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DYNAMIC DAMPING – COMPARISON OF DIFFERENT CONCEPTS FROM THE POINT OF VIEW OF THEIR PHYSICAL NATURE AND EFFECTS ON CIVIL ENGINEERING STRUCTURES

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Abstract: Sources of dynamic damping may be various. Mostly, the damping is implemented into calculations in a form of introduction of damping forces, as a product of the velocity vector and the damping matrix in an equation of motion. In practice, the damping matrix is usually assumed to be a linear combination of the mass matrix and the stiffness matrix (so called Rayleigh's damping). This kind of damping primarily assumes the external environment viscosity as the source of damping, even though the part of Rayleigh's damping with the stiffness matrix implies the internal damping of the material. Explicitly, the internal viscosity of the material is respected using the appropriate material models. The relation between the Rayleigh damping and the Kelvin-Voight viscosity is shown in the paper. Dynamic damping occurs even when using non-elastic materials, where the unloading takes place in a different path from the loading and thus it leads to dissipation during loading cycles. The paper deals with the comparison of different types of damping of the oscillation of a building structure. The main aim of the paper is to recommend to apply the viscous material model instead of the obsolete and physically unjustified Rayleigh damping in the nonlinear dynamic time analysis of structures.

Keywords: dynamic damping, Rayleigh damping, material viscosity

MSC: 74H05, 74H45, 74C10

1. Introduction

There are a variety of sources of dynamical damping of vibration in civil engineering structures. Generally, damping can be caused either by the external environment

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or by structural energy dissipation. In the equation of motion, damping is most often introduced in the form of damping forces obtained as the product of the damping matrix and the velocity vector. In civil engineering practice the damping matrix is usually assumed to be a linear combination of the stiffness and mass matrices (see [4], [5] and [6]), which is known as Rayleigh damping. In practice, mainly damping proportional to mass is applied in analyses, which implies that mainly the influence of the external environment is assumed. The influence of the stiffness matrix (the β Rayleigh coefficient), which is not always rarely used in civil engineering practice, could also imply that internal friction in the structure of material has an influence on damping. The paper shows the similarity (for 1D even identity) of the β Rayleigh coefficient and the viscosity η of the Kelvin-Voight material model.

This article discusses the physical justification for the use of Rayleigh damping and suggests another, better physically justified, solution based on the real physical sources of damping in civil engineering structures. Explicitly, internal material viscosity is considered when using appropriate, time-dependent material models (see [1], [2] and [3]). Dynamical damping also occurs in the case of using inelastic materials, when the loading and unloading parts of the strain-stress diagram differ, and therefore dissipation occurs in loading cycles, as described in [1]- [3]. Another source of damping is friction in structural connections. The aim of the paper is to compare the influence of different kinds of damping on vibration in civil engineering structures and mainly strongly recommend to use in nonlinear dynamic calculations a proper viscous material model, e.g. standard linear solid, instead of the physically unjustified Rayleigh damping, mostly used in the design practice today.

2. Theoretical background

2.1. FEM discretization of the equation of motion

Let us derive the equation of motion by the demand that the virtual work of external forces is equal to the sum of the work of the inertial, dissipative and internal forces for arbitrary virtual deformation, i.e. for an arbitrary small virtual deformation which satisfies the continuity and boundary conditions. For one element with the volume Ω_e and the surface boundary Γ_e we can write this equilibrium of virtual work as follows:

$$\int_{\Omega_e} \{\delta\mathbf{u}\}^T \{\mathbf{b}\}\,\mathrm{d}\Omega_e + \int_{\Gamma_e} \{\delta\mathbf{u}\}^T \{\mathbf{t}\}\,\mathrm{d}\Gamma_e + \sum_{i=1}^{n} \{\delta\mathbf{u}\}_i^T \{\mathbf{f}\}_i = \\ = \int_{\Omega_e} \left(\{\delta\mathbf{u}\}^T \rho\,\{\ddot{\mathbf{u}}\} + \{\delta\mathbf{u}\}^T c\,\{\dot{\mathbf{u}}\} + \{\delta\varepsilon\}^T \{\sigma\}\right) \mathrm{d}\Omega_e, \tag{1}$$

