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SCALABLE ALGORITHMS FOR CONTACT PROBLEMS WITH GEOMETRICAL AND MATERIAL NON-LINEARITIES*

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1. Introduction

Contact modelling is still a challenging problem of non-linear computational mechanics. The complexity of such problems is related to the a priori unknown contact interface and contact tractions. Their evaluations have to be part of the solution. In addition, the solution across the contact interface is non-smooth.

FETI (Finite Element Tearing and Interconnecting) method [1] belongs to the class of non-overlapping spatial domain decomposition method. Its key concept stems from the idea that the spatial sub-domains, into which the domain is partitioned, are 'glued' by Lagrange multipliers. After eliminating the primal variables, which are displacements, the original problem is reduced to a small, relatively well conditioned, typically equality constrained quadratic programming problem that is solved iteratively. The CPU time that is necessary for both the elimination and iterations can be reduced nearly proportionally to the number of the processors, so that the algorithm exhibits parallel scalability. Observing that the equality constraints may be used to define so called 'natural coarse grid', Farhat, Mandel and Roux modified the basic FETI algorithm so that they were able to prove its numerical scalability, i.e. asymptotically linear complexity.

If the FETI method is applied to the contact problems, the same methodology can be used to prescribe conditions of non-penetration between bodies.

After brief theoretical introduction, this paper is concerned with demonstration of scalability of a new variant of the FETI domain decomposition method, called TFETI (Total FETI) method, and application of the classic FETI method to the solution to contact problems with other non-linearities.

2. Theoretical background

Let us consider a contact problem between two solid deformable bodies. This is basically the boundary value problem known from the solid mechanics. The problem is depicted in Figure 1. Two bodies are denoted by $(\Omega_1, \Omega_2) \subset \mathbf{R}^n, n = 2$ or $n = 3$ where n stands for number of spatial dimensions. Γ stands for boundaries of the bodies that are sub-divided into three disjoint parts. The Dirichlet and Neumann boundary conditions are prescribed on the parts Γ^u and Γ^f, respectively. The third

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type of the boundary condition, Γ^c, is defined along the contact interface. The mathematical description of the problem is given by the governing equations expressing equilibrium conditions of the system, along with the boundary conditions.

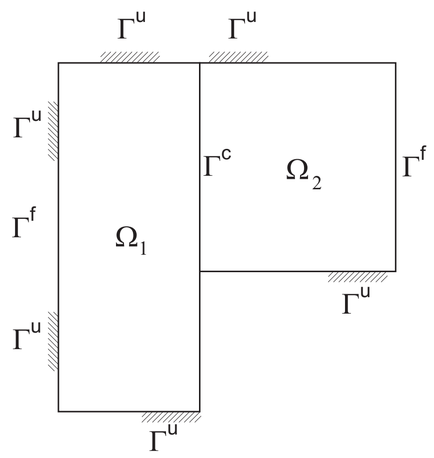

Fig. 1: *Contact problem.*

The result of application of the classic FETI method to the system of bodies from Figure 1 is depicted in Figure 2. The sub-domain Ω_1 is decomposed into two sub-domains with fictitious interface between them.

The fundamental idea of the FETI method is that the compatibility between sub-domains is ensured by means of the Lagrange multipliers or forces in this context. In Figure 2, λ^E denotes the forces along the fictitious interface and λ^I stands for the forces generated by contact.

The original FETI method assumes that Dirichlet boundary conditions are inherited from the original problem, which is shown in Figure 2. This fact implies that the defect of the stiffness matrices of individual sub-domains may vary from zero, for the sub-domains with enough Dirichlet conditions, to the maximum (6 for 3D solid mechanics problems and 3 for 2D ones) in the case of sub-domains exhibiting some rigid body modes. General solution to such systems requires computation of a generalised inverse and a basis of the null spaces of the underlying singular matrices. The problem is that the magnitudes of the defects are difficult to obtain because this computation is disposed to the round off errors [2].

To circumvent the problem, Dostál came up with a novel solution [3]. His idea was to release all prescribed Dirichlet boundary conditions and enforce them by the Lagrange multipliers as it is shown in Figure 3. The effect of the procedure on the stiffness matrices of the sub-domains is that their defects are the same and its magnitude is known beforehand.

