RENDICONTI DEL CIRCOLO MATEMATICO DI PALERMO
Serie II, Suppl. 72 (2004), pp. 151-157

ON ALMOST GEODESIC MAPPINGS $\pi_2(e)$ ONTO RIEMANNIAN SPACES

JOSEF MIKEŠ, OLGA POKORNÁ, GALINA STARKO

ABSTRACT. In this paper almost geodesic mappings of type $\pi_2(e)$ from a space A_n with an affine connection onto Riemannian spaces $\bar{V}_n$ will be investigated. We find more precise fundamental equations of almost geodesic mappings of type $\pi_2(e)$: $A_n \to \bar{A}_n$. We prove that the set of all Riemannian spaces $\bar{V}_n$ $(n > 4)$, for which A_n admits almost geodesic mappings $\pi_2(e)$, where $e = -1$, depends on at most $\frac{1}{2}n^2(n+1) + 2n + 3$ real parameters.

1. INTRODUCTION

N.S. Sinyukov ([12], [13]) introduced the notion of almost geodesic mappings of a space A_n with an affine connection without torsion onto $\bar{A}_n$ and found three types of these mappings: π_1, π_2 and π_3. J. Mikeš and V. Berezovsky ([2], [3]) proved completeness of this classification for $n > 5$. Almost geodesic mappings were studied by many authors (see e.g. [1] - [6], [10] - [18]). In this work, we present a study of almost geodesic mappings $\pi_2(e)$ from A_n onto a Riemannian space $\bar{V}_n$.

2. ALMOST GEODESIC MAPPINGS

A curve ℓ defined in the space A_n with affine connection is called *almost geodesic* if there exists a two-dimensional parallel distribution along ℓ, to which the tangent vector of this curve belongs at every point.

A diffeomorphism f: $A_n \to \overline{A}_n$ is an *almost geodesic mapping* if, as a result of f, every geodesic of the space A_n maps into an almost geodesic curve of the space $\overline{A}_n$.

A mapping f: $A_n \to \overline{A}_n$ is almost geodesic if and only if, in the common coordinate system $x \equiv (x^1, x^2, \dots, x^n)$ with respect to the mapping f, the deformation tensor of an affine connection $P^h_{ij}(x) \equiv \overline{\Gamma}^h_{ij}(x) - \Gamma^h_{ij}(x)$ satisfies the relations

$$A^h_{\alpha\beta\gamma}\lambda^\alpha\lambda^\beta\lambda^\gamma = a\,P^h_{\alpha\beta}\lambda^\alpha\lambda^\beta + b\,\lambda^h,$$

2000 *Mathematics Subject Classification*: 53B05, 53B20, 53C15.

Key words and phrases: almost geodesic mapping, Riemannian space, e-structure, almost complex structure.

Supported by grant No. 201/02/0616 of The Grant Agency of Czech Republic.

The paper is in final form and no version of it will be submitted elsewhere.

where $A^h_{ijk} \equiv P^h_{ij,k} + P^\alpha_{ij} P^h_{\alpha k}$, Γ^h_{ij} ($\overline{\Gamma}^h_{ij}$) are objects of an affine connection of spaces A_n ($\overline{A}_n$), λ^h is any vector, a and b are some functions of values x^h and λ^h. Hereafter "," denotes a covariant derivative with respect to the connection of the space A_n.

Almost geodesic mappings of the type π_2 are characterized by the following conditions ([11], [12], [13]):

$$
(1) \qquad \begin{aligned}
&\text{(a)}\ \ P^h_{ij} &&= \delta^h_{(i}\psi_{j)} + F^h_{(i}\varphi_{j)}\,;\\
&\text{(b)}\ \ F^h_{(i,j)} &&= F^h_\alpha F^\alpha_{(i}\sigma_{j)} + F^h_{(i}\mu_{j)} + \delta^h_{(i}\varrho_{j)}\,,
\end{aligned}
$$

where δ^h_i is the Kronecker symbol, F^h_i is an affinor, ψ_i, φ_i, μ_i, ϱ_i, σ_i are covectors, and $(i\,j)$ is the symmetrization of indices.

