Stanisław Formella

Generalized Einstein manifolds

In: Jarolím Bureš and Vladimír Souček (eds.): Proceedings of the Winter School "Geometry and Physics". Circolo Matematico di Palermo, Palermo, 1990. Rendiconti del Circolo Matematico di Palermo, Serie II, Supplemento No. 22. pp. [49]--58.

Persistent URL: http://dml.cz/dmlcz/701461

GENERALIZED EINSTEIN MANIFOLDS

Stanisław Formella

INTRODUCTION. Let (M,g) and $(M,\bar{g})$ be two n-dimensional Riemannian manifolds of class C^∞ with not necessarily positive definite metrics g and $\bar{g}$ respectively. A diffeomorphism $\gamma: (M,g) \longrightarrow (M,\bar{g})$ which maps geodesic lines into geodesic lines is called geodesic mapping. The following theorems are well-known. A mapping $\gamma: (M,g) \longrightarrow (M,\bar{g})$ is geodesic if and only if the Christoffel symbols are related by

$$(1) \qquad \bar{\nabla}_X Y = \nabla_X Y + \psi(X) Y + \psi(Y) X \quad ,$$

where $\psi(X)$ is locally a gradient. There is a geodesic correspondence between (M,g) and $(M,\bar{g})$ iff there exists a vector field $\psi(X)$ on M with the property

$$(2) \qquad (\nabla_X \bar{g})(Y,Z) = 2\,\psi(X)\bar{g}(Y,Z) + \psi(Y)\bar{g}(X,Z) + \psi(Z)\bar{g}(X,Y)$$

for any vector fields X,Y and Z. In the sequel the geodesic mapping γ determined by vector field $\psi(X)$ will be denoted by $\gamma: (M,g) \xrightarrow{\psi} (M,\bar{g})$. As it was shown in [8] this theorem is equivalent to the following one: a manifold (M,g) admits a non-trivial geodesic mapping iff there exists a non-singular symmetric covariant tensor field a of degree 2 satisfying

$$(3) \qquad (\nabla_X a)(Y,Z) = \lambda(Y) g(X,z) + \lambda(Z) g(X,Y) \; ,$$

where $\lambda(X)$ is a certain 1-form. The tensor field a we can take as a new metric tensor on M.

In [8] N.S. Sinyukov has proved that if (M,g) admits geodesic mappimg onto $(M,\bar{g})$, then (M,a) admits geodesic mapping onto $(M,\hat{a}=\exp(2\psi)g)$ with the same 1-form $\psi(X)$. For better under-

This paper is in final form and no version of it will be submitted for publication elsewhere.

standing of manifolds and mappings among them, we introduce the following diagram

$$\begin{array}{ccc} (M,g) & \xrightarrow{\Psi} & (M,\bar{g}) \\ \big\downarrow \text{conf.} & & \\ (M,\tilde{a}=\exp(2\psi)g) & \xleftarrow{\Psi} & (M,a) \end{array} .$$

This process can indefinitely being continued. In this way we obtain an infinite sequence of Riemannian manifolds admitting geodesic mappings.

J. Mikeš has proved that if it is possible to map geodesically (M,g) onto an Einstein manifold $(M,\bar{g})$ then (M,g) is also an Einstein manifold.

A manifold (M,g) is said to be generalized Einstein manifold if the following condition is satisfied

$$(\nabla_X S)(Y,Z) = \sigma(X)g(Y,Z) + \nu(Y)g(X,Z) + \nu(Z)g(X,Y), \tag{4}$$

where $S(X,Y)$ is the Ricci tensor of (M,g) and $\sigma(X)$, $\nu(X)$ are certain 1-forms. The generalized Einstein manifold is manifold with harmonic conformal curvature tensor.

It is known ([1] , [8] , [10]) that if an Einstein manifold (M,g) can be geodesically mapped onto $(M,\bar{g})$, then $(M,\tilde{a})$ is a generalized Einstein manifold.

In this paper we shall studied properties of conformal and geodesic mappings of generalized Einstein manifolds. We shall give the local classification of generalized Einstein manifolds when $g(\Psi(X), \Psi(X)) \neq 0$. **If $\Psi(X)$ is a null vector on a generalized Einstein manifold (M,g) then (M,g) is an Einstein manifold [2] .**

1. PRELIMINARIES. **Let (M,g) be a Riemannian manifold with a metric g. If $\tilde{g}$ is an another metric on M and if there exists a function φ on M such that $\tilde{g} = \exp(2\varphi)g$, then we say that the metrics g and $\tilde{g}$ are conformally related. It is well known that the Christoffel symbols, the Riemannian curvatures and the Ricci tensors of (M,g) and $(M,\tilde{g})$ are related by**

$$\tilde{\nabla}_X Y = \nabla_X Y + \pi(X)Y + \pi(Y)X - g(X,Y)U \tag{5}$$

with 1-form $\pi = d\varphi$ and vector field U which is defined by

$g(X,U) = \pi(X)$,

(6) $\tilde{R}(X,Y)Z = R(X,Y)Z + s(Y,Z)X - s(X,Z)Y + g(Y,Z)TX - g(X,Z)TY$,

where s and T are the tensor fields defined by

(7) $s(X,Y) = (\nabla_X \pi)(Y) - \pi(X)\pi(Y) + \frac{1}{2}\pi(U)g(X,Y)$,
$g(TX,Y) = s(X,Y)$.

