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PEAK FUNCTIONS ON CONVEX DOMAINS

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ABSTRACT. Let $\Omega \subseteq \mathbf{C}^n$ be a bounded smooth convex domain and $p \in b\Omega$ be a point of finite type. This paper constructs continuous and Hölder continuous peak functions at p on Ω. The construction uses Bergman kernel estimates on and off the diagonal, due to J.D.McNeal.

INTRODUCTION

Let $\Omega \subseteq \mathbf{C}^n$ be a domain with smooth boundary and p be a boundary point. Let $H(\Omega)$ denote the space of holomorphic functions on Ω. A holomorphic function $f \in H(\Omega) \cap C^k(\bar{\Omega})$, where $k = 0, 1, ..., \infty$, is called a C^k-peak function at p if $f(p) = 1$ and $|f(q)| < 1$ for $q \in \bar{\Omega} \setminus \{p\}$.

If a peak function exists, it gives a powerful tool for analysis on Ω. For various applications of peak functions see e.g. [H] and [Ch].

If f is a peak function, then $1/(1-f)$ is a holomorphic function on Ω which blows up at p. Hence if Ω has a peak function at every boundary point, it is necessarily pseudoconvex.

The question of existence of peak functions on pseudoconvex domains is well understood for strongly pseudoconvex domains. In this case one can introduce local holomorphic coordinates around p such that the boundary is strongly convex in the induced linear space, and that easily gives a local C^∞-peak function. Then one can pass to a global peak function by solving the $\bar{\partial}$-problem and using appropriate regularity results (see [K]).

On the other hand, much less is known for weakly pseudoconvex domains. Most attention here is paid to domains of finite type, introduced by Kohn and D'Angelo in [K] and [D]. The first negative result is due to Fornaess, who proved that on certain domains in $\mathbf{C}^2$, analogous to the Kohn-Nirenberg example (see [KN]), there exists no C^1-peak function (see [F]). This contrasts with a later result of Bedford

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and Fornaess in [BF] which shows that on pseudoconvex finite type domains in $\mathbf{C}^2$ there is always a continuous peak function.

In this paper we construct a continuous and a Hölder continuous peak function at a point of finite type on a convex domain in $\mathbf{C}^n$. This construction, which links peak functions to the assymptotic behaviour of the Bergman kernel, is originally used in [FM] for an alternative proof of the result of Bedford and Fornaess. The Bergman kernel estimates used in the construction were proved by McNeal (see [M1]). We obtain a semiglobal peak function and then pass to a global peak function by a geometric argument.

1. The Bergman kernel estimates

We will assume that $\Omega \subseteq \mathbf{C}^n$ is a bounded, smooth convex domain and $p \in b\Omega$ is a point of finite type T.

Recall that the Bergman kernel $K(w,z)$, $w, z \in \mathbf{C}^n$ is the integral kernel associated to the orthogonal projection B from $L^2(\Omega)$ onto its subspace of holomorphic functions $H^2(\Omega)$,

$$Bf(w) = \int_\Omega K(w,z)f(z)\, dz.$$

$K(w,z)$ is holomorphic in the first n variables and antiholomorphic in the second n variables.

The construction of peak functions is based on the holomorphic functions $h_z(w) = \dfrac{K(w,z)}{K(z,z)}$. We will prove the following two properties:

1. h_z are bounded uniformly in z,
2. each h_z is small outside of a certain neighbourhood of p, whose diameter goes to zero as $z \to p$.

To prove the first property, we will use Bergman kernel estimates on and off the diagonal, proved in [M1]. The second property will follow from pseudolocality of the $\bar\partial$-Neumann operator N, proved in [M2]. Both these results rely on the fact that there is a subelliptic estimate for the $\bar\partial$-Neumann problem in a neighbourhood of p, since p is of finite type.

From h_z we obtain a sequence of approximate peak functions, and then, by Bishop's technique, a peak function at p.

In the following, U will denote a neighbourhood of p which is sufficiently small to allow all the local constructions.

First we introduce notation and state the necessary results from [M1].

By a rotation of the canonical coordinates we arrange that the normal direction to $b\Omega$ at p is given by the $\Re z_1$-axis. Then using the implicit function theorem we obtain a local defining function of the form $r(z_1, \dots, z_n) = \Re z_1 - F(\Im z_1, \dots, \Re z_n, \Im z_n)$, where F is a convex function. For $q \in U$ and $\epsilon > 0$ we will consider the level sets

$$b\Omega_{q,\epsilon} = \{z \in U;\ r(z) = \epsilon + r(q)\}$$

which are also convex, by the choice of r.