where $\{\mathbf{b}\}$ and $\{\mathbf{t}\}$ are the volume and surface forces respectively, $\{\mathbf{f}\}_i$ and $\{\delta\mathbf{u}\}_i$ represent the concentrated forces and pertinent generalized displacement, respectively, ρ is the material density, c is the viscous damping parameter, and the dot stands for the time derivative. Regarding the fact that c is the multiplier of the vector field of the velocity of mass points in space and the pertinent term of the equation of motion is independent of the deformation of the body, it does not relate to material viscosity, which would only have an impact in the case of a nonzero rate of deformation in the given mass point. Neither does it relate to the viscosity of the external environment, because the stress vector $c\{\dot{\mathbf{u}}\}$, which acts against the motion $\{\mathbf{u}\}$, arises even in the internal points of the bodies. $\{\delta\mathbf{u}\}$ and $\{\delta\varepsilon\}$ represent the virtual displacement and pertinent strain, respectively. When discretizing the finite element method we obtain the following relations:

$$\{\mathbf{u}\} = [\mathbf{N}]\{\mathbf{d}\} \quad \{\dot{\mathbf{u}}\} = [\mathbf{N}]\left\{\dot{\mathbf{d}}\right\} \quad \{\ddot{\mathbf{u}}\} = [\mathbf{N}]\left\{\ddot{\mathbf{d}}\right\} \quad \{\varepsilon\} = [\mathbf{B}]\{\mathbf{d}\}. \tag{2}$$

Combining equations (1) and (2) we obtain:

$$\{\delta\mathbf{d}\}^T \left[\int_{\Omega_e} \rho\,[\mathbf{N}]^T[\mathbf{N}]\,\mathrm{d}\Omega_e \left\{\ddot{\mathbf{d}}\right\} + \int_{\Omega_e} c\,[\mathbf{N}]^T[\mathbf{N}]\,\mathrm{d}\Omega_e \left\{\dot{\mathbf{d}}\right\} + \right.$$

$$\left. + \int_{\Omega_e} [\mathbf{B}]^T\{\sigma\}\,\mathrm{d}\Omega_e - \int_{\Omega_e} [\mathbf{N}]^T\{\mathbf{b}\}\,\mathrm{d}\Omega_e - \int_{\Gamma_e} [\mathbf{N}]^T\{\mathbf{t}\}\,\mathrm{d}\Gamma_e - \sum_{i=1}^{n}\{\mathbf{f}\}_i \right] = 0, \tag{3}$$

where it is assumed that the concentrated forces $\{\mathbf{f}\}_i$ act at nodes. Let us denote the first two integrals in the equation as the consistent mass matrix and the damping matrix:

$$[\mathbf{M}_e] = \int_{\Omega_e} \rho\,[\mathbf{N}]^T[\mathbf{N}]\,\mathrm{d}\Omega_e, \tag{4}$$

$$[\mathbf{C}_e] = \int_{\Omega_e} c\,[\mathbf{N}]^T[\mathbf{N}]\,\mathrm{d}\Omega_e. \tag{5}$$

The word consistent means that the matrix follows directly from the discretization of a finite element with corresponding shape functions $[\mathbf{N}]$. Let us define the vectors of the internal and external nodal forces:

$$\left\{\mathbf{f}_e^{\mathrm{int}}\right\} = \int_{\Omega_e} [\mathbf{B}]^T\{\sigma\}\,\mathrm{d}\Omega_e, \tag{6}$$

$$\left\{\mathbf{f}_e^{\mathrm{ext}}\right\} = \int_{\Omega_e} [\mathbf{N}]^T\{\mathbf{b}\}\,\mathrm{d}\Omega_e + \int_{\Gamma_e} [\mathbf{N}]^T\{\mathbf{t}\}\,\mathrm{d}\Gamma_e + \sum_{i=1}^{n}\{\mathbf{f}\}_i. \tag{7}$$