The mathematical description of the FETI method can be found, e.g., in [4] and the TFETI method in [3].

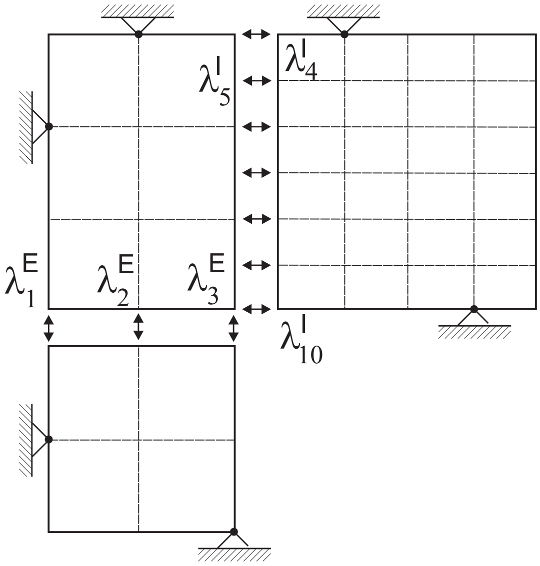

Fig. 2: *FETI method.*

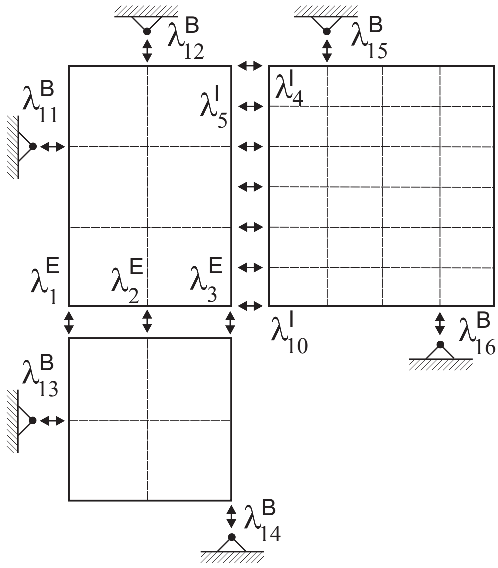

Fig. 3: *TFETI method.*

Application of the FETI and TFETI methods to the contact problems converts the original problem to the quadratic programming one with simple bounds and equality constraints. This problem is further transformed by Semi-Monotonic Augmented Lagrangians with Bound and Equality constraints (SMALBE) method to the sequence of simply bounded quadratic programming problems. These auxiliary problems may be solved efficiently by the Modified Proportioning with Reduced Gradient Projection (MPRGP) method which is described in more details in [5]. It was proved in [6] that application of combination of both these methods to solution to contact problems benefit the numerical and parallel scalability.

We extended the FETI and TFETI method to problems with the geometric and material non-linearities. The above mentioned approach is directly applicable to solution to the contact problems, but with other conditions linear, i.e. for linear elasticity with small displacements and rotations, and frictionless contact. Any additional non-linearity necessitates employment of the nested iteration strategy, where the outer loop is concerned with the material and geometric non-linear effects, contact geometry update, and equilibrium iterations.

3. Numerical experiments

We shall show results of three sets of numerical experiments we carried out. The first one documents numerical scalability of the FETI and TFETI methods. The second case is concerned with contact problem of two cylinders, and the third one with contact problem of the pin in hole with small clearance.

Numerical experiments in the second and third cases were carried out with our general purpose finite element package PMD [7].

3.1. Poisson's problem

Consider a Poisson's problem $\triangle u = 1 \quad \text{in} \quad \Omega$, where $\Omega = (0,1) \times (0,1)$. Dirichlet boundary conditions are prescribed along one edge of the domain, and Neumann conditions along remaining edges. This scalar boundary value problem can be interpreted as the deformation perpendicular to the domain for a thin membrane under lateral pressure, while the physical meaning of the right hand side is the applied pressure divided by membrane tension per unit length. We used bilinear quadrilateral elements for discretisation of the problem.

We carried out a series of computations with changing decomposition parameter H and discretisation parameter h. The results are summarised in Table 1.

