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A Note on $\mathscr{I}$-Ultrafilters and P-Points

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We consider the question whether P-points can be characterized as $\mathscr{I}$-ultrafilters for $\mathscr{I}$ an ideal on ω and show that (consistently) it is not possible if $\mathscr{I}$ is an F_σ-ideal or a P-ideal.

1. Introduction

Definition 1.1 (Baumgartner [2]). Let $\mathscr{I}$ be a family of subsets of a set X such that $\mathscr{I}$ contains all singletons and is closed under subsets. An ultrafilter $\mathscr{U} \in \omega^*$ is called an *$\mathscr{I}$-ultrafilter* if for any $F : \omega \to X$ there is $A \in \mathscr{U}$ such that $F[A] \in \mathscr{I}$.

Several classes of $\mathscr{I}$-ultrafilters for $X = 2^\omega$ were defined by Baumgartner [2], e.g. discrete or nowhere dense ultrafilters, and some other classes were defined by Brendle [4] and Barney [1]. All those ultrafilters exist under some additional set-theoretic assumptions, but they cannot be constructed in ZFC because they are nowhere dense ultrafilters and Shelah proved in [10] that it is consistent with ZFC that there are no nowhere dense ultrafilters. For $X = \omega_1$ ordinal ultrafilters were introduced by Baumgartner [2] as $\mathscr{I}$-ultrafilters for $\mathscr{I} = \{A \subseteq \omega_1 : \text{order type of } A \leq \alpha\}$ for some indecomposable ordinal $\alpha < \omega_1$.

In this paper, we consider $\mathscr{I}$-ultrafilters for $X = \omega$ and the situation is slightly different here since $\mathscr{I}$-ultrafilters exist in ZFC for some particular families $\mathscr{I}$ (see

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Proposition 2.1). Though most of the results in the paper remain consistency results.

Throughout the article we assume that the family $\mathscr{I}$ is an ideal on ω which contains all finite subsets of ω. We can do this without loss of generality because if we replace an arbitrary family $\mathscr{I}$ in the definition of $\mathscr{I}$-ultrafilter by the ideal generated by $\mathscr{I}$, we get the same concept (first noticed in [1]).

An ideal $\mathscr{I} \subseteq \mathscr{P}(\omega)$ is called *tall* if every $A \notin \mathscr{I}$ contains an infinite subset that belongs to the ideal $\mathscr{I}$. (Some authors call ideals with this property *dense*.)

For $A, B \subseteq \omega$ we say that A is *almost contained* in B and we write $A \subseteq^* B$ if $A \setminus B$ is finite. Let us also recall that an ideal $\mathscr{I}$ is called a *P-ideal* if whenever $A_n \in \mathscr{I}$, $n \in \omega$, then there is $A \in \mathscr{I}$ such that $A_n \subseteq^* A$ for every n.

As additional set-theoretic assumptions we will use two instances of Martin's Axiom — Martin's Axiom for countable posets and Martin's Axiom for σ-centered posets which is equivalent to the assumption $\mathfrak{p} = \mathfrak{c}$. Let us recall that the *pseudointersection number* $\mathfrak{p}$ is defined by:

$$\mathfrak{p} = \min \left\{ |\mathscr{F}| : \mathscr{F} \subseteq [\omega]^{\omega} \text{ centered}, \neg \left(\exists A \in [\omega]^{\omega}\right)\left(\forall F \in \mathscr{F}\right) A \subseteq^* F \right\}$$

2. The existence of $\mathscr{I}$-ultrafilters

For some ideals on ω the existence of $\mathscr{I}$-ultrafilters can be established in ZFC in contrast to the above mentioned result of Shelah. We shall recall that the *character of* $\mathscr{I}$, $\chi(\mathscr{I})$, is the minimal cardinality of a base for $\mathscr{I}$, i.e.

$$\chi(\mathscr{I}) = \min \left\{ |\mathscr{B}| : \mathscr{B} \subseteq \mathscr{I} \wedge \left(\forall I \in \mathscr{I}\right)\left(\exists B \in \mathscr{B}\right) I \subseteq^* B \right\}.$$