Substituting from equations (4), (5), (6) and (7) into equation (3), and taking into account the fact that variation $\{\delta\mathbf{d}\}$ can be arbitrary, and hence the second form of the product must be equal to zero, we obtain:

$$[\mathbf{M}_e]\left\{\ddot{\mathbf{d}}\right\} + [\mathbf{C}_e]\left\{\dot{\mathbf{d}}\right\} + \left\{\mathbf{f}_e^{\mathrm{int}}\right\} = \left\{\mathbf{f}_e^{\mathrm{ext}}\right\}. \tag{8}$$

For a linear elastic material without viscosity we can write the following relation for the internal nodal forces:

$$\left\{\mathbf{f}_e^{\text{int}}\right\} = [\mathbf{K}_e]\{\mathbf{d}\}, \tag{9}$$

where

$$[\mathbf{K}_e] = \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{D}] [\mathbf{D}] \, \mathrm{d}\Omega_e \tag{10}$$

is the stiffness matrix of the element, with $[\mathbf{D}]$ being the constitutive matrix of the material. Then equation (8) can be rewritten:

$$[\mathbf{M}_e]\left\{\ddot{\mathbf{d}}\right\} + [\mathbf{C}_e]\left\{\dot{\mathbf{d}}\right\} + [\mathbf{K}_e]\{\mathbf{d}\} = \left\{\mathbf{f}_e^{\text{ext}}\right\}. \tag{11}$$

Equations (8) and (11) express in discretized form Newton's second law, or, more generally, the law of conservation of momentum. When writing these equations in the global form, i.e. for all degrees of freedom of the analyzed structure, we obtain:

$$[\mathbf{M}]\left\{\ddot{\mathbf{d}}\right\} + [\mathbf{C}]\left\{\dot{\mathbf{d}}\right\} + \left\{\mathbf{f}^{\text{int}}\right\} = \left\{\mathbf{f}^{\text{ext}}\right\}, \tag{12}$$

$$[\mathbf{M}]\left\{\ddot{\mathbf{d}}\right\} + [\mathbf{C}]\left\{\dot{\mathbf{d}}\right\} + [\mathbf{K}]\{\mathbf{d}\} = \left\{\mathbf{f}^{\text{ext}}\right\}. \tag{13}$$

2.2. Rayleigh damping

For Rayleigh damping the damping matrix $\left[\mathbf{C}_e^R\right]$ is defined as the linear combination of the consistent mass matrix $[\mathbf{M}_e]$ and the stiffness matrix $[\mathbf{K}_e]$:

$$\left[\mathbf{C}_e^R\right] = \alpha[\mathbf{M}_e] + \beta[\mathbf{K}_e]. \tag{14}$$

When substituting into equation (14) $[\mathbf{M}_e]$ from equation (4) and $[\mathbf{K}_e]$ from (10), we obtain the relation for the damping matrix in the following form:

$$\left[\mathbf{C}_e^R\right] = \alpha \int_{\Omega_e} \rho [\mathbf{N}]^T [\mathbf{N}] \, \mathrm{d}\Omega_e + \beta \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{D}] [\mathbf{B}] \, \mathrm{d}\Omega_e. \tag{15}$$

If we compare this with (5), one can see that the first part of the Rayleigh damping matrix $\left[\mathbf{C}_e^R\right]$ given here is identical with $[\mathbf{C}_e]$ as defined in (5), where $c = \alpha\rho$ and its physical nature is unclear. The second part of the expression has a different character. It does not correspond with relation (5), but is proportional to the stiffness matrix. A stiffness matrix in 3D space has a nullity of 6 and when an element moves as a rigid body no internal nodal forces arise. They arise only when the body deforms over time. The damping for $\alpha = 0$ and $\beta > 0$ is thus proportional to the rate of deformation of the body.

