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NUMERICAL SIMULATION OF GENERALIZED NEWTONIAN FLUIDS FLOW IN BYPASS GEOMETRY

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Abstract: The aim of this work is to present numerical results of non-Newtonian fluid flow in a model of bypass. Different angle of a connection between narrowed channel and the bypass graft is considered. Several rheology viscosity models were used for the non-Newtonian fluid, namely the modified Cross model and the Carreau-Yasuda model. The results of non-Newtonian fluid flow are compared to the results of Newtonian fluid. The fundamental system of equations is the generalized system of Navier-Stokes equations for incompressible laminar flow. Generalized Newtonian fluids flow in the bypass is numerically simulated by using an open source CFD tool, OpenFOAM.

Keywords: generalized Newtonian fluids, Navier-Stokes equations, OpenFOAM, bypass

MSC: 65L06, 65N08, 76A05, 76A10, 76D05

1. Introduction

The diseases of arteries causes approximately 31 % of all global deaths. Cardiovascular diseases belong to the category of the diseases that refer to the heart and blood vessels, e.g. hyperthension, heart attack, artherosclerosis or stroke etc. Cardiovascular diseases are mainly caused by a formation of sediments on the inner wall of the vessel, which can restrict the blood flow rate. In the case of a narrowing of a vessel, it is necessary to proceed with a medication. One way is to bridge the narrowing place by the graft, such a bridge is called bypass. The quality of the blood flow in the bypass can be influenced by geometry, e.g. by the angle of connection, see [10].

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Further, the flow in the bypass can be affected by characteristics of blood. Blood is a red coloured liquid in humans which is composed of blood cells (red blood cells and white blood cells and platelets) suspended in a plasma. The density of the blood is in the range between 1043 to 1066 $\mathrm{kg\,m^{-3}}$ depending on gender, health etc. The blood cells comprise 45 % of the blood fluid [2], [3]. Blood as fluid can be characterized as a shear thinning fluid, see e.g the non-Newtonian models in [4], [5], [17]. Nevertheless various researchers study the blood as the Newtonian fluid [6], [7].

This paper used the geometry of the bypass with the different angle of connection between the narrowing channel and the graft. Three rheological viscosity models are used: the Newtonian model, the Carreau-Yasuda model and the modified Cross model. The numerical results are shown.

2. Mathematical model

Let us consider the idealized case when the blood flow is laminar and the fluid is incompressible with the constant density ρ and the shear dependent dynamic viscosity $\mu = \mu(\dot{\gamma})$ depending on the shear rate $\dot{\gamma}$ (see [13]) defined by

$$\dot{\gamma} = 2\sqrt{\frac{1}{2}\mathrm{tr}\ \boldsymbol{D}^2}, \tag{1}$$

where $\boldsymbol{D} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$, $\boldsymbol{u}$ is the velocity vector. For the Newtonian fluid the viscosity model reads (for more details see [3], [16])

$$\mu(\dot{\gamma}) = \mu_\infty, \tag{2}$$

whereas for the generalized Newtonian fluid one of the following viscosity models can be applied, see [8]:

- the modified Cross model

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty)\left[1 + (\lambda\dot{\gamma})^b\right]^{-a}, \tag{3}$$

- the Carreau-Yasuda model

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty)\left[1 + (\lambda\dot{\gamma})^m\right]^{(n-1)/m}, \tag{4}$$

where μ_0 and μ_∞ are the asymptotic viscosity values at zero and infinite shear rates. The symbol λ denotes a relaxation time and a, b, m, n are parameters of the non-Newtonian viscosity models, see Table 1 (see [5], [11], [14], [15]). In Fig. 1 the relationship between the viscosity μ and the shear rate $\dot{\gamma}$ for selected viscosity models is presented.

Let us consider the blood flow in a bounded three dimensional computational domain $\Omega \subset \mathcal{R}^3$ with its boundary $\partial\Omega = \partial\Omega_I \cup \partial\Omega_O \cup \partial\Omega_W$, where $\partial\Omega_I$, $\partial\Omega_O$ and $\partial\Omega_W$ denote the inlet, the outlet and the wall parts of the boundary $\partial\Omega$, respectively.

viscosity model	parameters
Newtonian model	$\mu_\infty = 3.5 \times 10^{-3}\,\mathrm{Pa\,s}$
modified Cross model	$\mu_\infty = 3.5 \times 10^{-3}\,\mathrm{Pa\,s}$, $\mu_0 = 160 \times 10^{-3}\,\mathrm{Pa\,s}$, $\lambda = 8.2\,\mathrm{s}$, $a = 1.23$, $b = 0.64$
Carreau-Yasuda model	$\mu_\infty = 3.45 \times 10^{-3}\,\mathrm{Pa\,s}$, $\mu_0 = 56 \times 10^{-3}\,\mathrm{Pa\,s}$, $\lambda = 1.902\,\mathrm{s}$, $m = 1.25$, $n = 0.22$