To every $q \in U$ and a sufficiently small $\epsilon > 0$ we assign coordinates $(z_1, \dots, z_n)$, $z_i = x_i + ix_{n+i}$ centered at q, obtained by translating and rotating the canonical coordinates, and numbers $\tau_i(q, \epsilon)$ which measure the distance from q to $b\Omega_{q,\epsilon}$ along the complex line determined by the z_i-axis. First we choose z_1 so that $\tau_1(q, \epsilon) = dist(q, b\Omega_{q,\epsilon})$, and that this distance is achieved along the positive x_1-axis. The choice of the remaining coordinates is such that it guarantees that the polydisc

$$P(q, \epsilon) = \{|z_i| \le \tau_i(q, \epsilon),\ i = 1, \dots, n\}$$

is essentially the largest polydisc around z contained in the set $\{z \in U\ ;\ r(z) < r(q) + \epsilon\}$. Also the remaining z_i, $i = 2, \dots, n$, have the property that the distance from q to $b\Omega_{q,\epsilon}$ within the z_i-axis is achieved on the positive x_i-axis. (For the whole construction and for other properties of the coordinate system, which we will not need, see [M1]).

We will use the following notation. For two quantities X, Y we write $X \lesssim Y$ if there is a constant C such that $X \lesssim CY$ and C is independent of the variables entering X and Y, which are clear from the context.

By definition, $\tau_1(q, \epsilon) \approx \epsilon$. The numbers $\tau_i(q, \epsilon)$, $2 \le i \le n$ can be approximately calculated from the coefficients in the Taylor expansion of r, restricted to the x_i-axis. Let

$$r(0, \dots, x_i, \dots, 0) = r(q) + \sum_{k=2}^{T} a_k^i(q) x_i^k + O(|x_i|^{T+1})$$

and denote $A_k^i(q) = |a_k^i(q)|$. We have

$$\tau_i(q, \epsilon) \approx \min \left\{ \left(\frac{\epsilon}{A_k^i(q)} \right)^{\frac{1}{k}}\ ;\ 2 \le k \le T \right\}.$$

Now we state the lower estimates on the diagonal (see [M1]).

Proposition 1.1. *There exists a neighbourhood V of p such that for $q \in V \cap \Omega$*

$$K(q, q) \gtrsim \prod_{i=1}^{n} \tau_i(q, r(q))^{-2}.$$

Note that the right hand side is the volume of $P(q, r(q))$. Since $P(q, r(q)) \subseteq \Omega$, the reverse inequality with $\le$ is obvious, so in this sense the estimate is sharp.

We turn to the off-diagonal estimates, which are formulated in terms of a "pseudometric", determined by the polydiscs $P(q, \epsilon)$. We define

$$M(q^1, q^2) = \inf\{\epsilon > 0;\ q^2 \in P(q^1, \epsilon)\}.$$

M is not a metric, but symmetry and triangle inequality are satisfied with respect to the relation $\lesssim$, i.e. $M(z, w) \approx M(w, z)$ and $M(z_1, z_3) \lesssim M(z_1, z_2) + M(z_2, z_3)$. In terms of the Taylor expansion of r we have

$$M(q^1, q^2) \approx |q_1^1 - q_1^2| + \sum_{i=2}^{n} \sum_{k=2}^{T} A_k^i(q^1) |q_i^1 - q_i^2|^k \tag{1.1}$$

in the coordinates assigned to q^1 and $M(q^1, q^2)$.

In the following theorem, D_i denotes the differential operator $\frac{\partial}{\partial z_i}$ in the coordinates associated to q^1 and δ. For multiindices μ, ν let

$$D^\mu \bar{D}^\nu = D_1^{\mu_1} \dots D_n^{\mu_n} \bar{D}_1^{\nu_1} \dots \bar{D}_n^{\nu_n},$$

where the holomorphic derivatives act on the first n variables in K and the antiholomorphic derivatives act on the last n variables.

Proposition 1.2 ([M1]). *There exists a neighbourhood V of p so that for all multiindices μ, ν there exists a constant $C_{\mu\nu}$ such that for all $q^1, q^2 \in V \cap \Omega$*

$$|D^\mu \bar{D}^\nu K(q^1, q^2)| \le C_{\mu\nu} \prod_{i=1}^{n} \tau_i(q^1, \delta)^{-2-\mu_i-\nu_i}$$

where $\delta = |r(q^1)| + |r(q^2)| + M(q^1, q^2)$.