Proposition 2.1. *If $\mathscr{I}$ is a maximal ideal on ω such that $\chi(\mathscr{I}) = \mathfrak{c}$ then $\mathscr{I}$-ultrafilters exist.*

Proof. It is an immediate consequence of Theorem in [5]. □

There are of course many interesting ideals on ω to which we cannot apply Proposition 2.1. It seems that in general some additional set-theoretic assumptions are again necessary to construct the corresponding $\mathscr{I}$-ultrafilters. The next proposition states that for some ideals there are no $\mathscr{I}$-ultrafilters at all.

Proposition 2.2. *If $\mathscr{I}$ is not tall then $\mathscr{I}$-ultrafilters do not exist.*

Proof. Suppose that for $A \in [\omega]^{\omega} \setminus \mathscr{I}$ we have $\mathscr{I} \cap \mathscr{P}(A) = [A]^{<\omega}$ and let $e_A : \omega \to A$ be an increasing enumeration of the set A.

Now assume for the contrary that there exists an $\mathscr{I}$-ultrafilter $\mathscr{U} \in \omega^*$. According to the definition of an $\mathscr{I}$-ultrafilter there exists $U \in \mathscr{U}$ such that $e_A[U] \in \mathscr{I}$. Since $e_A[U] \subseteq A$ the set $e_A[U]$ is finite. It follows that U is finite because e_A is one-to-one — a contradiction to the assumption that $\mathscr{U}$ is a free ultrafilter. □

It turns out that under Martin's Axiom for σ-centered posets the necessary condition from Proposition 2.2 is also sufficient.

Proposition 2.3. $(\mathfrak{p} = \mathfrak{c})$ *If $\mathscr{I}$ is tall then $\mathscr{I}$-ultrafilters exist.*

Proof. Enumerate all functions from ω to ω as $\{f_\alpha : \alpha < \mathfrak{c}\}$. By transfinite induction on $\alpha < \mathfrak{c}$ we will construct filter bases $\mathscr{F}_\alpha$ satisfying:

(i) $\mathscr{F}_0$ is the Fréchet filter
(ii) $\mathscr{F}_\alpha \subseteq \mathscr{F}_\beta$ whenever $\alpha \leq \beta$
(iii) $\mathscr{F}_\gamma = \bigcup_{\alpha<\gamma} \mathscr{F}_\alpha$ for γ limit
(iv) $(\forall\alpha)\ |\mathscr{F}_\alpha| \leq |\alpha| \cdot \omega$
(v) $(\forall\alpha)(\exists F \in \mathscr{F}_{\alpha+1})\ f_\alpha[F] \in \mathscr{I}$

Suppose we already know $\mathscr{F}_\alpha$. If there is a set $F \in \mathscr{F}_\alpha$ such that $f_\alpha[F] \in \mathscr{I}$ then put $\mathscr{F}_{\alpha+1} = \mathscr{F}_\alpha$. Hence we may assume that $f_\alpha[F] \notin \mathscr{I}$, in particular $f_\alpha[F]$ is infinite, for every $F \in \mathscr{F}_\alpha$.

Since $|\mathscr{F}_\alpha| < \mathfrak{c} = \mathfrak{p}$ there exists $M \in [\omega]^\omega$ such that $M \subseteq^* f_\alpha[F]$ for every $F \in \mathscr{F}_\alpha$. The ideal $\mathscr{I}$ is tall, so there is $A \in \mathscr{I}$ which is an infinite subset of M, hence $A \subseteq^* f_\alpha[F]$, in particular $f_\alpha^{-1}[A] \cap F$ is infinite for every $F \in \mathscr{F}_\alpha$. It follows that $f_\alpha^{-1}[A]$ is compatible with $\mathscr{F}_\alpha$. To complete the induction step let $\mathscr{F}_{\alpha+1}$ be the filter base generated by $\mathscr{F}_\alpha$ and $f_\alpha^{-1}[A]$.