2.3. Damping caused by material viscosity

This damping depends on the pertinent time-dependent material model and arises only when a body is deformed. The damping is proportional to the rate of deformation. There are many viscous material models, and we have chosen the three most well-known viscoelastic models for this paper; they are described below.

2.3.1. The Maxwell material model

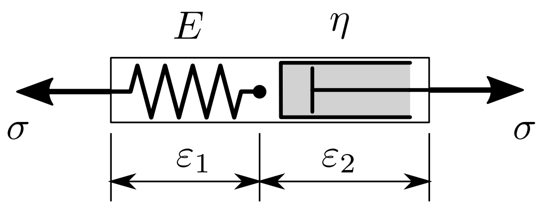

Figure 1: Scheme of the Maxwell model

For the Maxwell model the following relations are valid:

$$\sigma(t) = \sigma^e(t) = \sigma^v(t), \varepsilon(t) = \varepsilon^e(t) + \varepsilon^v(t), \varepsilon^e(t) = \frac{\sigma(t)}{E}, \dot{\varepsilon}^v(t) = \frac{\sigma(t)}{\eta}.$$

As a result we obtain the following linear non-homogenous ordinary differential equation which describes the constitutive relation between stress and strain:

$$\sigma(t) + \frac{\eta}{E}\dot{\sigma}(t) = \eta\dot{\varepsilon}(t).$$

2.3.2. The Kelvin-Voigt material model

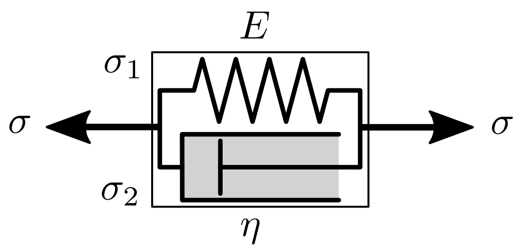

Figure 2: Scheme of the Kelvin-Voigt model

The Kelvin-Voigt model is based on the following relations:

$$\varepsilon(t) = \varepsilon^e(t) = \varepsilon^v(t), \sigma(t) = \sigma^e(t) + \sigma^v(t), \varepsilon^e(t) = \frac{\sigma(t)}{E}, \dot{\varepsilon}^v(t) = \frac{\sigma(t)}{\eta}.$$

Using these relations, we obtain as a result the following linear non-homogenous ordinary differential equation which describes the constitutive relation between stress and strain:

$$\sigma(t) = E\varepsilon(t) + \eta\dot{\varepsilon}(t).$$

2.3.3. The Standard Linear Solid (SLS) material model

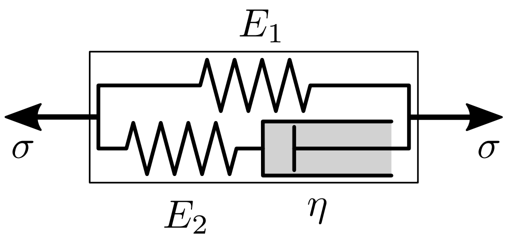

Figure 3: Scheme of the standard model of a solid material

For the SLS model the following relations are valid:

$$\varepsilon(t)=\varepsilon_1(t)=\varepsilon_2(t),\varepsilon_1(t)=\varepsilon_1^e(t)=\frac{\sigma(t)}{E_1},\varepsilon_2(t)=\varepsilon_2^e(t)+\varepsilon_2^v(t),\sigma(t)=\sigma_1(t)+\sigma_2(t),$$

$$\sigma_1(t)=E_1\varepsilon(t)\ \text{ and }\ \sigma_2(t)=E_2\left(\varepsilon(t)-\varepsilon_2^v(t)\right).$$

Then, for the resulting stress the following relations can be written:

$$\sigma(t)=E_1\varepsilon(t)+E_2\left(\varepsilon(t)-\varepsilon_2^v(t)\right).$$