Now let $\bar{N}$ denote the set of points in U lying on the inner normal to $b\Omega$ at p. In the following q will always denote a point on $\bar{N}$ and we denote $d(q) = dist(q, b\Omega) = dist(q, p)$. A consequence of the compatible estimates of Proposition 1.1 and 1.2 is the following lemma.

Lemma 1.3. *There exists a positive constant C such that*

$$\frac{|K(q, w)|}{K(q, q)} \le C$$

for all $q \in \bar{N}$ and $w \in \Omega$.

Proof. Follows immediately from proposition 3.1 and 3.2 and from the fact that there is a subelliptic estimate of order $\epsilon > 0$ in a neighbourhood of p, and so $K(z, w)$ is C^∞ off the boundary diagonal (see [Ke]). □

To prove the second property of $\frac{K(w, q)}{K(q, q)}$, we need the following theorem about pseudolocality of the $\bar{\partial}$-Neumann operator N. Recall that a subelliptic estimate of order $\epsilon > 0$ is said to hold in a neighbourhood V of $p \in b\Omega$, if there is a constant $C > 0$ such that

$$\|\alpha\|_\epsilon \le C(\|\bar{\partial}\alpha\| + \|\bar{\partial}^*\alpha\| + \|\alpha\|)$$

for all infinitely smooth $(0, 1)$ - forms supported in V, which are in the domain of $\bar{\partial}^*$. Here $\|.\|_\epsilon$ denotes the tangential Sobolev norm of order ϵ on $(0, 1)$-forms and the norms on the right are L^2-norms.

For $\eta > 0$ let $B(p, \eta)$ denote the ball centered at p with radius η.
Let α be a smooth $(0, 1)$-form with support in $B(p, \frac{\eta}{8})$ which is in the domain of the Kohn Laplacian $\Box = \bar{\partial}\bar{\partial}^* + \bar{\partial}^*\bar{\partial}$. Let $\xi \in C^\infty(\mathbf{C}^n)$ be a function satisfiing $\xi \equiv 1$ in $\Omega \setminus B(p, \eta)$ and $\xi \equiv 0$ in $B(p, \frac{\eta}{2})$.

Proposition 1.4 ([M2]). *Let Ω be a smooth bounded pseudoconvex domain in $\mathbb{C}^n$, and let $p \in b\Omega$ have a neighbourhood, where a subelliptic estimate of order $\epsilon > 0$ for the $\bar\partial$-Neumann problem on $(0,1)$-forms holds. For $s, t \in R^+$ there is a constant $C_{st} > 0$ so that*

$$\|\xi N\alpha\|_s^2 \leq C_{st}(\eta)^{-2(\frac{s+t}{\epsilon}+4)}\|\alpha\|_{-t}^2 .$$

Since p is of finite type, there is an $\epsilon > 0$ for which Proposition 1.4 applies, and we use it to prove the following property of h_q.

Lemma 1.5. *There exists a constant C independent of q such that*

$$\frac{|K(q,w)|}{K(q,q)} \leq Cd(q)$$

for $w \in \Omega \setminus B(p,\eta)$, where $\eta = d(q)^{\frac{\epsilon}{2(2n+4)}}$.

Proof. Following Kerzman, we will use Kohn's formula which relates the Bergman projection to the operator N:

$$B = I - \bar\partial^* N \bar\partial .$$

Let ϕ_q be a nonnegative radial function centered at q with support in Ω, and such that $\int \phi_q = 1$. By the mean value theorem

$$K(w,q) = B\phi_q(w) .$$

Note that ϕ_q is supported in $B(q, d(q))$. If η is appropriately larger than $d(q)$, then the support of $\bar\partial\phi_q$ is contained in $B(p, \frac{\eta}{8})$. By Kohn's formula

$$\xi(w)K(w,q) = \xi\phi_q - \xi\bar\partial^* N\bar\partial\phi_q = \xi\bar\partial^* N\bar\partial\phi_q .$$

Since $\|\bar\partial^* N\bar\partial\phi_q\|_{s-1} \leq \|N\bar\partial\phi_q\|_s$, Proposition 1.4 gives

$$\|\xi(w)K(w,q)\|_{s-1} \leq C_{st}\eta^{-2(\frac{s+t}{\epsilon}+4)}\|\bar\partial\phi_q\|_{-t} .$$