It is easy to see that every ultrafilter that extends the filter base $\mathscr{F} = \bigcup_{\alpha<\mathfrak{c}} \mathscr{F}_\alpha$ is an $\mathscr{I}$-ultrafilter. □

3. $\mathscr{I}$-ultrafilters and P-points

A free ultrafilter $\mathscr{U}$ is called a *P-point* if for all partitions of ω, $\{R_i : i \in \omega\}$, either for some i, $R_i \in \mathscr{U}$, or $(\exists U \in \mathscr{U})(\forall i \in \omega)\ |U \cap R_i| < \omega$.

There exist two characterizations of P-points as $\mathscr{I}$-ultrafilters: If $X = 2^\omega$ then P-points are precisely the $\mathscr{I}$-ultrafilters for the family $\mathscr{I}$ consisting of all finite and converging sequences; if $X = \omega_1$ then P-points are precisely the $\mathscr{I}$-ultrafilters for $\mathscr{I} = \{A \subseteq \omega_1 : A \text{ has order type } \leq \omega\}$ (see [2]).

Is there an ideal $\mathscr{I} \subseteq \mathscr{P}(\omega)$ such that P-points are precisely the $\mathscr{I}$-ultrafilters? In the next two propositions we prove (under additional set-theoretic assumptions) that such an ideal can be neither an F_σ-ideal nor a P-ideal.

The following description of F_σ-ideals is due to Mazur [9]: For every F_σ-ideal $\mathscr{I}$ there exists a lower semicontinuous submeasure $\varphi : \mathscr{P}(\omega) \to [0, \infty]$ such that $\mathscr{I} = \mathrm{Fin}(\varphi) = \{A \subseteq \omega : \varphi(A) < \infty\}$. Remember that a submeasure φ is called *lower semicontinuous* (*lsc* in short) if $\varphi(A) = \lim_{n\to\infty} \varphi(A \cap n)$.

Theorem 3.1. (MA_{ctble}) *For every F_σ-ideal $\mathscr{I} \subseteq \mathscr{P}(\omega)$ there exists a P-point that is not an $\mathscr{I}$-ultrafilter.*

Proof. Let φ be the lsc submeasure for which $\mathscr{I} = \mathrm{Fin}(\varphi)$. Enumerate all partitions of ω (into infinite sets) as $\{\mathscr{R}_\alpha : \alpha < \mathfrak{c}\}$. By transfinite induction on $\alpha < \mathfrak{c}$

we will construct filter bases $\mathscr{F}_\alpha$, $\alpha < \mathfrak{c}$, so that the following conditions are satisfied:

(i) $\mathscr{F}_0$ is the Fréchet filter

(ii) $\mathscr{F}_\alpha \subseteq \mathscr{F}_\beta$ whenever $\alpha \leq \beta$

(iii) $\mathscr{F}_\gamma = \bigcup_{\alpha<\gamma} \mathscr{F}_\alpha$ for γ limit

(iv) $(\forall\alpha)\,|\mathscr{F}_\alpha| \leq |\alpha|\cdot\omega$

(v) $(\forall\alpha)(\forall F \in \mathscr{F}_\alpha)\,\varphi(F) = \infty$

(vi) $(\forall\alpha)(\exists F \in \mathscr{F}_{\alpha+1})$ either $(\exists R \in \mathscr{R}_\alpha)\ F \subseteq R$
or $(\forall R \in \mathscr{R}_\alpha)\,|F \cap R| < \omega$

Assume we already know $\mathscr{F}_\alpha$ and we should define $\mathscr{F}_{\alpha+1}$.

Case A. $(\exists R \in \mathscr{R}_\alpha)(\forall F \in \mathscr{F}_\alpha)\,\varphi(F \cap R) = \infty$
Let $\mathscr{F}_{\alpha+1}$ be the filter base generated by $\mathscr{F}_\alpha$ and such a set R.

