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ENVELOPE CONSTRUCTION OF TWO-PARAMETERIC SYSTEM OF CURVES IN THE TECHNOLOGICAL PRACTICE

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Abstract

A two-parametric system of close planar curves is defined in the introduction of the presented article. Next a theorem stating the existence of the envelope is presented and proved. A mathematical model of the collecting mechanism of the Horal forage trailer is developed and used for practical demonstrations. The collecting mechanism is a double joint system composed of three rods. An equation describing the trajectory of a random point of the working rod is derived using Maple. The trajectories of two close points of the working rod create a planar system of close curves, of which the envelope can be computed. As this computation is extremely complex, MAPLE was used to optimize the computations. Through this optimization the computation needed less memory and the processing time was shorter. In the final part the working areas and the corresponding envelopes of all rods defining the collecting mechanism are plotted.

1. Theoretical introduction

Definition of the planar close curves: *Smooth and continuous two-parametric curves $[x(p,q), y(p,q), z]$ and $[x(P,Q), y(P,Q), z]$ are planar close curves if:*

$$(Q = q + \Delta q, \ \Delta q \to 0, \quad \text{or} \quad P = p + \Delta p, \ \Delta p \to 0) \quad \text{and} \quad z \equiv 0.$$

Theorem 1 (Condition of the envelope existence)**.** *Let $[x(p,q), y(p,q)]$ be a system of close planar curves. The envelope exists if and only if the parameters p and q satisfy (see [2]):*

$$\frac{\dfrac{\partial y(p,q)}{\partial p}}{\dfrac{\partial x(p,q)}{\partial p}} = \frac{\dfrac{\partial y(p,q)}{\partial q}}{\dfrac{\partial x(p,q)}{\partial q}} \iff \begin{vmatrix} \dfrac{\partial x(p,q)}{\partial q} & \dfrac{\partial y(p,q)}{\partial p} \\ \dfrac{\partial x(p,q)}{\partial q} & \dfrac{\partial y(p,q)}{\partial p} \end{vmatrix} = 0 \ .$$

Proof: Let us consider two curves belonging to above stated system. The second curve can be written as $[x(P,Q), y(P,Q)]$ and the parameters values p, P, q, Q correspond to their intersection point. If these curves are close, the intersection point is close to the contact point of the curves with their envelope. The intersection point must satisfy:

$$x(p,q) = x(P,Q) \quad \text{and} \quad y(p,q) = y(P,Q).$$

The curves are close, thus satisfying the definition (1),

$$x(P,Q) = x(p + \Delta p, q + \Delta q) \quad \text{and} \quad y(P,Q) = y(p + \Delta p, q + \Delta q). \tag{1}$$

Expanding (1) in the Taylor series leads to:

$$
\begin{aligned}
x(p,q) &= x(p,q) + \sum_{i=1}^{\infty} \frac{\partial^i x(p,q)}{i! \, \partial p^i} (\Delta p)^i + \sum_{i=1}^{\infty} \frac{\partial^i x(p,q)}{i! \, \partial q^i} (\Delta q)^i \\
y(p,q) &= y(p,q) + \sum_{i=1}^{\infty} \frac{\partial^i y(p,q)}{i! \, \partial p^i} (\Delta p)^i + \sum_{i=1}^{\infty} \frac{\partial^i y(p,q)}{i! \, \partial q^i} (\Delta q)^i
\end{aligned}
\tag{2}
$$

If $i \geq 2$ we can neglect the powers $(\Delta p)^i, (\Delta q)^i$. Therefore the condition simplifies (2) to

$$\frac{\partial x(p,q)}{\partial p} \Delta p + \frac{\partial x(p,q)}{\partial q} \Delta q = 0 \quad \text{and} \quad \frac{\partial y(p,q)}{\partial p} \Delta p + \frac{\partial y(p,q)}{\partial q} \Delta q = 0. \tag{3}$$

From the equations (3) the binding condition follows easily.

