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OSCILLATION OF BÔCHER'S PAIRS WITH RESPECT TO HALFLINEAR SECOND ORDER DIFF. EQU.-S

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The equation in question is

$$(1)\qquad (py')' + qf(y,py') = 0 \quad ('=\frac{d}{dx}),\quad x\in I=(-\infty,\infty)$$

$$p,q\in C(I),\ p>0,\ f(u,v)\in C(R\times R),\ uf\geq 0 \quad (f(o,v)=0)$$
$$f(\lambda u,\lambda v) = \lambda f(u,v), \forall\ \lambda,u,v$$
(uniqueness assumed, too).

In a former paper of mine the pair of functions

$$\Phi = \varphi_1 y-\varphi_2 py',\quad \Psi = \psi_1 y-\psi_2 py'$$

was investigated under the conditions

a) y is a solution of (1)

b) $\varphi_i,\ \psi_i\in C_1(I)$ $(i=1,2)$ and $\begin{vmatrix}\varphi_1 & \varphi_2\\ \psi_1 & \psi_2\end{vmatrix} \neq 0\quad x\in I$

c) $\{\varphi_1,\varphi_2\}\neq 0,\ \{\psi_1,\ \psi_2\}\neq 0,\quad x\in I$

where the symbol $\{g,h\}$ $(g,h\in C_1(I))$ is defined by

$$\{g,h\} = p(g'h-h'q)+g^2+pqhf(h,g).$$

The results stated there were the following:

1° Φ and Ψ have no common zeros,

2° they have no multiple zeros,

3° their zeros do not accumulate at a finite point,

4° the zeros of Φ and Ψ - if any - separate each other,

5° if $\{\varphi_1;\varphi_2\}\cdot\{\psi_1,\psi_2\}<0$, then only one of Φ and Ψ can vanish, moreover once at most,

6° assuming simple conditions - not detailed here - y and Φ are oscillatory or not oscillatory simultaneously (i.e. in the same time), expressed otherwise: (1) and Φ (or Ψ) are oscillatory or not oscillatory simultaneously.

These statements involve a lot of theorems (old and new) concerning oscillations and non-oscillations.

In a recent paper - to appear - these results (except 5°) have been extended to the pair of functions

$$U = \varphi y_1 - \phi p y_1' \quad , \quad V = \varphi y_2 - \phi p y_2'$$

where y_i $(i=1,2)$ are linear independent solutions of (1), i.e.

$$y_1'y_2 - y_2'y_1 \neq 0 \qquad x \in I$$

and

$$\{\varphi, \phi\} \neq 0, \qquad \varphi, \phi \in C_1(I) \; .$$

Let be formulated here only the corresponding of 6°.

Theorem I (of the simultaneous oscillations): Under the above conditions y_i (i.e.(1)) and U (or V) oscillate or do not oscillate in the same time provided φ and ϕ are chosen in a suitable way, namely in such a manner that

$$(2) \qquad 2\varphi\eta - \phi p\eta' \neq 0, \qquad x \in I \quad \eta = y_1^2 + y_2^2 .$$

Since $\frac{\eta'}{\eta}$ remains finite inside I, this choice is possible very easily, however it is connected to the given pair (y_1, y_2) of solutions of (1).

In the linear case - when $f(u,v) \equiv u$ - this choice is of universal validity, that means: once the pair (φ, ϕ) is suitable chosen in the above sense to a pair (y_1, y_2), the same pair (φ, ϕ) is appropriate to any other pair $(\tilde{y}_1, \tilde{y}_2)$ of linear independent solutions of (1), i.e. if y_i and U are simultaneously oscillatory or non-oscillatory, then so are $\tilde{y}_i$ and $\tilde{U}$, too.

In the oscillatory case $\frac{p\eta'}{\eta}$ can be unbounded as $x \to \pm\infty$, then - according to (2) - $\frac{\varphi}{\phi}$ must behave in the same way. However in the non-oscillatory case the situation is more advantageous, namely the inequality

$$z < z_o - 2\int_{x_o}^{x} q\,dx + 2c^2 \int_{x_o}^{x} \frac{dx}{p\eta^2} \quad , \quad z = \frac{p\eta'}{\eta} , \quad z_o = z(x_o)$$

$$c = p(y_1'y_2 - y_2'y_1) = \text{const}$$

holds, where the second integral is convergent by a theorem of Hartman and Wintner and the first one cannot converge to $+\infty$ (by a theorem of Wintner), consequently if this integral does not converge to $-\infty$, then z is bounded and thus also $\frac{\varphi}{\phi}$ can be chosen to be bounded.

All the above results may be extended to half-linear systems, too.