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Erik G. F. Thomas
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Path Integrals in Finite Sets

E. G. F. THOMAS

Groningen*)

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The lectures concern the construction of an analogue of the Feynman path integral for the case of a system of differential equations,

$$\frac{1}{i}\frac{\partial}{\partial t}\varphi_t = H_0\varphi_t$$

in which H_0 is a self-adjoint operator in the space $L^2(M) = \mathbb{C}^M$, where M is a finite set, the paths being functions on $\mathbb{R}$ with values in M. The path integral is a family of measures $F_{t',t}$ with values in the operators on $L^2(M)$, or equivalently, a family of complex measures corresponding to matrix coefficients.

It is shown that these measures on path space are in some sense dominated by the measure of a Markov process. This implies that $F_{t',t}$ is concentrated on the set of step functions $S[t, t']$.

This allows one to make sense of, and prove, the analogue of Feynman's formula for the propagator of the Hamiltonian $H = H_0 + V$, where V is a potential, namely the formula:

$$e^{i(t'-t)H} = \int_{S[t,t']} \exp\left[i\int_t^{t'} V(x(s))\,ds\right] F_{t',t}(dx)$$

and the corresponding formulas for the matrix coefficients, in which the integral extends over the paths beginning and ending in the appropriate points. We show that the measures $F_{t',t}$ are completely determined by these equations and by a certain multiplicative property.

The path integral coresponding to a "two particle system without interaction" is the direct product of the corresponding path integrals. The propagator for a "two particle system with interaction" can be obtained by repeated integration.

Finally we show that the above integral formula can be generalized to the case where the potential is time dependent.

*) Department of Mathematics, University of Groningen, Postbus 800, 9700 AV Groningen, Holland