Case B. $(\forall R \in \mathscr{R}_\alpha)(\exists F \in \mathscr{F}_\alpha)\,\varphi(F \cap R) < \infty$
Enumerate $\mathscr{R}_\alpha$ as $\{R_n : n \in \omega\}$. The assumption of Case B. implies that $(\forall K \in [\omega]^{<\omega})\ (\exists F_K \in \mathscr{F}_\alpha)\ \varphi(F_K \cap \bigcup_{n\in K} R_n) < \infty$.

Consider $P = \{\langle K,n\rangle \in [\omega]^{<\omega} \times \omega : K \subseteq \bigcup_{i\leq n} R_i,\ K \cap R_n \neq \emptyset\}$ and define order $\leq_P$ by $\langle K,n\rangle \leq_P \langle L,m\rangle$ if $\langle K,n\rangle = \langle L,m\rangle$ or $K \supset L$, $\min(K\setminus L) > \max L, n > m$ and $(K\setminus L) \cap \bigcup_{i\leq m} R_i = \emptyset$. Obviously, $(P, \leq_P)$ is a countable poset. Now, for $F \in \mathscr{F}_\alpha$ and $k,j \in \omega$ define $D_{F,k} = \{\langle K,n\rangle \in P : \varphi(K \cap F) \geq k\}$ and $D_j = \{\langle K,n\rangle \in P : n \geq j\}$.

Claim: $D_{F,k}$ is dense in $(P, \leq_P)$ for every $F \in \mathscr{F}_\alpha$ and $k \in \omega$; D_j is dense in $(P, \leq_P)$ for every $j \in \omega$.

Proof of the claim. Consider $\langle L, m\rangle \in P$ arbitrary. Since L is finite there exists $p \geq m$ such that $[0, \max L] \subseteq \bigcup_{i\leq p} R_i$. According to the assumption there is $F_p \in \mathscr{F}_\alpha$ such that $\varphi(F_p \cap \bigcup_{i\leq p} R_i) < \infty$. It follows that $\varphi((F_p \cap F)\setminus \bigcup_{i\leq p} R_i) = \infty$. We can choose a finite set $L' \subseteq (F_p \cap F)\setminus\bigcup_{i\leq p} R_i$ such that $\varphi(L') \geq k$ because φ is lower semicontinuous. Let $n = \max\{i : L' \cap R_i \neq \emptyset\}$ and $K = L \cup L'$. Note that the choice of p implies $\min L' > \max L$. It follows that $\langle K,n\rangle \leq_P \langle L,m\rangle$ and $\langle K,n\rangle \in D_{F,k}$. So $D_{F,k}$ is dense. For $j \leq m$ we have $\langle L,m\rangle \in D_j$ and for any $j > m$ we can choose arbitrary $r \in R_j$ such that $r > \max L$. Let $K' = L \cup \{r\}$. Of course, $\langle K',j\rangle \leq_P \langle L,m\rangle$ and $\langle K',j\rangle \in D_j$. So D_j is dense. □

The family $\mathscr{D} = \{D_{F,k} : F \in \mathscr{F}_\alpha, k \in \omega\} \cup \{D_j : j \in \omega\}$ consists of dense subsets in P and $|\mathscr{D}| < \mathfrak{c}$. Therefore there is a $\mathscr{D}$-generic filter $\mathscr{G}$. Let $U = \bigcup\{K : (\exists n)\langle K,n\rangle \in \mathscr{G}\}$. It remains to check that:

- $(\forall F \in \mathscr{F}_\alpha)\,\varphi(U \cap F) = \infty$

Take $k \in \omega$ arbitrary. For every $\langle K,n\rangle \in \mathscr{G} \cap D_{F,k}$ we have $U \supseteq K$ and $k \leq \varphi(K \cap F) \leq \varphi(U \cap F)$ (submeasure φ is monotone). Hence $\varphi(U \cap F) = \infty$.