2. Kinematics of pick up vehicle Horal collector mechanism

The Horal pick up vehicle is a drawn forage wagon for collecting grass, hay or straw. The basic constructive scheme of the collecting mechanism is depicted in the Figure 1, see [3] for the technical parameters. The axis of the drive shaft was chosen as origin of the coordinate system. The propelling handle r, which is at the point $B \equiv [B_x(t), B_y(t)]$, is linked by a joint to the two-part work rod $L2$–$L3$. These two parts form an angle f. The end of the upper part of the work rod with length $L2$ is joined at the point $C \equiv [C_x(t), C_y(t)]$ with an additional handle of length $L1$ which can freely rotate with its other end around point of coordinates $A \equiv [A_x, A_y]$. The free working point, marked with P creates the curve $[X(t), Y(t)]$. First we would like to determine the position of the additional point $D \equiv [D_x(t), D_y(t)]$ which corresponds to the position of the work point P when the two parts $L2$ and $L3$ are parallel. Then we can rotate the point D around point B by the angle f to calculate the position of work point P.

3. Determination of the trajectories of individual points of the work rod

All necessary calculations are carried out in program MAPLE, [1]. Considering the page restrictions for this article the output of the program is suppressed apart from a few exceptions. The specifications of the variables, important points and the
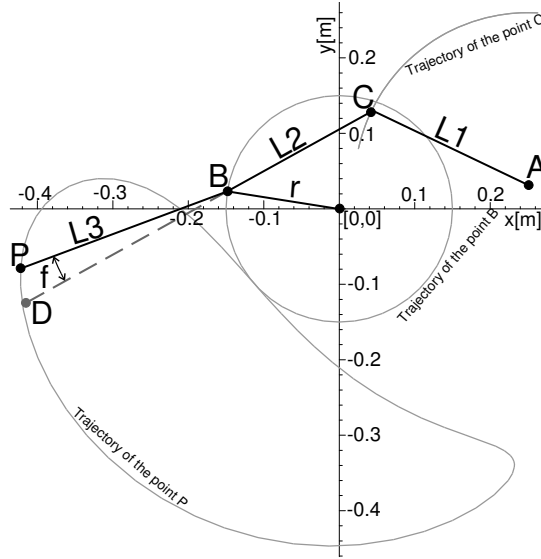
Figure 1: The constructive schema

coordinate system can be seen in Figure 1. The dimensions and the coordinates saved in the variable `Ksu` are set in meters. The driving handle rotates to left direction with speed of one revolution per second.

```
> restart; with(plots): Ksu:=[Ax=0.25,Ay=0.03,r=0.15,L1=0.23,L2=0.22,L3=0.30,
    f=55*Pi/1800,omega=-2*Pi];
> Dx:=Bx+(Bx-Cx)*L3/L2: Dy:=By+(By-Cy)*L3/L2:
> X:=(Dx-Bx)*cos(f)+(Dy-By)*sin(f)+Bx;Y:=(Bx-Dx)*sin(f)+(Dy-By)*cos(f)+By;
```

The current coordinates of the joint C are determined as the intersection point of the circles $c_1 \equiv ([B_x(t), B_y(t)], L2)$ and $c_2 \equiv ([A_x, A_y], L1)$. The point with y positive is chosen from both intersections.

```
> CB:=(x-Bx)^2+(y-By)^2=L2^2; CA:=(x-Ax)^2+(y-Ay)^2=L1^2;
> SolC:=[allvalues(solve({CB,CA},{x,y}))]: SolCf:=subs(Bx=r,By=0,Ksu,SolC);
> SolC:=zip((u,v)->`if`(subs(u,y)>0,v,NULL),SolCf,SolC)[]:
> Cx:=subs(SolC,x): Cy:=subs(SolC,y): Bx:=r*cos(omega*t); By:=r*sin(omega*t);
```

Now we choose ten points on both parts of the work rod and draw their trajectories. For the upper part $L2$ these trajectories are marked light gray, for lower part $L3$ in dark gray, see Figure 2. The parameter λ determines the positions of the points.

```
> Lambda1:=1/N1*[$1..N1-1]: Lambda2:=1/N2*[$1..N2-1]: RL3x:=Bx+(X-Bx)*lambda:
>  RL3y:=By+(Y-By)*lambda: RL2x:=Bx+(Cx-Bx)*lambda: RL2y:=By+(Cy-By)*lambda:
> G7:=plot([seq(subs(Ksu,[RL3x,RL3y,t=0..1]),lambda=Lambda1)],color=red):
> G8:=plot([seq(subs(Ksu,[RL2x,RL2y,t=0..1]),lambda=Lambda2)], color=blue):
> G9:=plot(evalf([seq(subs(Ksu,[Bx,By]),t=T)]),style=point,color=black):
> G10:=textplot(evalf([seq(subs(Ksu,[Bx,By,cat(" ",convert(evalf(t,2),string))]),
        t=T)]),color=black,align=right, font=[HELVETICA,BOLD,12]):
> display({G4,G5,G6,G7,G8,G9,G10},labels=["x [m]","y [m]"]);
```

From Figure 2 it is obvious that two envelopes really exist for both systems of curves which delimit the work area of rods $L2$ and $L3$.