If $s > n$, then by Sobolev's lemma, $\sup|\xi(w)K(w,q)| \leq C\|\xi(.)K(.,q)\|_s$. Also, if $t > n$, another application of Sobolev's lemma gives

$$\|\bar\partial\phi_q\|_{-t}^2 \leq \|\phi_q\|_{-t+1}^2 = \sup\{|(\phi_q, f)|;\ \ f \in C_0^\infty,\ \|f\|_{t-1} \leq 1\} \leq \int |\phi_q| \sup|f| \leq C .$$

Together we get

$$\sup_w |\xi(w)K(w,q)| \leq C\eta^{-2(\frac{s+t}{\epsilon}+4)} .$$

Now we take $\eta = d(q)^{\frac{\epsilon}{2(2n+4)}}$, which gives $|K(w,q)| \leq Cd(q)^{-1}$ for w outside of $B(p,\eta)$. On the other hand, from Proposition 1.1 we obtain $K(q,q) \geq Cd(q)^{-2}$, since $\tau_1(q, r(q)) = d(q)$ and all other τ_i are bounded, say by 1. That proves the lemma. □

2. The construction of a continuous peak function

By Lemma 1.3 and 1.5, the holomorphic functions

$$h_q(w) = \frac{K(w,q)}{K(q,q)}$$

have the following properties:

(i) $|h_q(w)| \leq C$ for $w \in \Omega$ and a constant C independent of q.

(ii) $|h_q(w)| \leq \frac{1}{2}$ in $\Omega \setminus B(p,\eta)$

(iii) $h_q(q) = 1$.

Properties (i) and (iii) are immediate, (ii) follows from Lemma 1.5 if $d(q) \leq \frac{1}{2C}$.

Now we translate the functions h_q to p. Define $h'_q(w) = h_q(w+q-p)$.

The functions h'_q provide approximate peak functions at p, and Bishop's technique can be applied to construct a continuous local peak function.

Proposition 2.1. *There exists a sequence of points $\{q_n\}_{n=0}^{\infty}$ converging to p, and a real number c, $0 < c < 1$, such that the function*

$$H(w) = (1-c)\sum_{n=0}^{\infty} c^n h'_{q_n}(w)$$

is a local continuous peak function at p.

Proof. $\{q_n\}_{n=0}^{\infty}$ will be a sequence of points on $\bar{N}$ converging monotonically to p. It is defined inductively as follows. Let $h_n = h'_{q_n}$ and let U_n denote the neighbourhood of p, outside of which $h_n < \frac{1}{2}$.

1. Choose an arbitrary point on $\bar{N}$ to be q_0. If w is outside U_0, we can estimate $H(w)$ by $|H(w)| \leq \frac{1}{2}(1-c)\sum c^k = \frac{1}{2}$.

2. Let $n \geq 1$, and suppose that q_k are already chosen for $k < n$. We choose q_n in such a way that $|h_k(w)| < r_n$ for $w \in U_n$ and $k < n$, where $r_n > 1$ is a number close to 1, to be determined later. Since the h_k are continuous and $h_k(p) = 1$, a point sufficiently close to p will satisfy the requirement.

Let us estimate $H(w)$ on $U_n \setminus U_{n-1}$, to see how to choose c.

$$|H(w)| \leq (1-c)(r_n \sum_{k<n} c^k + c^n C + \frac{1}{2}\sum_{k>n} c^k) =$$

$$1 + (1-c)(C-1)c^n - \frac{c^{n+1}}{2} + (r_n - 1).$$

In order to get $|H(w)| < 1$, we need

$$(1-c)(C-1)c^n < \frac{c^{n+1}}{2}$$

(neglecting temporarily $r_n - 1$), which gives $c > 1 - \frac{1}{2C-1}$. So we choose $c = 1 - \frac{1}{2C}$, and make r_n sufficiently close to 1. □

Notice that h'_q is defined only on $\bar{\Omega} \cap \Omega_{p-q}$, where Ω_{p-q} is the translate of Ω by $p-q$, and so H is not a global peak function on Ω. Using the fact that Ω is convex, we can construct a global peak function by a geometric argument, without having to solve the $\bar{\partial}$-problem.