- $(\forall R_n \in \mathscr{R}_\alpha)\,|U \cap R_n| < \omega$

Fix $\langle K_n, j_n\rangle \in \mathscr{G} \cap D_n$. Observe that $j_n \geq n$ and for $\langle K,m\rangle \in \mathscr{G}$ we have $K \cap R_n = \emptyset$ if $m < n$ and that $K \cap R_n = K_n \cap R_n$ if $m \geq n$. To see the latter consider $\langle L, m'\rangle \in \mathscr{G}$ such that $\langle L, m'\rangle \leq_P \langle K,m\rangle$ and $\langle L, m'\rangle \leq_P \langle K_n, j_n\rangle$ (such a condition exists because $\mathscr{G}$ is a filter) for which we get $L \cap R_n = K \cap R_n$ and $L \cap R_n = K_n \cap R_n$. It follows that $U \cap R_n = K_n \cap R_n$ is finite.

To complete the induction step let $\mathscr{F}_{\alpha+1}$ be the filter base generated by $\mathscr{F}_\alpha$ and the set U.

It follows from condition (vi) that every ultrafilter which extends the filter base $\mathscr{F} = \bigcup_{\alpha<\mathfrak{c}} \mathscr{F}_\alpha$ is a P-point. Because of condition (v) there exists an ultrafilter extending $\mathscr{F}$ which extends also the dual filter to $\mathrm{Fin}(\varphi) = \mathscr{I}$, in particular it is not an $\mathscr{I}$-ultrafilter. □

Theorem 3.2. $(\mathfrak{p} = \mathfrak{c})$ *If $\mathscr{I}$ is a tall P-ideal on ω then there is an $\mathscr{I}$-ultrafilter that is not a P-point.*

Proof. It was proved in Proposition 2.3 that assuming $\mathfrak{p} = \mathfrak{c}$ there exist $\mathscr{I}$-ultrafilters for every tall ideal $\mathscr{I}$. We will show that if $\mathscr{I}$ is a tall P-ideal then the square of an $\mathscr{I}$-ultrafilter is again an $\mathscr{I}$-ultrafilter and it is not a P-point.

So let us first recall the definition of the product of ultrafilters (see [6]): If $\mathscr{U}$ and $\mathscr{V}$ are ultrafilters on ω then $\mathscr{U} \cdot \mathscr{V} = \{A \subseteq \omega \times \omega : \{n : \{m : \langle n,m\rangle \in A\} \in \mathscr{V}\} \in \mathscr{U}\}$ is an ultrafilter on $\omega \times \omega$ which is isomorphic to (and can be identified with) an ultrafilter on ω. By the square of ultrafilter $\mathscr{U}$ we mean the ultrafilter $\mathscr{U} \cdot \mathscr{U}$.

Notice that the partition $\{\{n\} \times \omega : n \in \omega\}$ of $\omega \times \omega$ witnesses the fact that no product of free ultrafilters on ω is a P-point. Hence to complete the proof it remains to check that if $\mathscr{U}$ is an $\mathscr{I}$-ultrafilter then $\mathscr{U} \cdot \mathscr{U}$ is again an $\mathscr{I}$-ultrafilter, i.e. for every $f : \omega \times \omega \to \omega$ there is $U \in \mathscr{U} \cdot \mathscr{U}$ such that $f[U] \in \mathscr{I}$.

To this end define for arbitrary function $f : \omega \times \omega \to \omega$ and for every $n \in \omega$ a function $f_n : \omega \to \omega$ by $f_n(m) = f(\langle n,m\rangle)$. If $\mathscr{U}$ is an $\mathscr{I}$-ultrafilter then there exists $V_n \in \mathscr{U}$ such that $f_n[V_n] \in \mathscr{I}$ for every n. Now we can find a set $A \in \mathscr{I}$ such that $f_n[V_n] \subseteq^* A$ for every n because $\mathscr{I}$ is P-ideal. It is obvious that $f_n^{-1}[f_n[V_n]] \in \mathscr{U}$ for every $n \in \omega$. Hence either $f_n^{-1}[f_n[V_n] \cap A]$ or $f_n^{-1}[f_n[V_n] \setminus A]$ belongs to $\mathscr{U}$. Let $I_0 = \{n \in \omega : f_n^{-1}[f_n[V_n] \cap A] \in \mathscr{U}\}$ and $I_1 = \{n \in \omega : f_n^{-1}[f_n[V_n] \setminus A] \in \mathscr{U}\}$. Since $\mathscr{U}$ is an ultrafilter it contains one of the sets I_0, I_1.

