed-hereditary reflective in $\mathcal{Top}$, then K fulfils the conditions of Theorem 5. The next example shows that if $K \subset \mathcal{Top}_{T_2}$, K is epireflective (hence closed-hereditary) then yet K may contain a nontrivial epireflective and coreflective subcategory (an, thus, contains a space which is not locally K).

<u>EXAMPLE 4.</u> Denote by S_{ω_1+1} the set of all countable ordinals together with ω_1 endowed with such a topology that T_{ω_1} is an open subspace of S_{ω_1+1} and a basis of neighborhoods of ω_1 consists of the sets $[\eta, \omega_1] \cap T^0_{\omega_1} \cup (\omega_1)$, where $\eta < \omega_1$ and $T^0_{\omega_1}$ is the set of all isolated ordinals of T_{ω_1} . The category K is the epireflective

hull in Top_{T_2} of S_{ω_1+1} (i.e., K is composed of all homeomorphs to closed subspaces of powers $(S_{\omega_1+1})^{\alpha}$, [Ke]) and L is the epireflective hull in Top_{T_2} of T_{ω_1}. Since T_{ω_1} can be embedded as a closed subspace into S_{ω_1+1} (onto limit countable ordinals), L is epireflective in K. It remains to prove that L is coreflective in K. The space $T'_{\omega_1+1} = T_{\omega_1} + (\omega_1)$ belongs to L and the basic thing to show is the following: If $X \in L$, $f : X \longrightarrow S_{\omega_1+1}$ then $f : X \longrightarrow T'_{\omega_1+1}$, i.e. the set $A = f^{-1}[\omega_1]$ is clopen in X. Denote $A_\xi = f^{-1}[\xi]$ for isolated countable ξ and pick out an $x \in A$. The set $A \cup \bigcup A_\xi$ is a neighborhood of x in X and, because X is regular, there is a closed neighborhood F of x in X, $F \subset A \cup \bigcup A_\xi$. Since $F \in L$, only finite number of intersections $F \cap A_\xi$ is nonempty since for any $\eta < \omega_1$, the space $F \cap \bigcup_{\xi < \eta} A_\xi$ (homeomorphic to $\sum_{\xi<\eta}(F \cap A_\xi)$) belongs to L and, therefore, is countably compact. Now it follows that $F \cap A$ is a neighborhood of x in F and that A is open in X.

A coreflection of a $Y \in K$ in L can be constructed as follows: let $h : Y \longrightarrow S^{\alpha}_{\omega_1+1}$ be a closed embedding and put cY to be the set $h[Y]$ but with the topology as the subspace of $(T'_{\omega_1+1})^{\alpha}$, the coreflection map $g : cY \longrightarrow Y$ be the inverse of h. We must prove that any $f : X \longrightarrow Y$, $X \in L$, can be factorised via g. But that follows easily from the preceding investigation because $h \circ f$ followed by any projection onto S_{ω_1+1} is continuous as a mapping into $T'_{\omega+1}$ and, thus, $h \circ f$ is continuous into $(T'_{\omega_1+1})^{\alpha}$.

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