For viscous strain we can write:

$$\dot{\varepsilon}_2^v(t)=\frac{\sigma_2(t)}{\eta}=\frac{\sigma(t)-\sigma_1(t)}{\eta}=\frac{\sigma(t)-E_1\varepsilon(t)}{\eta}.$$

And assuming $\sigma_2(t)=E_2\left(\varepsilon(t)-\varepsilon_2^v(t)\right)$, the following holds:

$$\dot{\sigma}_2(t)=E_2\dot{\varepsilon}(t)-E_2\frac{\sigma(t)-E_1\varepsilon(t)}{\eta}.$$

Using the relation derived above and the relation:

$$\varepsilon(t)=\varepsilon_1(t)=\varepsilon_1^e(t)=\frac{\sigma(t)}{E_1},$$

we obtain

$$\dot{\varepsilon}(t)=\frac{1}{E_1}\dot{\sigma}_1(t)=\frac{1}{E_1}\left[\dot{\sigma}(t)-\dot{\sigma}_2(t)\right]=\frac{1}{E_1}\left[\dot{\sigma}(t)-E_2\dot{\varepsilon}(t)+E_2\frac{\sigma(t)-E_1\varepsilon(t)}{\eta}\right].$$

After several modifications we obtain the final relation for expressing strain rate as a function of stress rate, stress and actual strain (it is a differential equation describing the constitutive relation between stress and strain):

$$\dot{\varepsilon}(t)=\frac{1}{E_1+E_2}\left[\dot{\sigma}(t)+\frac{E_2}{\eta}\sigma(t)-\frac{E_2E_1}{\eta}\varepsilon(t)\right].$$

Let us now demonstrate how the damping caused by the Kelvin-Voigt model will manifest itself in the equation of motion. Let us substitute for $\{\sigma(t)\}$ from (18) into equation (6). For the vector of internal nodal forces we obtain:

$$\begin{aligned}\left\{\mathbf{f}_e^{\text{int}}\right\} &= \int_{\Omega_e} [\mathbf{B}]^T \{\{\sigma^e\} + \eta\{\dot{\varepsilon}\}\} \,\mathrm{d}\Omega_e = \int_{\Omega_e} [\mathbf{B}]^T \left\{\{\sigma^e\} + \eta [\mathbf{B}] \left\{\dot{\mathbf{d}}\right\}\right\} \mathrm{d}\Omega_e = \\ &= \int_{\Omega_e} [\mathbf{B}]^T \{\sigma^e\} \,\mathrm{d}\Omega_e + \eta \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{B}] \left\{\dot{\mathbf{d}}\right\} \mathrm{d}\Omega_e. \end{aligned} \tag{16}$$

Taking into consideration the fact that the damping will be caused by the material, we will not expect damping to take place with the help of matrix $[\mathbf{C}]$ here. The equation of motion shall then read:

$$[\mathbf{M}_e]\left\{\ddot{\mathbf{d}}\right\} + \int_{\Omega_e} [\mathbf{B}]^T \{\sigma^e\} \,\mathrm{d}\Omega_e + \eta \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{B}] \left\{\dot{\mathbf{d}}\right\} \mathrm{d}\Omega_e = \left\{\mathbf{f}_e^{\text{ext}}\right\}. \tag{17}$$

The equation can then be rewritten for a linear elastic material as:

$$\left\{\mathbf{f}_e^{\text{int}}\right\} = [\mathbf{K}_e]\{\mathbf{d}\} + \eta \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{B}] \left\{\dot{\mathbf{d}}\right\} \mathrm{d}\Omega_e, \tag{18}$$

$$[\mathbf{M}_e]\left\{\ddot{\mathbf{d}}\right\} + \left\{\mathbf{f}_e^{int}\right\} = \left\{\mathbf{f}_e^{ext}\right\} \tag{19}$$

and after the substitution for $\left\{\mathbf{f}_e^{\text{int}}\right\}$, we can rewrite the equation of motion in the form

$$[\mathbf{M}_e]\left\{\ddot{\mathbf{d}}\right\} + [\mathbf{K}_e]\{\mathbf{d}\} + \eta \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{B}] \left\{\dot{\mathbf{d}}\right\} \mathrm{d}\Omega_e = \left\{\mathbf{f}_e^{\text{ext}}\right\}. \tag{20}$$