A mapping π_2: $A_n \to \bar{A}_n$ was called *a mapping* $\pi_2(e)$ by N. S. Sinyukov ([11], [12], [13]), if its inverse $\pi_2(e)^{-1}$ is one of the mappings $\tilde{\pi}_2$: $\bar{A}_n \to A_n$. The mapping $\pi_2(e)$ is characterized by equations:

$$
(2) \qquad \begin{aligned}
&\text{(a)}\ \ P^h_{ij} &&= \delta^h_{(i}\psi_{j)} + F^h_{(i}\varphi_{j)}\,;\\
&\text{(b)}\ \ F^h_{(i,j)} &&= F^h_{(i}\mu_{j)} + \delta^h_{(i}\varrho_{j)}\,;\\
&\text{(c)}\ \ F^h_\alpha F^\alpha_i &&= e\,\delta^h_i\,, \quad e = \pm 1, 0\,.
\end{aligned}
$$

The equation (c) characterizes the affinor F^h_i as an *e-structure.*

Now we shall show that the equations (2) characterizing the mapping $\pi_2(e)$: $A_n \to \bar{A}_n$, can be precized. The following theorem holds:

Theorem 1. *The mapping f: $A_n \to \overline{A}_n$ is almost geodesic $\pi_2(e)$ if and only if, in the common coordinate system x with respect to the mapping f, the deformation tensor of an affine connection $P^h_{ij}(x)$ satisfies the relations*

$$
(3) \qquad \begin{aligned}
&\text{(a)}\ \ P^h_{ij} &&= \delta^h_{(i}\psi_{j)} + F^h_{(i}\varphi_{j)}\,;\\
&\text{(b)}\ \ F^h_{(i,j)} &&= F^h_{(i}\mu_{j)} - \delta^h_{(i}F^\alpha_{j)}\mu_\alpha\,;\\
&\text{(c)}\ \ F^h_\alpha F^\alpha_i &&= e\,\delta^h_i\,, \quad e = \pm 1, 0\,.
\end{aligned}
$$

where δ^h_i is the Kronecker symbol, F^h_i is an affinor, ψ_i, φ_i, μ_i are covectors, and $(i\,j)$ is the symmetrization of indices.

The proof of this theorem follows from an analysis of conditions (1b) and (1c). As these conditions have their own meaning the following section is devoted to their analysis.

3. On e-structures generated by almost geodesic mappings $\pi_2(e)$

Affinor F^h_i satisfying conditions (2b) and (2c) will be called *e-structure generated by almost geodesic mappings* $\pi_2(e)$.

Theorem 2. *e-structure F^h_i generated by almost geodesic mappings $\pi_2(e)$ satisfies the relations*

$$
(4) \qquad \begin{aligned}
&\text{(b)}\ \ F^h_{(i,j)} &&= F^h_{(i}\mu_{j)} - \delta^h_{(i}F^\alpha_{j)}\mu_\alpha;\\
&\text{(c)}\ \ F^h_\alpha F^\alpha_i &&= e\,\delta^h_i, \quad e = \pm 1\,.
\end{aligned}
$$

where μ_i is a covector.

Proof. Let us have an e-structure F_i^h generated by almost geodesic mappings $\pi_2(e)$ which is characterized by conditions (2b) and (2c).

We differentiate covariantly the algebraic conditions (2c) by x^j and then we use the symmetrization by indices i and j:

$$F^h_{\alpha,i}F^\alpha_j + F^h_{\alpha,j}F^\alpha_i + F^h_\alpha F^\alpha_{(i,j)} = 0\,.$$

After exclusion of $F^\alpha_{(i,j)}$ by means of (2b) we obtain

$$F^h_{\alpha,i}F^\alpha_j + F^h_{\alpha,j}F^\alpha_i + e\delta^h_{(i}\mu_{j)} + F^h_{(i}\varrho_{j)} = 0\,.$$

After contraction with F^j_k and symmetrization by indices i and k we have:

$$\begin{aligned} eF^h_{(i,k)} + F^h_{(\alpha,\beta)}F^\alpha_i F^\beta_k + F^h_i(F^\alpha_k\varrho_\alpha + e\mu_k) + F^h_k(F^\alpha_i\varrho_\alpha + e\mu_i) \\ + e\delta^h_i(F^\alpha_k\mu_\alpha + \varrho_k) + e\delta^h_k(F^\alpha_i\mu_\alpha + \varrho_i) = 0\,. \end{aligned}$$