Proposition 2.2. *If $\Omega \subseteq \mathbb{C}^n$ is a bounded, smooth convex domain and $p \in b\Omega$ is a point of finite type, then there exists a global peak function at p, continuous in $\bar{\Omega}$.*

Proof. Consider the dilation by a factor of two with center at p, and let $\tilde{\Omega}$ be the resulting image of Ω. We have $p \in b\tilde{\Omega}$ and $\Omega \subseteq \tilde{\Omega}$, from convexity. Convexity of Ω also implies the following property of $\tilde{\Omega}$:
(P) Let $q \in \Omega$ be a point on the inner normal at p. Then Ω translated by $q - p$ is still contained in $\tilde{\Omega}$.
In fact, $\tilde{\Omega}$ is still the double dilated domain of the translated domain, this time with center at $p - 2(q - p)$, i.e. the reflection of p around q.
Now we can consider the Bergman kernel of $\tilde{\Omega}$. If $q \in \Omega$ lies on $\bar{N}$, we define h_q and h_q' as before, using $K_{\tilde{\Omega}}$. From the previous observation, h_q' is defined in all of $\bar{\Omega}$. Therefore H, constructed for $\tilde{\Omega}$ as in Proposition 2.1 is a global peak function for Ω. □

3. The construction of a Hölder continuous peak function

To construct a Hölder continuous peak function, we will use the following properties of h_q. Let $U_q = B(p, \eta)$ for η from Lemma 1.5.

Lemma 3.1. *There exists a constant $C > 0$ independent of q such that*
(i) $h_q(q) = 1$
(ii) $h_q(w) \le C$
(iii) $h_q(w) \le Cd(q)$ *in* $\Omega \setminus U_q$
(iv) $\dfrac{\partial}{\partial w_j} h_q(w) \le C \frac{1}{\tau_j(q,d(q))} \qquad j = 1, \dots, n$
(v) $\dfrac{\partial}{\partial w_j} h_q(w) \le C \frac{1}{|w|^T} \qquad j = 1, \dots, n$

Proof. (i) - (iii) are immediate; (iv) follows from Proposition 1.1 and 1.2, since $\tau_j(q,\delta) \ge \tau_j(q, d(q))$. (v) follows from Propositions 1.1 and 1.2 and from (1.1), since $\tau_j(q,\delta) \ge \tau_j(q, M(q,w)) \ge |w|^T$. □

Now we choose q_k so that $d(q_k) = \frac{s}{2^k}$, where s is a small number to be determined later, and denote $d_k = d(q_k)$. As before, we define $h_k(z) = h_{q_k}(z + q - p)$, and denote $U_n = U_{q_n}$.

Proposition 3.2. *For a suitable positive constant $c < 1$, the function*

$$H(z) = (1 - c) \sum_{k=0}^{\infty} c^k h_k(z)$$

is a local Hölder continuous peak function at p, with Hölder exponent $\nu = \frac{-\log c}{T \log 2}$

Proof. First we show that for suitable c and s, H is a local peak function at p. We will estimate the size of H at a point $z \in U$. If z lies outside of U_0, then we get from (iii) of Lemma 3.1 that $|H(z)| \le (1 - c) \sum C s \frac{c^k}{2^k} \le \frac{1}{2}$, if $s \le \frac{1}{2C}$.

Now assume that $z \in U_n \backslash U_{n-1}$. Let m be the largest integer such that $M(q_k, z) < d_m$ for all $k \leq m$. For $k < m$ we can use (iv) to estimate

$$|h_k(z) - 1| \leq C\left(\frac{|z_1|}{d_k} + \sum_{i=2}^{n} \frac{|z_i|}{\tau_i(q_k, d_k)}\right).$$

By (1.1) and (1.2) we have

$$\sum_{i=2}^{n} \frac{|z_i|}{\tau_i} \lesssim \sum_{i=2}^{n} |z_i| \sum_{j-2}^{T} \left(\frac{A_i^j(q_k)}{d_k}\right)^{\frac{1}{j}} \lesssim \sum_{j=2}^{T} \left(\frac{M(z, q_k)}{d_k}\right)^{\frac{1}{j}} \lesssim \left(\frac{d_m}{d_k}\right)^{\frac{1}{T}}.$$

Together we obtain

$$|h_k(z) - 1| \lesssim \frac{d_m}{d_k} + \left(\frac{d_m}{d_k}\right)^{\frac{1}{T}}.$$

For $m < k < n$ we get from Proposition 1.1 and 1.2

$$\begin{aligned} |h_k(z)| = \frac{|K(z, q_k)|}{K(q_k, q_k)} &\lesssim \frac{\tau(q_k, M(z, q_k))^{-2}}{\tau_1(q_k, d_k)^{-2}} \\ &\lesssim \frac{d_k^2}{(M(z, q_k) + M(q_k, q_m))^2} \lesssim \frac{d_k^2}{M(z, q_m)^2}. \end{aligned}$$

The same estimate holds with m replaced by $m+1$.