<u>Case A.</u> $I_0 \in \mathscr{U}$

Put $U = \{\{n\} \times f_n^{-1}[f_n[V_n] \cap A] : n \in I_0\}$. It is easy to see that $U \in \mathscr{U} \cdot \mathscr{U}$ and $f[U] = \bigcup_{n \in I_0} f_n[V_n] \cap A \subseteq A \in \mathscr{I}$.

<u>Case B.</u> $I_1 \in \mathscr{U}$

Since $f_n[V_n] \setminus A$ is finite and $\mathscr{U}$ is an ultrafilter, there exists $k_n \in f_n[V_n] \setminus A$ such that $f_n^{-1}\{k_n\} \in \mathscr{U}$ for every $n \in I_1$. Fix arbitrary $g : \omega \to \omega$ such that $g(n) = k_n$ for each $n \in I_1$. Since $\mathscr{U}$ is an $\mathscr{I}$-ultrafilter there exists $V \in \mathscr{U}$ such that $g[V] \in \mathscr{I}$. Now put $U = \{\{n\} \times f_n^{-1}\{k_n\} : n \in I_1 \cap V\}$. It is easy to check that $U \in \mathscr{U} \cdot \mathscr{U}$ and $f[U] \subseteq g[V] \in \mathscr{I}$. □

For ideals which are neither (analytic) P-ideals nor F_σ-ideals there is no 'nice' description. So it is rather difficult to prove any general statements about $\mathcal{I}$-ultrafilters and P-points in this case. We will conclude by one example of such an ideal and show that it cannot be used to characterize P-points via the corresponding $\mathcal{I}$-ultrafilters.

Definition 3.3. A set $A \in [\omega]^\omega$ with an (increasing) enumeration $A = \{a_n : n \in \mathbb{N}\}$ is called *thin* (see [3]) if $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 0$.

Obviously, thin sets do not form an ideal (consider for example the sets $\{n! : n \in \omega\}$ and $\{n! + 1 : n \in \omega\}$), but they generate an ideal which we denote by $\mathcal{T}$. We refer to $\mathcal{T}$-ultrafilters as thin ultrafilters. Borel complexity of the ideal $\mathcal{T}$ is $F_{\sigma\delta\sigma}$ since $\mathcal{T} = \bigcup_{r\in\mathbb{N}}\bigcap_{k\in\mathbb{N}}\bigcup_{m\in\mathbb{N}}\bigcap_{n\geq m}\{A \subseteq \omega : \frac{a_n}{a_{n+r}} < \frac{1}{k}\}$ and the ideal $\mathcal{T}$ is not a P-ideal (if $A_k = \{n! + k : n \in \omega\}$ then there is no set $A \in \mathcal{T}$ such that $A_k \subseteq^* A$ for each $k \in \omega$). Thus Theorems 3.1 and 3.2 do not apply to thin ultrafilters. However, assuming Martin's Axiom for countable posets it is possible to prove that there is no inclusion between thin ultrafilters and P-points.

Theorem 3.4. (MA_{ctble})

(1) *There exists a P-point that is not a thin ultrafilter.*

(2) *There exists a thin ultrafilter that is not a P-point.*

Proof. The ideal generated by thin sets is contained in the F_σ-ideal $\mathcal{I}_{1/n} =$ $= \{A \subseteq \mathbb{N} : \sum_{n\in A}\frac{1}{n} < \infty\}$. Statement (1) follows from the obvious fact that for $\mathcal{I} \subseteq \mathcal{J}$ every $\mathcal{I}$-ultrafilter is a $\mathcal{J}$-ultrafilter and from Theorem 3.1.

As for (2), a thin ultrafilter which is not a P-point was constructed in [8] assuming Martin's Axiom for countable posets (published in [7] as Proposition 4 assuming Continuum Hypothesis). □

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