Substituting $F^\alpha_{(i,j)}$ by (2b) we obtain the expression

$$F^h_i(F^\alpha_k\varrho_\alpha + e\mu_k) + F^h_k(F^\alpha_i\varrho_\alpha + e\mu_i) + e\delta^h_i(F^\alpha_k\mu_\alpha + \varrho_k) + e\delta^h_k(F^\alpha_i\mu_\alpha + \varrho_i) = 0\,.$$

Analyzing this expression, we see that $F^\alpha_i\mu_\alpha + \varrho_i = 0$, i.e. $\varrho_i = -F^\alpha_i\mu_\alpha$. By substitution to (2b) we get equation (4b), resp. (3b). This finishes the proof of Theorem 1 and 2. □

Futher we shall analyze equations (4b). A covariant derivative in the direction x^k gives

$$F^h_{i,jk} + F^h_{i,kj} = F^h_{(i}\mu_{j)k} - \delta^h_{(i}m_{j)k} + \overset{1}{\Theta}{}^h_{ijk}\,, \tag{5}$$

where $\overset{1}{\Theta}{}^h_{ijk} \equiv \mu_{(i}F^h_{j)k} - \delta^h_{(i}F^\alpha_{j)k}\mu_\alpha$, $\mu_{ij} \equiv \mu_{i,j}$ and $m_{ij} \equiv F^\alpha_i\mu_{\alpha j}$.

Then we shall alternate (5) by indices i and k and use the Ricci identities:

$$F^h_{i,jk} - F^h_{k,ji} = F^h_i\mu_{jk} - F^h_k\mu_{ji} + F^h_j\mu_{[ik]} - \delta^h_i m_{jk} + \delta^h_k m_{ji} - \delta^h_j m_{[ik]} + \overset{2}{\Theta}{}^h_{ijk}\,,$$

where $\overset{2}{\Theta}{}^h_{ijk} \equiv \overset{1}{\Theta}{}^h_{ijk} - \overset{1}{\Theta}{}^h_{kji} + F^\alpha_j R^h_{\alpha ik} - F^h_\alpha R^\alpha_{jik}$, R^h_{ijk} is the Riemannian tensor of A_n and $[i\,k]$ denotes the alternation by the corresponding indices.

In the formula obtained we will change indices j and k:

$$F^h_{i,kj} - F^h_{j,ki} = F^h_i\mu_{kj} - F^h_j\mu_{ki} + F^h_k\mu_{[ij]} - \delta^h_i m_{kj} + \delta^h_j m_{ki} - \delta^h_k m_{[ij]} + \overset{2}{\Theta}{}^h_{ikj}\,.$$

After addition to the original formula (5) and some corrections we obtain

$$2F^h_{i,jk} = F^h_i\mu_{(jk)} + F^h_j\mu_{[ik]} + F^h_k\mu_{[ij]} - \delta^h_i m_{(jk)} - \delta^h_j m_{[ik]} - \delta^h_k m_{[ij]} + \overset{3}{\Theta}{}^h_{ikj}\,, \tag{6}$$

where $\overset{3}{\Theta}{}^h_{ijk} \equiv \overset{1}{\Theta}{}^h_{ijk} + \overset{1}{\Theta}{}^h_{kji} - \overset{1}{\Theta}{}^h_{jki} + 2F^h_\alpha R^\alpha_{kji} - F^\alpha_i R^h_{\alpha jk} + F^\alpha_j R^h_{\alpha ik} + F^\alpha_k R^h_{\alpha ij}$.

Hence we have

Theorem 3. *e-structure F^h_i generated by almost geodesic mappings $\pi_2(e)$, $e = \pm 1$, satisfies the equations* (6).

4. On almost complex structures generated by almost geodesic mappings $\pi_2(e)$

This part is devoted to the analysis of number of parameters which describe the set of all almost complex structure F_i^h generated by almost geodesic mappings $\pi_2(e)$, for $e=-1$, in a given space A_n with an affine connection.