The Weyl conformal curvature tensor field

(8) $C(X,Y)Z = R(X,Y)Z + L(Y,Z)X - L(X,Z)Y + g(Y,Z)PX - g(X,Z)PY$,

where

(9) $L(X,Y) = \frac{1}{n-2}\left[S(X,Y) - \frac{r}{2(n-1)} g(X,Y)\right]$,
$g(PX,Y) = L(X,Y)$ and r is the scalar curvature of g, is an invariant of the conformal transformation. We also have

(10) $D(X,Y,Z) = \tilde{D}(X,Y,Z) + \varphi(C(X,Y)Z)$,

where

(11) $D(X,Y,Z) = (\nabla_X L)(Y,Z) - (\nabla_Y L)(X,Z)$,
$\tilde{D}$ having a similar expression.

2. SOME PROPERTIES OF CONFORMAL AND GEODESIC MAPPINGS OF GENERALIZED EINSTEIN MANIFOLDS.

LEMMA 1 ([1], [10]) . Let g be a generalized Einstein metric on a manifold M. Then (M,g) is a manifold with harmonic conformal curvature tensor, i.e., L(X,Y) is the Codazzi tensor.

LEMMA 2 ([8]) . Let the relation (4) holds on (M,g) . Then

(12)
$$\sigma(X) = \frac{n}{(n-1)(n+2)} \nabla_X r$$
$$\nu(X) = \frac{n-2}{2(n-1)(n+2)} \nabla_X r ,$$

where r is the scalar curvature of (M,g) .

From (3), employing the Ricci identities, in view of Lemma 1 and Lemma 2, we obtain

LEMMA 3. If (M,g) is a generalized Einstein manifold then the relation

$$\lambda(C(X,Y)Z) = 0$$

holds on M.

From (3) and (4) we have

LEMMA 4 ([8]) . For an arbitrary generalized Einstein manifold (M,g), there always exists a Riemannian manifold which is geodesically equivalent to the given manifold (M,g)

LEMMA 5 ([2]). The condition $g(\Psi(X),\Psi(X)) \neq 0$ holds on (M,g) if and only if $g(\lambda(X),\lambda(X)) \neq 0$.

As an immediate consequence of Lemma 3 and (10) we obtain the following

PROPOSITION 1. Let (M,g) be a generalized Einstein manifold. Then the manifold $(M,\tilde{g}=\exp(2\lambda)g)$, where $\nabla_X\lambda = \lambda(X)$, is a manifold with harmonic conformal curvature tensor.

Suppose that there is given a geodesic mapping $\gamma : (M,g) \xrightarrow{\Psi} (M,\tilde{g})$ satisfying the condition $g(\Psi(X),\Psi(X)) \neq 0$. Then we have

THEOREM 1. A necessary and sufficient condition for a Riemannian manifold (M,g) to be a generalized Einstein manifold is that

$$S(X,Y) = \omega \cdot a(X,Y) + (\sigma + C_1)\, g(X,Y) \quad , \tag{13}$$

where

$C_1, \omega = \text{const.}$, $\omega \neq 0$, σ is a function such that $\nabla_X\sigma = \sigma(X)$ (see (4)).

Proof. In the local coordinate system (U,x^i), the conditions of integrability of equations (3) are

$$a_{ti}R^t_{jkl} + a_{tj}R^t_{ikl} = \lambda_{,li}g_{jk} + \lambda_{,lj}g_{ik} - \lambda_{,ki}g_{jl} - \lambda_{,kj}g_{il} \quad , \tag{14}$$

where the comma indicates covariant differentiation with respect to the metric g. Contracting now (14) with g^{jk} we obtain

$$a_{it}S^t_k = a_{kt}S^t_i \quad . \tag{15}$$

Hence by the covariant differentiation, in view of (3) and (4), we find

$$a^t_i\,\nu_{,t} - \frac{a}{n}\,\nu_{,i} = \lambda^t S_{ti} - \frac{r}{n}\,\lambda_i \tag{16}$$

and

$$\lambda_k(S_{ij} - \tfrac{r}{n}g_{ij}) - \lambda_i(S_{jk} - \tfrac{r}{n}g_{jk}) = \nu_{,k}(a_{ij} - \tfrac{a}{n}g_{ij}) - \nu_{,i}(a_{kj} - \tfrac{a}{n}g_{kj}) \quad , \tag{17}$$

where

$$a = a_{pt}g^{pt} \quad .$$

Transvecting (14) with λ^l and using Lemma 3 and (17), after straightforward calculations, we obtain