2.4. Comparison of Rayleigh damping with the damping of the Kelvin-Voigt material

From the character of the term of the Rayleigh damping with the coefficient α which was discussed earlier, it follows that damping depends only on the velocities of mass points in space, and that damping also arises in the case of the movement of a rigid body. Because in the case of damping caused by material viscosity no damping effect arises, the comparison of Rayleigh damping due to coefficient α with the damping of the Kelvin-Voigt material does not make sense. For a comparison between Rayleigh and the Kelvin-Voigt material let us assume $\alpha = 0$. The equation of motion then will read:

$$[\mathbf{M}_e]\left\{\ddot{\mathbf{d}}\right\} + \beta \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{D}] [\mathbf{B}] \,\mathrm{d}\Omega_e \left\{\dot{\mathbf{d}}\right\} + \int_{\Omega_e} [\mathbf{B}]^T \{\sigma\} \,\mathrm{d}\Omega_e = \left\{\mathbf{f}_e^{\text{ext}}\right\}. \tag{21}$$

The comparison of this equation with the equation of motion for the Kelvin-Voigt material shows that the difference between them lies only in the term with the first derivative of the deformation parameters by time, i.e. the velocities. For 1D this term will be simplified, substituting Young's modulus E for the constitutive matrix $[\mathbf{D}]$. Comparing the terms with $\left\{\dot{\mathbf{d}}\right\}$ from the equations of motion for the Rayleigh and Kelvin-Voigt material we obtain the relation:

$$\beta E \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{B}] \, \mathrm{d}\Omega_e \left\{\dot{\mathbf{d}}\right\} = \eta \int_{\Omega_e} [\mathbf{B}]^T [\mathbf{B}] \, \mathrm{d}\Omega_e \left\{\dot{\mathbf{d}}\right\}. \tag{22}$$

And finally we obtain a simple relation between the parameters β and η of both models:

$$\beta E = \eta. \tag{23}$$

For 1D and a linear material, these two models, i.e. Rayleigh damping and the Kelvin-Voight material model, are identical. Through numerical comparisons it can be shown that formula (23) is sufficiently precise for a linear material even for 2D or 3D models.

2.5. Damping caused by the material plasticity

Fig. 4 shows a loading and unloading diagram for plastic material for the 1D stress and strain state. There it can be seen that a substantial part of deformation work is connected with plastic work, w^p:

$$w^p = \int_0^t \sigma : \dot{\varepsilon}^p \, \mathrm{d}t. \tag{24}$$

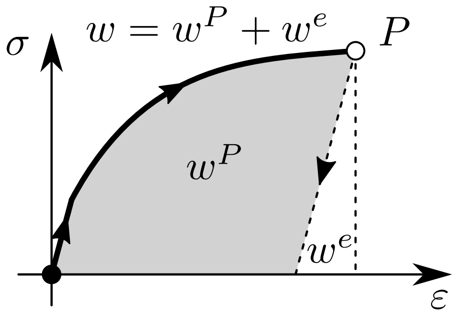

Figure 4: Loading and unloading diagram for plastic material, with division of energy into elastic and plastic (dissipative)

The total energy used for deformation work can be divided into elastic energy w^e and plastic energy w^p, which dissipate in the form of heat. This part of the energy is lost from the mechanical system, a process which manifests itself in dynamics as damping. This damping occurs at the time when plastic deformation arises, and later vibration is no longer damped unless the maximum strain achieved so far is overcome once again.