H	h	prim.	dual FETI	dual TFETI	CG steps FETI	CG steps TFETI
1/2	1/4	36	11	17	7	4
1/4	1/8	144	63	75	12	5
1/8	1/16	576	287	311	13	7
1/16	1/32	2304	1215	1263	15	11
1/2	1/8	100	19	29	9	9
1/4	1/16	400	111	131	16	12
1/8	1/32	1600	511	551	18	16
1/16	1/64	6400	2175	2255	20	21
1/2	1/16	324	35	53	14	9
1/4	1/32	1296	207	243	22	14
1/8	1/64	5184	959	1031	24	20
1/16	1/128	20736	4095	4239	23	23

Tab. 1: *Scalability of FETI and TFETI.*

The table also shows numbers of primal variables and numbers of dual variables for both FETI and TFETI. We observe from the last two columns that performances of FETI and TFETI are close and that both algorithms exhibit the numerical scalability as can be seen from number of the conjugate gradient (CG) steps.

Figure 4 shows the case corresponding to the first line in Table 1, i.e. $H = 1/2$ and $h = 1/4$. There are four sub-domains there, each with nine primal variables so that the total number is 36. The FETI dual variables are explicitly depicted. The number of the TFETI dual variables is obtained as the sum of the FETI dual variables and the Dirichlet boundary conditions, which are indicated by triangles.

3.2. Contact problem of two cylinders

Consider contact of two cylinders with parallel axes. The diameter of the upper cylinder $R_u = 1$ m and of the lower one $R_l = \infty$. In spite of the fact that it is a 2D problem, it is modelled with 3D continuum trilinear elements with two layers

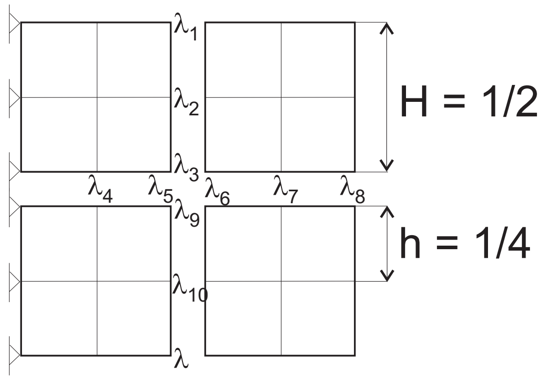

Fig. 4: *Decomposition and discretisation of the domain.*

of them along the axis of symmetry of the upper cylinder. Nevertheless, it is clear that number of layers is irrelevant. The boundary conditions are imposed in such a way that from the physical viewpoint it is the plane strain problem. The model consists of 8904 elements and 12765 nodes. The upper cylinder is loaded by 40 MN/m along the upper line of the upper cylinder.

Figure 5 shows solution to linearly elastic and linearly geometric problem in terms of the deformed mesh. The material properties are as follows: Young's modulus $E = 2.0 \times 10^{11}$ Pa and Poisson's ratio $\nu = 0.3$.

The second problem was computed on the same mesh with the same loading, but we considered linearly–elastic–perfectly–plastic material with yield stress $\sigma_Y =$ 800 MPa. We also considered the geometric non-linearity, i.e. large displacements and finite rotations. The deformed mesh is depicted in Figure 6.