(18) $$\lambda_{,ij} = \frac{1}{n-2}\left[a_j^t S_{it} - \frac{r}{n(n-1)} a_{ij} - \frac{a}{n} S_{ij} + (n-2)\rho_1 g_{ij}\right],$$

where

$$\rho_1 = \frac{\lambda}{n} - \frac{1}{n(n-2)} a^{tr} S_{tr} + \frac{r a}{n(n-1)(n-2)}, \lambda = \lambda_{,tr} g^{tr}.$$

Similarly, from (4), we find

(19) $$\nu_{,ik} = \frac{1}{n-2}\left[S_i^t S_{kt} - \frac{r}{n-1} S_{ik} + (n-2)\rho_2 g_{ik}\right],$$

where

$$\rho_2 = \frac{1}{n}\left[\nu + \frac{r^2}{(n-1)(n-2)} - \frac{1}{n-2} S^{rt} S_{rt}\right], \nu = \nu_{,rt} g^{rt}.$$

Differentiating covariantly (16) and alternating the resulting equality, in view of (3),(4),(18) and (19), we obtain

(20) $$\nu_i = \omega \lambda_i .$$

The formula (17), in virtue of the above equality and Lemma 5, implies

(21) $$S_{ij} = \omega a_{ij} + \mu g_{ij} .$$

Hence by the covariant differentiation and making use of (3) and (4), we obtain $\omega = \text{const} \neq 0$ and $\mu_i = \sigma_i$. The converse part of the theorem is obvious.

Let g be a generalized Einstein metric on a manifold M. The manifold (M,g) admits a geodesic mapping $\gamma : (M,g) \xrightarrow{\Psi} (M,\bar{g})$. According to the theorem of Sinyukov the manifold $(M,\tilde{g} = \exp(2\Psi)g)$ admits the geodesic mapping $(M,\tilde{g}) \xrightarrow{-\Psi} (M,a)$, where a is a tensor field satisfying (3). We shall prove

PROPOSITION 2. If (M,g) is a generalized Einstein manifold, then

(i) the relation $\Psi(C(X,Y)Z) = 0$ holds on (M,g) ,

(ii) the relation $(\tilde{\nabla}_X \tilde{g})(Y,Z) = \tilde{\lambda}(Y)\tilde{g}(X,Z) + \tilde{\lambda}(Z)\tilde{g}(X,Y)$ holds on $(M,\tilde{g})$ (s.(3)) ,

(iii) the manifold $(M,\tilde{g})$ is a manifold with harmonic conformal curvature tensor.

Proof. (i) From (2), by Ricci-identity and making use of (13), we obtain (s. [7] p.294)

$$\Psi^t_{,k} a_{tj} = \Psi^t_{,j} a_{tk} \quad \text{and} \quad \Psi^t_{,k} S_{tj} = \Psi^t_{,j} S_{tk} .$$

Transvecting (13) with Ψ^j, differentiating covariantly and alternating the resulting equality, in view of (4), we obtain

(22) $\lambda_i = - \theta \psi_i$

and

(23) $a_{it} \lambda^t = \theta \lambda_i$.

Differentiating covariantly this equation and alternating the resulting relation, we obtain $\theta_i = \theta'(\lambda) \lambda_i$. From (23), by covariant differentiation and making use of (13), we have

(24) $\lambda_t \lambda^t \cdot (\theta' - 2) = 0$.

Now, from Lemma 4 and (22) we obtain our assertion.

(ii) This relation is an immediate consequence of (5) and (1)

(iii) This follows from (i) .

THEOREM 2. A manifold (M,g) is a generalized Einstein manifold if and only if one of the following two conditions is satisfied :

(i) $(M,\tilde{g}=\exp(2\Psi)\, g)$ is an Einstein manifold admitting a geodesic mapping,

(ii) $(M,\tilde{g})$ is also a generalized Einstein manifold.

Moreover, if (M,g) and $(M,\tilde{g})$ are Einstein manifolds, then $(M,\tilde{g})$ and (M,a) are Ricci-flat manifolds. In this case $(M,\tilde{a}=\exp(2\Psi)a)$ is a generalized Einstein manifold.