2.6. Damping caused by friction

Damping by friction most often comes about in screw or rivet connections in steel structures. In contrast to plastic behavior, mechanical work is performed with each relative forward and backward motion, and the pertinent damping is then permanent during vibration. Mechanical work and pertinent dissipation arise in the case of any relative motion in the connections. The friction force is usually proportional to the pressure force in connections (Coulomb friction).

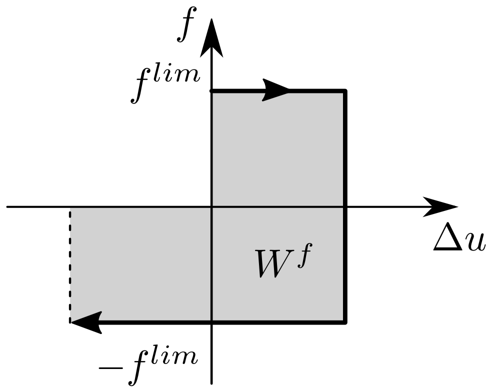

Figure 5: Friction diagram

3. Numerical results

The simulation tests were performed on high, slim structure, which could represent e.g. chimney. The example structure is shown in Fig. 6 together with distribution of the damping forces pertaining to the Rayleigh α and β coefficient along the height of the structure.

The equivalence between the Rayleigh parameter β and the Kelvin-Voight parameter η, see (23), is shown in the Fig. 7. When the relation (23) is applied, the vibration course is almost identical even for 2D.

Typical course of vibration for the Maxwell viscous model is shown in the Fig. 8. In the time course we can see the yielding which would for infinite time converge to infinity.

Unlike the Kelvin-Voigt material, which can well model damping of vibration, and the Maxwell material with unrealistic yielding, the SLS model can very well model the real behavior of materials and can be recommended by the authors of the paper for nonlinear dynamic analysis of buildings. Typical course for the SLS material, with damping and also with the global yielding in the beginning, is shown in the Fig. 9.

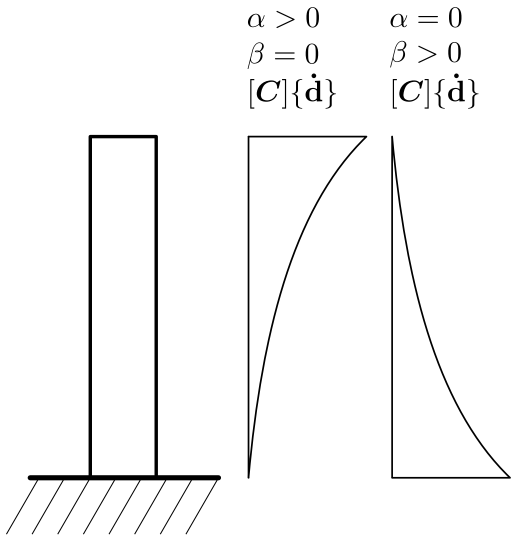

Figure 6: Scheme of the solved structure and the distribution of the damping forces

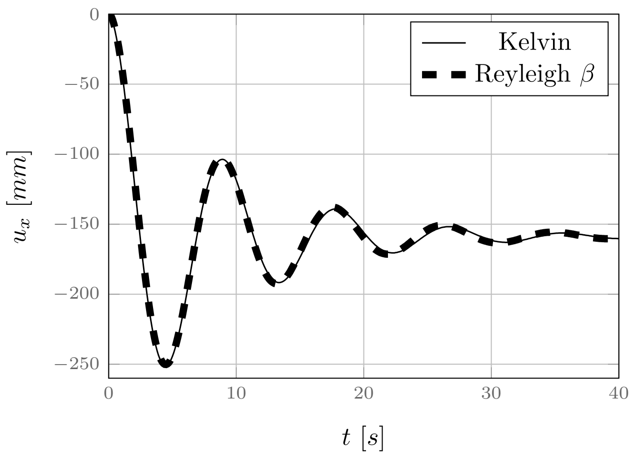

Figure 7: Vibration for the Rayleigh parameter β and the Kelvin-Voight param. η

Conclusion

The use of the most commonly employed damping model in the field of the dynamical analysis of buildings, Rayleigh damping, seems after more detailed investigation to be unjustified from the physical point of view. In linear mechanics its use has certain advantages because it does not influence the frequency of vibration, which remains the same as when damping does not occur. However, its use today in nonlinear dynamics is unjustified and a transition to the introduction of the real