The following assertion holds:

Theorem 4. *Let A_n be a space with an affine connection. The set of all complex structures $F_i^h(x)$, generated by almost geodesic mappings $\pi_2(e)$, depends on at most $\frac{1}{2}n(n^2-1)$ real parameters.*

To prove this theorem we will analyze equations (4) and (6). We shall apply contraction of algebraic conditions (4c) : $F^\alpha_\beta F^\beta_\alpha = -n$, and then covariant derivative in direction x^j and x^k:

$$F^\alpha_\beta F^\beta_{\alpha,jk} + F^\alpha_{\beta j} F^\beta_{\alpha k} = 0\,.$$

Substituting (6), we have

$$(n-1)\mu_{(jk)} - \mu_{(\alpha\beta)} F^\alpha_j F^\beta_k = \overset{4}{\Theta}_{jk}\,, \tag{7}$$

where $\overset{4}{\Theta}_{jk} \equiv F^\alpha_\beta \overset{3}{\Theta}{}^\beta_{\alpha jk} + 2F^\alpha_{\beta j} F^\beta_{\alpha k}$.

Contracting (7) with $F^j_{j'} F^k_{k'}$, we further obtain

$$(n-1)\mu_{(\alpha\beta)} F^\alpha_j F^\beta_k - \mu_{(jk)} = \overset{4}{\Theta}_{\alpha\beta}\, F^\alpha_j F^\beta_k\,, \tag{7'}$$

From formulas (7) and (7') follows

$$\mu_{(i,j)} = \overset{5}{\Theta}_{ij}\,, \tag{8}$$

where $\overset{5}{\Theta}_{ij} \equiv \dfrac{1}{n(n-2)}\Big((n-1)\,\overset{4}{\Theta}_{ij} + \overset{4}{\Theta}_{\alpha\beta}\, F^\alpha_i F^\beta_j\Big)$.

Futher we covariantly derive formula (8) in the direction x^k:

$$\mu_{i,jk} + \mu_{j,ik} = \overset{5}{\Theta}_{ij,k}\,, \tag{9}$$

and then alternate by indices i a k:

$$\mu_{i,jk} - \mu_{k,ji} + \mu_\alpha R^\alpha_{jik} = \overset{5}{\Theta}_{ij,k} - \overset{5}{\Theta}_{kj,i}\ .$$

Changing somewhere indices j a k, we have:

$$\mu_{i,kj} - \mu_{j,ki} + \mu_\alpha R^\alpha_{kij} = \overset{5}{\Theta}_{ik,j} - \overset{5}{\Theta}_{jk,i}\,,$$

After addition with the origin formula (9) and after some reductions we get

$$\mu_{ij,k} = \mu_\alpha R^\alpha_{kji} + \frac{1}{2}\big(\overset{5}{\Theta}_{ij,k} + \overset{5}{\Theta}_{ik,j} - \overset{5}{\Theta}_{jk,i}\big)\,. \tag{10}$$

Finally we obtain a system of differential equations of Cauchy type in covariant derivatives with respect to unknown functions F_i^h, F_{ij}^h, μ_i, μ_{ij}:

$$(11)\qquad \begin{aligned} F^h_{i,j} &= F^h_{ij}\,;\\ F^h_{ij,k} &= \overset{6}{\Theta}_{ijk}\,;\\ \mu_{i,j} &= \mu_{ij}\,;\\ \mu_{ij,k} &= \overset{7}{\Theta}_{ijk}\,, \end{aligned}$$

where

$$2\,\overset{6}{\Theta}{}^h_{ijk} \equiv F_i^h\mu_{(jk)} + F_j^h\mu_{[ik]} + F_k^h\mu_{[ij]} - \delta_i^h m_{(jk)} - \delta_j^h m_{[ik]} - \delta_k^h m_{[ij]} + \overset{3}{\Theta}{}^h_{ikj}\,;$$

$$\overset{7}{\Theta}_{ijk} \equiv \mu_\alpha R^\alpha_{kji} + \frac{1}{2}(\overset{5}{\Theta}_{ij,k} + \overset{5}{\Theta}_{ik,j} - \overset{5}{\Theta}_{jk,i})\,.$$

Rides sides of equations (11) depend on functions F_i^h, F_{ij}^h, μ_i, μ_{ij} and objects of an affine connection of space A_n. From the other hand, this functions satisfy algebraic formulas (4b) a (8):

$$(12)\qquad \begin{aligned} F^h_{(ij)} &= F^h_{(i}\mu_{j)} - \delta^h_{(i}F^\alpha_{j)}\mu_\alpha\,;\\ \mu_{(ij)} &= \overset{5}{\Theta}_{ij}\,. \end{aligned}$$

The system (11) has at most one solution for an initial condition. Initial conditions are limited by algebraic conditions (12). It can be easily seen that initial conditions have at most

$$\frac{1}{2}n(n^2-1)$$

independent parameters. This proves Theorem 4. □

5. On F-planar mappings onto Riemannian spaces

This section is concerned with certain questions of F-planar mapping from affine-connected spaces A_n onto (pseudo-) Riemannian spaces $\bar{V}_n$.