Table 1: Characteristics of the presented viscosity models

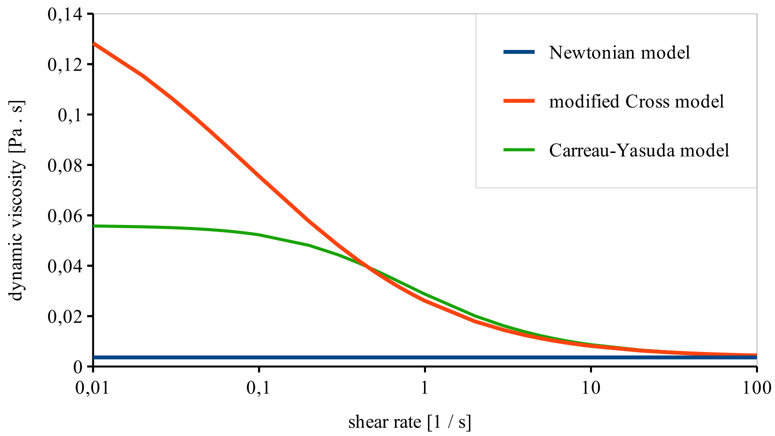


Figure 1: The relationship between the dynamic viscosity μ and the shear rate $\dot{\gamma}$ for the chosen viscosity models

The fundamental system of equations describing the motion of blood in the arteries is based on the system of balance laws of mass and momentum. The generalized system of Navier-Stokes equations can be written in the form as

$$\begin{aligned} \operatorname{div}\,\boldsymbol{u} &= 0, \\ \rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} &= -\nabla P + \operatorname{div}\,(2\mu(\dot{\gamma})\boldsymbol{D}), \end{aligned} \tag{5}$$

where P is the dynamic pressure, ρ is the constant density, $\boldsymbol{u}$ is the velocity vector, $\mu(\dot{\gamma})$ denotes the dynamic viscosity of the generalized Newtonian fluid given by one of the Eqs. (2)–(4) and $\boldsymbol{D}$ is the symmetric part of the velocity gradient, see [1], [9], [18].

System of equations (5) is equipped with an initial condition $\boldsymbol{u}(x, 0) = \boldsymbol{u}_0(x)$ and with the boundary conditions specified at $\partial\Omega$. At the inlet, a Dirichlet boundary condition for the velocity vector is used. At the outlet part, the pressure value is prescribed and the no-slip boundary condition for the velocity vector is used on the wall.

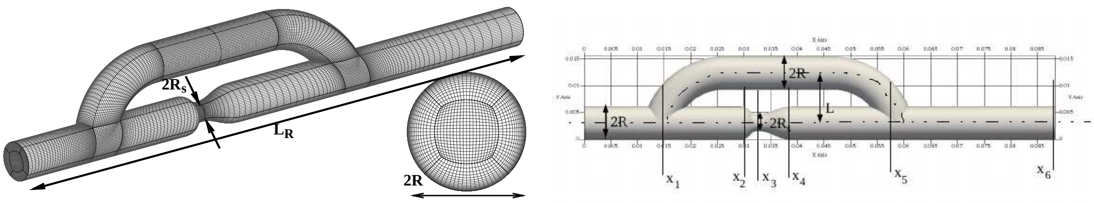


Figure 2: Unstructured hexahedral computational mesh (left) and the sketch of the computational domain (right)

3. Numerical results

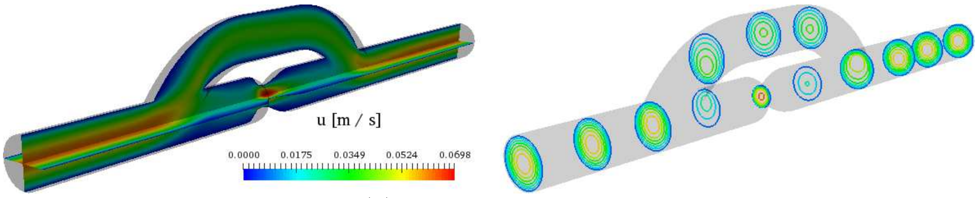
The generalized Newtonian fluid flow in the bypass tube is numerically simulated by using the open source CFD tool, OpenFOAM, where the SIMPLE algorithm is used for the numerical solution [12].