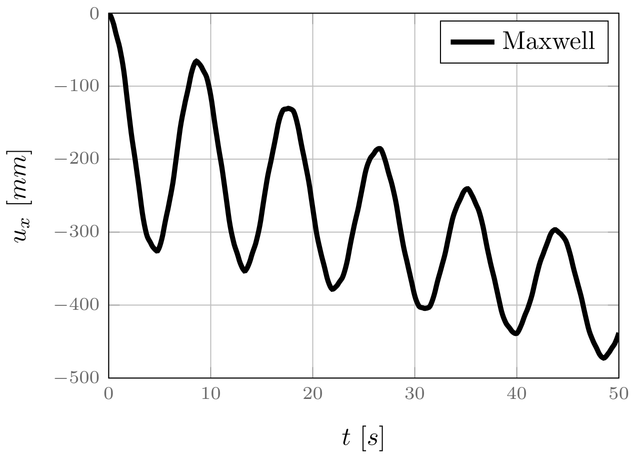

Figure 8: Typical course of vibration for the Maxwell viscous model

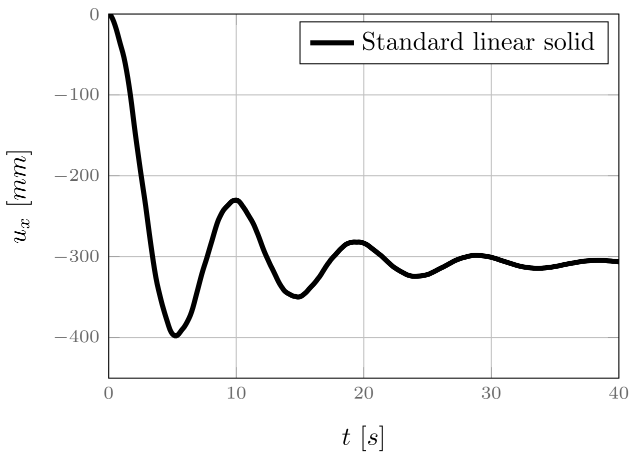

Figure 9: Typical course of vibration for the SLS model

physical causes of damping is desirable. Structures often consist of different materials which are defined by different plastic and time-dependent material models. The use of traditional Rayleigh damping does not provide sufficiently good results such as those that it is possible to obtain via contemporary state of the art methods. Moreover, the setting the Rayleigh damping parameters is in design praxis estimated for particular type of structures, instead of a material property as it should be. Behavior of material elements depends only on the used material and on the rate of deformation rather than on a type of the structure. The paper shows that the Rayleigh β coefficient has the similar physical meaning as the viscosity η of the Kelvin-Voigt material. For 1D these coefficients are identical. But the Rayleigh α coefficient is physically unjustified. The viscous damping of the moving rigid body cannot be proportional to the moving mass quantity. Behavior of real materials is more complex, than it is possible to model by the Kelvin-Voigt model and a better material model, e.g. standard linear solid, should be applied in nonlinear time dependent analysis. Regarding the fact that some computer programs for structural analysis (e.g. RFEM) which are widely used in structural design enable more advanced analysis by taking into account the real sources of damping, i.e. material viscosity, material plasticity and friction in connections and supports, it is strongly recommended that the traditional modeling of damping be replaced in dynamical time analysis by more modern and more exacting solutions currently available to structure designers.

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