A curve ℓ: $x^h = x^h(t)$ is said to be *F-planar* (J. Mikeš, N.S. Sinyukov [8]) if, under the parallel translation along it, the tangent vector $\lambda^h \equiv dx^h/dt$ lies in the tangent 2-plane formed by the tangent vector λ^h and its conjugate $F^h_\alpha\lambda^\alpha$, i.e.

$$\nabla_t\lambda^h \equiv d\lambda^h/dt + \Gamma^h_{\alpha\beta}\lambda^\alpha\lambda^\beta = \varrho_1\lambda^h + \varrho_2F^h_\alpha\lambda^\alpha\,,$$

where ϱ_1 and ϱ_2 are functions of the parameter t.

Let in spaces A_n and $\bar{A}_n$, together with objects of affine connections Γ^h_{ij} and $\bar{\Gamma}^h_{ij}$, affinor structures F_i^h and $\bar{F}_i^h$ be defined.

A diffeomorfism f: $A_n \to \bar{A}_n$ is said to be an *F-planar mapping* [8] if, under this mapping, any F-planar curve A_n maps into the $\bar{F}$-planar curve $\bar{A}_n$.

Under the condition $\mathrm{Rank}\|F_i^h - \varrho\delta_i^h\| > 1$ the mapping of A_n onto $\bar{A}_n$ is F-planar if and only if the conditions

$$(13)\qquad \begin{aligned} &(a)\quad P^h_{ij} = \delta^h_{(i}\psi_{j)} + F^h_{(i}\varphi_{j)}\,;\\ &(b)\quad \bar{F}^h_i = \alpha F^h_i + \beta\delta^h_i\,, \end{aligned}$$

holds ([6], [8]), where $\psi_i(x)$, $\varphi_i(x)$ are covectors, $\alpha(x)$, $\beta(x)$ are functions in the coordinate system x which is general with respect to the mapping. F-planar mappings generalize geodesic (if $\varphi_i \equiv 0$ or $F_i^h \equiv a\delta_i^h$), quasigeodesic, holomorphically projective, planar, and almost geodesic of the type of π_2 mappings (see [6], [12], [13], [19]).

The following assertion [7] holds.

Theorem 5. *Let A_n be a space with affine connection, where an affinor $F_i^h(x)$ is defined such that* $\mathrm{Rank}\|F_i^h - \varrho\delta_i^h\| > 5$. *The set of all Riemannian spaces $\bar{V}_n$, for which A_n admits F-planar mappings, depends on at most $\frac{1}{2}n(n+5)+3$ real parameters.*

6. On almost geodesic mappings $\pi_2(e), e = -1$, onto Riemannian spaces

From analysis of equations (1a) and (13a) obviously follows that almost geodesic mappings π_2 are a special case of F-planar mappings.

Thus we could come to a false conclusion that Theorem 5, which is formulated for F-planar mappings, automatically holds for mappings π_2.

The problem lies in the fact that the structure F is apriori defined for F-planar mappings, but in the case of mappings π_2 the structure affinor F is unknown.

Thus we must not automatically transfer facts holding for F-planar mappings $A_n \to \bar{V}_n$ (see [5] – [7]), to the case of almost geodesic mappings π_2 onto Riemannian spaces.

For the almost complex structure F with $n > 4$ the condition $\mathrm{Rank}\|F_i^h - \varrho\delta_i^h\| > 5$ holds. Hence with respect to Theorems 4 and 5, we have the followings assertion:

Theorem 6. *Let A_n $(n > 4)$ be a space with affine connection. The set of all Riemannian spaces $\bar{V}_n$, for which A_n admits an almost geodesic mappings $\pi_2(e)$, $e = -1$, depends on at most $\frac{1}{2}n^2(n+1)+2n+3$ real parameters.*

References

[1] Adamów, A., *On reduced almost geodesic mappings in Riemannian spaces*, Demonstr. Math. **15** (1982), 925–934.