The computational domain Ω as a model of the bypass geometry is shown in Fig. 2. It is described by the parameters R, L, R_s, L_R. R denotes the radius of the main channel, $R = 0.0031\,\mathrm{m}$ and L_R denotes the length of the channel with bypass, $L_R = 28.5\,R$. The radius of the narrow part of the channel is R_s, ($R_s = 0.5\,R$). The sketch of the computational domain is presented in Fig. 2 (right). The geometry of the bypass channel is described by the parameters L, x_1, x_2, x_3, x_4, x_5, x_1 given as $L = 6\,R$, $x_1 = 5\,R$, $x_2 = x_1 + 4.5\,R$, $x_3 = x_2 + R$, $x_4 = x_3 + 2\,R$, $x_5 = x_4 + 6\,R$ and $x_6 = x_5 + 10\,R$. The angle of the connection between the narrowed channel and the bypass were considered in the range from 20 to 70 degrees. The computational domain is discretized using an unstructed mesh composed of hexahedral cells, see Fig. 2 (left).

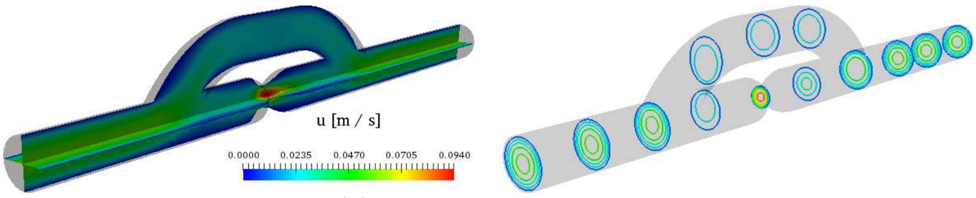
The fluid is described by the constant density $\rho = 1050\,\mathrm{kg\,m^{-3}}$ and the viscosity model specified by the parameters summarized in Table 1. At the inlet a fully developed flow is assumed. In the case of the Newtonian fluid, the parabolic velocity profile with the maximum velocity value U_0, $U_0 = 0.0615\,\mathrm{m\,s^{-1}}$, is defined at the inlet. A constant pressure value is prescribed at the outlet.

Figs. 3 and 4 show the axial velocity distribution in 3D (left) and the velocity isolines in the cross sections of the bypass and of the stenosed vessel (right) for the considered viscosity models. The results are shown for the two angles of connection (Fig. 3 shows 30 degrees and Fig. 4 shows 60 degrees).

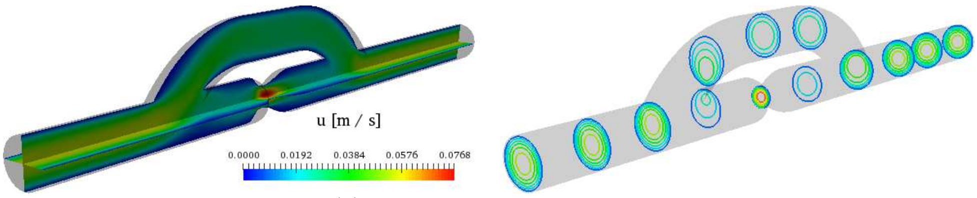
Fig. 5 represents the velocity distribution along the axis of the main channel for the tested viscosity models and in dependence on the angle of the connection. It can be observed that for a smaller angle the numerical results are very similar for the Newtonian and the Carreau-Yasuda viscosity models, whereas for the modified Cross model the differences are obvious for any angle. In the case of Newtonian viscosity model the peak of the velocity distribution has the same value for all tested angles. Some differences between the non-Newtonian viscosity models appear with higher angles, namely the values of the maximal velocity are different. In the case of the



(a) Newtonian model



(b) modified Cross model



(c) Carreau-Yasuda model

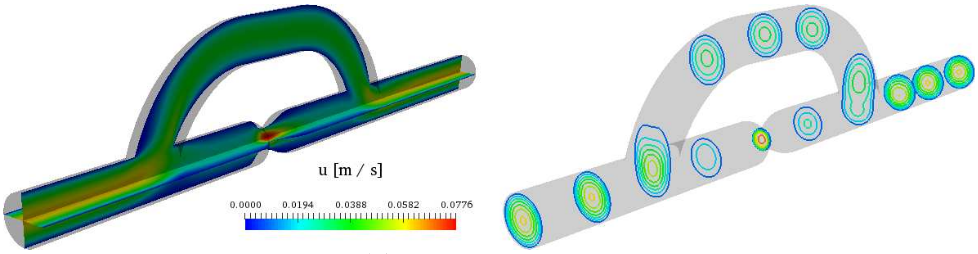
Figure 3: Axial velocity distribution in the center-plane area (left) and velocity isolines in the selected cross section (right) for the angle of 30 degrees

modified Cross model the peak of the velocity distribution along the axis is higher than for the other viscosity models.