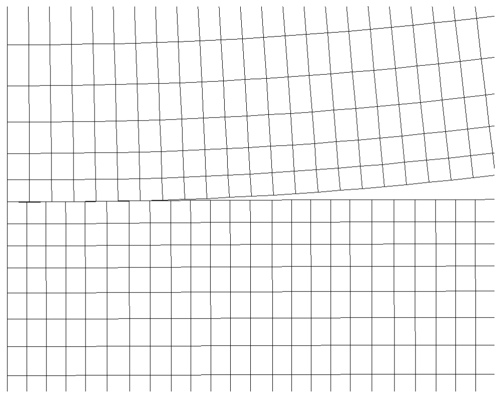

Fig. 5: *Deformed mesh, linear problem.*

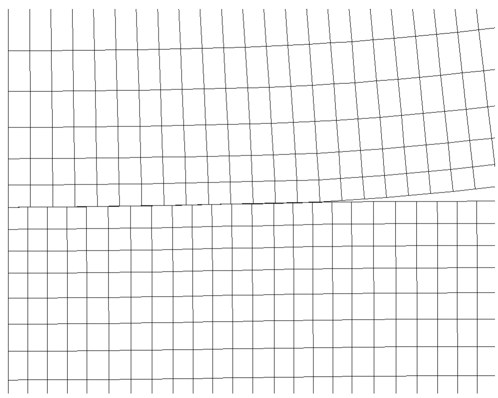

Fig. 6: *Deformed mesh, non-linear problem.*

3.3. Pin-in-hole contact problem

Consider problem of a circular pin in circular hole with small clearance. The radius of the hole is 1 m and the pin has its radius by 1% smaller. Again, the 2D problem is modelled with 3D trilinear elements. The model consists of 15844 elements and 28828 nodes. The pin is loaded along its centre line by 133 MN/m. The geometric non-linearity was considered. The material properties are the same as in the previous case.

Figure 7 shows von Mises stress distribution on the deformed mesh.

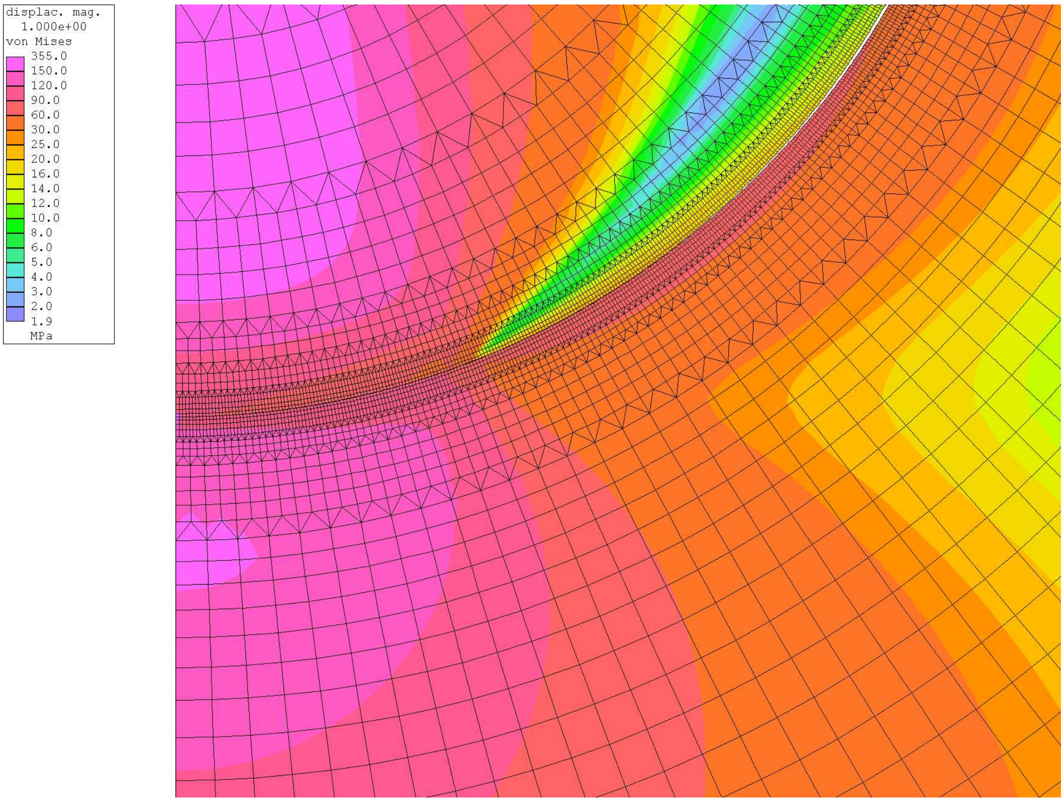

Fig. 7: *Deformed mesh, non-linear problem, von Mises stress.*

4. Conclusion

A new variant of the original FETI domain decomposition method was presented. It is called TFETI and its basic idea, in comparison with FETI, consists in replacement of Dirichlet boundary conditions by Lagrange multipliers or forces in this context. It is of great importance from the computational point of view, because the defect of stiffness matrices of all sub-domains is the same and its magnitude is known beforehand. Numerical experiments show that algorithm stemming from TFETI exhibits the numerical scalability. We also show results of solution to contact problems by the FETI method.

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