[2] Berezovskij, V., Mikeš, J., *On the classification of almost geodesic mappings of affine-connected spaces*, Differential Geometry and its Applications, Proc. Conf., Dubrovnik/Yugosl. 1988, 41–48 (1989).

[3] Berezovskij, V., Mikeš, J., *On a classification of almost geodesic mappings of affine connection spaces*, Acta Univ. Palacki. Olomuc., Fac. Rerum Nat., Math. **35** (1996), 21–24.

[4] Ehsenov, K. R., *On properties of generalized equidistant Kählerian spaces admitting special, almost geodesic mappings of the second type*, Studies on topological and generalized spaces, Collect. Sci. Works, Frunze, 81–84 (1988).

[5] Mikeš, J., *Special F-planar mappings of affinely connected spaces onto Riemannian spaces*, Mosc. Univ. Math. Bull. **49**, No. 3 (1994), 15–21 (1994); translation from Vestn. Mosk. Univ., Ser. I, No. 3, (1994), 18–24.

[6] Mikeš, J., *Holomorphically projective mappings and their generalizations*, J. Math. Sci., New York **89**, 3 (1998), 1334–1353.

[7] Mikeš, J., *F-planar mappings onto Riemannian spaces*, Sb. trudov mezhd. konf. Invariantnye metody issl. na mnogoobrazijah struktur geometrii, analiza i mat. fiziki. Tom 2. Mosc. Univ. (2001), 138–145.

[8] Mikeš, J., Sinyukov, N. S., *On quasiplanar mappings of spaces with affine connection*, Sov. Math. **27**, No. 1 (1983), 63–70; translation from Izv. Vyssh. Uchebn. Zaved., Mat. 1983, No. 1 (248), 55–61.

[9] Petrov, A. Z., *New Methods in Theory of General Relativity*, Nauka, Moscow, 1966.

[10] Shadnyj, V. S., *Almost-geodesic maps of riemannian spaces onto spaces of constant curvature*, Math. Notes **25** (1979), 151–153.

[11] Sinyukov, N. S., *Almost geodesic mappings of spaces with affine connection and e-structures*, (Russian) Mat. Zametki **7** (1970), 449–459.

[12] Sinyukov, N. S., *Geodesic mappings of Riemannian spaces*, Nauka, Moscow 1979.

[13] Sinyukov, N. S., *Almost geodesic mappings of affine connected and Riemannian spaces*, Math. Notes **17** (1975), 450–454; translation from Itogi Nauki Tekh., Ser. Probl. Geom. **13** (1982), 3–26.

[14] Sobchuk, V. S., *Interior almost geodesic mappings*, Sov. Math. **33**, No. 5 (1989), 97–101; translation from Izv. Vyssh. Uchebn. Zaved., Mat. No. 5 (324) (1989), 62–64.

[15] Sobchuk, V. S., *Intrinsic almost geodesic mapping*, Izv. Vyssh. Uchebn. Zaved., Mat. No. 5 (324) (1989), 62–64.

[16] Sobchuk, V. S., *On almost geodesic mappings in the class of semi-symmetric pseudo-Riemann spaces*, (Ukrainian. English summary) Nauk. Visn. Chernivets'kogo Univ., Mat. **76** (2000), 107–108.

[17] Sobchuk, V. S., Mikeš, J., Pokorná, O., *On almost geodesic mappings π_2 between semisymmetric Riemannian spaces*, Novi Sad J. Math. **29**, No.3 (1999), 309–312.

[18] Stankovic, M. S., *On canonic almost geodesic mappings of the second type of affine spaces*, Filomat **13** (1999), 105–144.

[19] Yano, K., *Differential Geometry on Complex and Almost Complex Spaces*, Pergamon Press, Oxford, 1965.

DEPARTMENT OF ALGEBRA AND GEOMETRY, FAC. SCI., PALACKY UNIVERSITY
TOMKOVA 40, 779 00 OLOMOUC, CZECH REPUBLIC
E-MAIL: Mikes@risc.upol.cz

DEPARTMENT OF MATHEMATICS, CZECH UNIVERSITY OF AGRICULTURE
KAMÝCKÁ 129, PRAHA 6, CZECH REPUBLIC
E-MAIL: Pokorna@tf.czu.cz

DEPARTMENT OF MATHEMATICS, BUILDING ACADEMY OF ODESSA
UL. KANATNAYA 100/3/30, ODESSA, UKRAINE