Next we have $h_n(z) \leq C$ and $h_k(z) \leq d_k$ for $k > n$. Together we obtain

$$\begin{aligned} |H(z)| \lesssim &(1-c)\left(\sum_{k \leq m}\left(1 + C\frac{2^{\frac{k}{T}}}{2^{\frac{m}{T}}}\right)c^k + \sum_{m<k<n} c^k C \frac{d_k^2}{d_{m+1}^2} + Cc^n + \sum_{k>n} c^k C \frac{s}{2^k}\right) \\ \leq &(1 - c^{m+1}) + (1-c)\, C \frac{1 - c^{m+1} 2^{\frac{m+1}{T}}}{1 - c2^{\frac{1}{T}}} 2^{-\frac{m}{T}} \\ &+ (1-c)\frac{4}{3} Cc^{m+1} + (1-c)\, Cc^n + C(\frac{c}{2})^{n+1} s \\ < &(1 - c^{m+1}) + (1-c) C \frac{c^{m+1} - 2^{-\frac{m+1}{T}}}{c - (\frac{1}{2})^{\frac{1}{T}}} \\ &+ (1-c)\frac{4}{3} Cc^{m+1} + (1-c)\, Cc^n + (\frac{1}{2})^{n+1} \frac{1}{4} \end{aligned}$$

if $s < \frac{1}{4C}$. We take c so that $c > (\frac{1}{2})^{\frac{1}{T}}$ and $\frac{(1-c)C}{c - (\frac{1}{2})^{\frac{1}{T}}} < \frac{1}{4}$, i.e. $c > 1 - \frac{1 - (\frac{1}{2})^{\frac{1}{T}}}{4C}$. Then each of the four last summands is $< \frac{1}{4} c^{m+1}$, and so $|H(z)| < 1$.

Now we turn to Hölder continuity. Let $x, y \in U$. Without loss of generality we can assume that $0 < |y| \leq |x|$. Fix $m \leq n$ so that $d_{n+1} \leq |y| \leq d_n$ and $d_{m+1} \leq |x| \leq d_m$. First we estimate the first m terms of the series. From (iv) we get

$$(1-c)\sum_{j<m} c^j |h_j(y) - h_j(x)| \leq (1-c) C \sum c^j |x-y| 2^j \leq C|x-y|(2c)^m.$$

To get $|x-y|(2c)^m \leq C|x-y|^\nu$ we need $|x-y|^{1-\nu} < \frac{1}{(2c)^m}$. Since $|x-y| \leq \frac{2s}{2^m}$, this holds when $\nu \leq \frac{-\log c}{\log 2}$.
Now we consider the remaining part of the series. First let $n > m+4$. Then we can estimate from (ii):

$$(1-c)\sum_{k=m}^{\infty} c^k |h_k(y) - h_k(x)| \leq Cc^m.$$

We have $|x-y| \gtrsim \frac{1}{2^m}$, so $c^m \leq C|x-y|^\nu$ holds again for $\nu < \frac{-\log c}{\log 2}$.
Now let $n \leq m+3$.

Case A: Let $|x-y| \leq |y|^T$. By (v) we have

$$(1-c)\sum_{k=m}^{\infty} c^k |h_k(x) - h_k(y)| \leq (1-c)\sum_{k=m}^{\infty} c^k \frac{|x-y|}{|y|^T} \leq Cc^m|x-y|2^{mT}.$$

This will be less than $C|x-y|^\nu$ if we take $\nu < \frac{-\log c}{T\log 2}$.

Case B: Let $|x-y| > |y|^T$.
Now we have $|x-y| \gtrsim \frac{1}{2^{mT}}$ and we can again use the estimate $\sum_{k>m} c^k|h_k(y)-h_k(x)| \leq Cc^m$. In order to get $c^m \leq C|x-y|^\nu$ we need again $\nu < \frac{-\log c}{T\log 2}$.

Remark. Using convexity of Ω, we get a global Hölder continuous peak function at p as in Proposition 2.2.

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