In order to estimate when individual points of both rods reach contact with the envelope, the position of the joint B is highlighted in the time period of 1/10 of the work period.
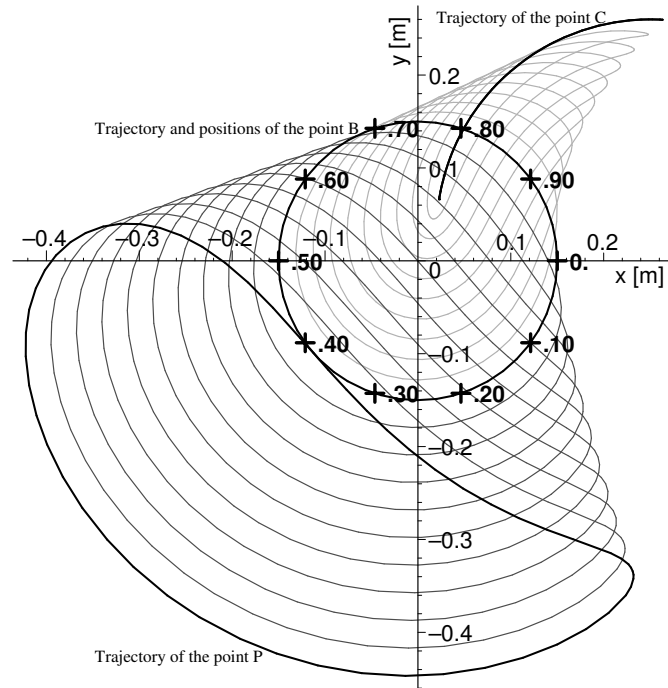

Figure 2: Trajectories of the 10 points on the rods $L2$ and $L3$

4. Envelope construction

The systems of the light and dark gray curves are two-parametric systems satisfying the definition of the planar close curves, see page 17. The first parameter is the time t, which determines the position of work rod, the second parameter λ determines position of point on the rod. By using the binding condition between both parameters we can determine the points that constitute the envelopes of both systems of curves according to equation (1). The binding conditions O2 for rod $L2$ and O3 for rod $L3$ are very complicated expressions and therefore it pays to use the MAPLE optimization facility.

Because the binding conditions O2 and O3 are non-linear implicit equations, it is necessary to execute the following calculations numerically.

```
> t:='t': lambda:='lambda': Digits:=24: C31:=cost(O3): C21:=cost(O2):
> PO3:=optimize(makeproc(O3,t)): PO2:=optimize(makeproc(O2,t)):
> C32:=cost(PO3): C22:=cost(PO2): Savings=C31+C21-C32-C22;
```

Savings = 55178 multiplications + 16198 functions + 9372 additions − 239 storage − 239 assignments

It is obvious that the optimization of MAPLE is very effective. Newton's iteration is used to determine numerically the time t for each point when this point reaches the envelope. The use of automatic derivatives is a great advantage for computing this step.

```
ILT:=proc(PROC,VAR) local var, dvar;
 var:=VAR; dvar:=1;
 while abs(dvar)>1e-6 do; dvar:=-PROC(var)/D(PROC)(var); var:=var+dvar; end do;
 var;
end proc;
```

From Figure 2 it is obvious that point B whose parameter $\lambda = 0$ for both rods, reaches the left envelope at time $t \approx 0.7\,\mathrm{s}$ and the right shell at time $t \approx 1\,\mathrm{s}$. These time values can be used as an input data of iterative procedure. If there is a shift from point B to a close point (a little shift corresponds to $\Delta\lambda \to 0$), then it is possible to determine the time from binding condition (1) by means of the iterative procedure ILT when this close point reaches the envelope. The final time t used by the previous point for reaching the envelope is used as initial value for the next iteration.