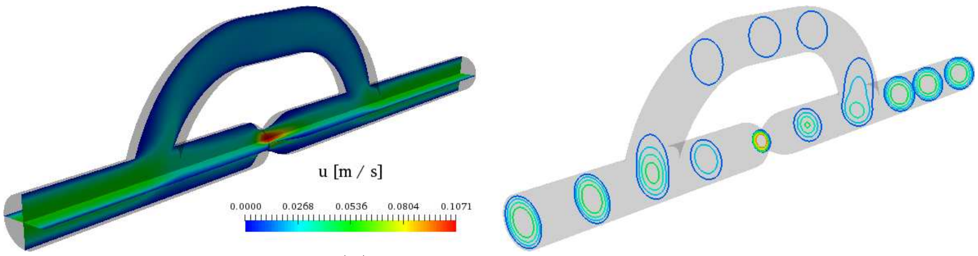
4. Conclusion

In this paper the numerical results for generalized Newtonian fluids flow in the bypass geometry were presented. The tests were performed on a model of bypass geometry, where the different angles of the connection between the narrowed channel and the bypass graft were considered. Several viscosity models were used as the Newtonian model, the modified Cross model and the Carreau-Yasuda model. Numerical results were obtained using the SIMPLE algorithm included in the OpenFOAM and the generalized Newtonian fluid model was used. Two selected viscosity models were implemented into the OpenFOAM, namely the modified Cross model and the Carreau-Yasuda model.

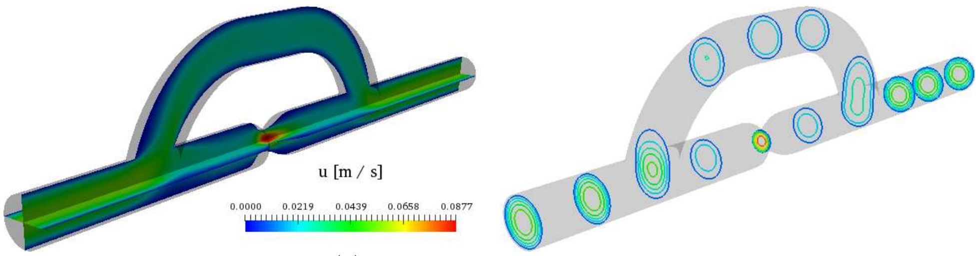
The results show that for the considered angle of connection (40–50 degrees) the differences between Newtonian and non-Newtonian models are not significant. And thus the use of Newtonian model is reasonable there. On the other hand for other angles (less than 40 deg or higher than 50 deg) the influence of the non-Newtonian character of the fluid becomes more important and thus needs to be taken into account.



(a) Newtonian model



(b) modified Cross model



(c) Carreau-Yasuda model

Figure 4: Axial velocity distribution in the center-plane area (left) and velocity isolines in the selected cross section (right) for the angle of 60 degrees

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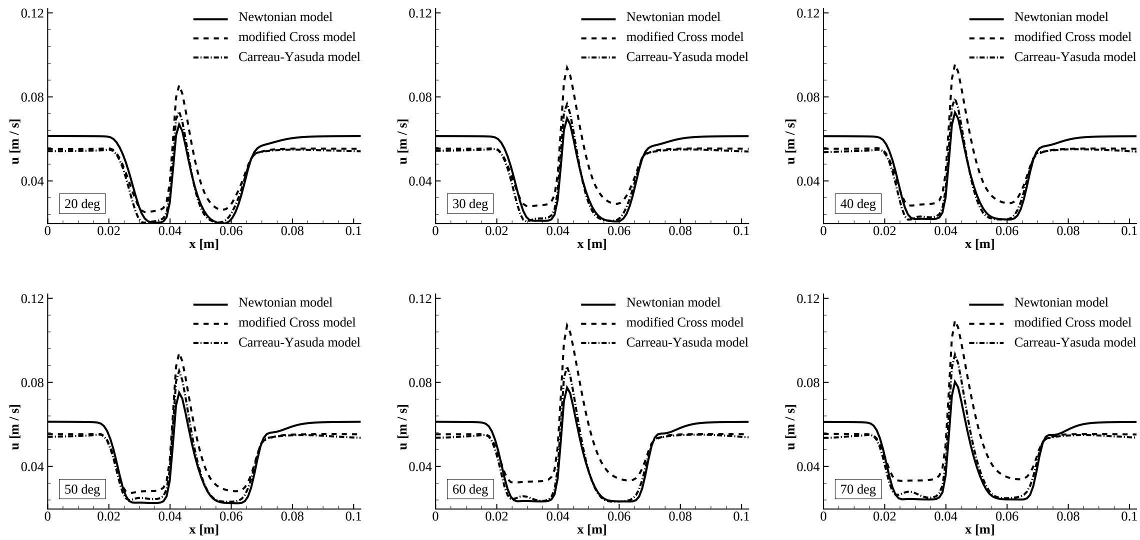

Figure 5: Velocity distribution along the axis of the main channel for the different angles

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