The time values t_k of reaching the envelope for individual values of λ will be saved as ordered pairs $[\lambda_k, t_k]$ which will be used to draw envelope in detail, at the end of the calculation, see Figure (3).

```
> lambda:=0; tau31:=ILT(PO3,0.7); tau32:=ILT(PO3,1.0);
> tau21:=ILT(PO2,0.7); tau22:=ILT(PO2,1.0); LT31:=[[0,tau31]];
> LT32:=[[0,tau32]]; LT21:=[[0,tau21]]; LT22:=[[0,tau22]];
> for lambda from 0.01 to 1 by 0.01 do;
    tau31:=ILT(PO3,tau31);tau32:=ILT(PO3,tau32);tau21:=ILT(PO2,tau21);
    tau22:=ILT(PO2,tau22);LT31:=[LT31[],[lambda,tau31]];
    LT32:=[LT32[],[lambda,tau32]];LT21:=[LT21[],[lambda,tau21]];
    LT22:=[LT22[],[lambda,tau22]];
> end do:
> Digits:=10; lambda:='lambda';
> O31:=map(u->evalf(subs(Ksu,lambda=u[1],t=u[2],[RL3x,RL3y])),LT31):
> O32:=map(u->evalf(subs(Ksu,lambda=u[1],t=u[2],[RL3x,RL3y])),LT32):
> O21:=map(u->evalf(subs(Ksu,lambda=u[1],t=u[2],[RL2x,RL2y])),LT21):
> O22:=map(u->evalf(subs(Ksu,lambda=u[1],t=u[2],[RL2x,RL2y])),LT22):
> G11:=plot([O31,O32,O21,O22],color=black,thickness=3):
> display({G4,G5,G6,G7,G8,G11},labels=["x [m]","y [m]"]);
```

5. Conclusion

The process described in this article can be generalized to compute areas and envelopes for other curves. This method can be used to avoid contacts of different moving parts. For example, Figure 3 demonstrates that there exists an area inside the Horal collector mechanism passed by three different components.
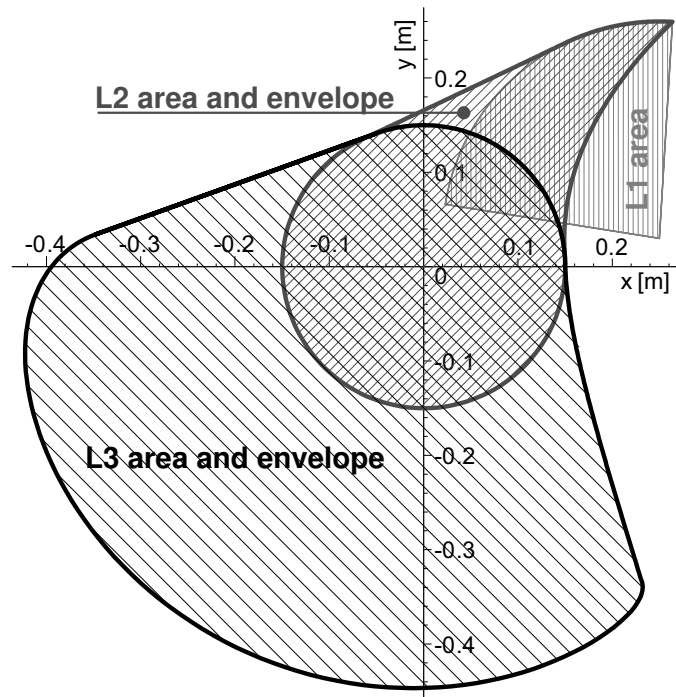
Figure 3: The envelopes and rods workspaces

It can also be used to find points of maximal deflection of the trajectory, so called *dead points of working mechanisms*. If a component passes through this point some of its points have maximum acceleration, *tangent or normal*. Taking these accelerations into account is essential for proper construction of the components.

Acknowledgments

References

[1] *Maple User Manual.* Maplesoft, 2011, Waterloo Canada, ISBN 978-1-926902-07-4.

[2] Rutter, J. W.: *Geometry of curves.* CRC Press, 2000, ISBN 1-58488-166-6, 243–270.

[3] *Sběrací vozy Horal SP 3-341.*
http://www.vozy-biso.cz/cs/produkty/sberaci-vozy-horal-sp-3-341/