Proof. Differentiating covariantly (14) and contracting with g^{lm} the resulting equality, in view of (3) and Lemma 1, we obtain

$$\lambda_t R^t_{jki} + \lambda_t R^t_{ikj} + \frac{1}{2(n-1)}(r^t a_{ti} g_{kj} + r^t a_{tj} g_{ik}) - $$
$$- \frac{1}{2(n-1)} (a_{ki} r_j + a_{kj} r_i) = a_i g_{jk} + a_j g_{ik} - \lambda_{,kij} - \lambda_{,kji}$$

where

$$r_i = \nabla_i r \ , \quad a_i = \lambda_{,ipt} g^{pt} \quad ,$$

which, by antisymmetrization in i,k and application of (23) and (13), gives

(25) $$\lambda_{,kij} = \frac{1}{n-2} \left[(S_{ij} - \eta g_{ij}) \cdot \lambda_k + 2(S_{ik} - \eta g_{ik}) \cdot \lambda_j + (S_{jk} - \eta g_{jk}) \cdot \lambda_i \right] \quad ,$$

where

$$\eta = \frac{r}{n-1} - (\omega \theta + \sigma + C) \ .$$

The above equation, together with (4), (5), (6), (7), (9), (22), (20) and (24), gives

(26) $\tilde{L}_{ij,k} = \tilde{\nu}_k \tilde{g}_{ij} + \tilde{\nu}_i \tilde{g}_{jk} + \tilde{\nu}_j \tilde{g}_{ik}$,

where the comma denotes covariant differentiation with respect to $\tilde{g}$ and $\tilde{\nu}_k = \frac{n-2}{2(n-1)(n+2)} \tilde{\nabla}_k \tilde{r}$. If the scalar curvature $\tilde{r}$ = const. then, according to the paper [5], $(M,\tilde{g})$ is an Einstein manifold. The converse follows from (26) and [1], [10].

3. CLASSIFICATION OF GENERALIZED EINSTEIN MANIFOLDS. We consider the following cases:

(i) the manifold $(M,\tilde{g})$ is an Einstein manifold. Then, as an immediate consequence of [1] and [3], we have

THEOREM 3. A manifold (M,g) is a generalized Einstein manifold iff in some coordinate system a metric form of M takes one of the following forms

$$(27)\qquad ds^2 = \frac{A}{(KA-\bar{K}(x^1)^2)\cdot(cA(x^1)^2-1)}(dx^1)^2 + \frac{cA(x^1)^2-1}{2c} g^*_{\alpha\beta}dx^\alpha dx^\beta,$$

$$(28)\qquad ds^2 = \frac{A}{(KA-K(x^1)^2)\cdot(cA(x^1)^2-1)}(dx^1)^2 + \frac{cA(x^1)^2-1}{2c} g^*_{\alpha_1\beta_1}dx^{\alpha_1}dx^{\beta_1} + \frac{\bar{K}(x^1)^2-AK}{2cAK} g^*_{\alpha_2\beta_2}dx^{\alpha_2}dx^{\beta_2},$$

where

$c, A, K, \bar{K}$ = const. $\neq 0$, $g^*_{\alpha\beta}dx^\alpha dx^\beta$ is a metric of $(n-1)$-dim. Einstein manifold with $R^*_{\alpha\beta} = (n-2)K^* g^*_{\alpha\beta}$, $K^* = \frac{cA^2K-\bar{K}}{2cA}$, $g^*_{\alpha_1\beta_1}dx^{\alpha_1}dx^{\beta_1}$ is a metric of $(m-1)$-dim. $(m<n)$ Einstein manifold with $R^*_{\alpha_1\beta_1} = (m-2)K^*_1 g^*_{\alpha_1\beta_1}$, $K^*_1 = \frac{cA^2K-K}{2cA}$, $g^*_{\alpha_2\beta_2}dx^{\alpha_2}dx^{\beta_2}$ is a metric of $(n-m)$-dim. Einstein manifold with $R^*_{\alpha_2\beta_2} = (n-m-1)K^*_2 g^*_{\alpha_2\beta_2}$, $K^*_2 = \frac{\bar{K}-cA^2K}{2cA}$,
$\alpha,\beta = 2,\dots,n$, $\alpha_1,\beta_1 = 2,\dots,m$, $\alpha_2,\beta_2 = m+1,\dots,n$,

$$(29)\qquad ds^2 = \frac{h}{K(x^1)^2(cA(x^1)^2-1)}(dx^1)^2 + \frac{hA(cA(x^1)^2-1)}{2} g^*_{\alpha\beta}dx^\alpha dx^\beta$$

where

$R^*_{\alpha\beta} = (n-2)\frac{AK}{2}g^*_{\alpha\beta}$, h = const. $\neq 0$, $\alpha,\beta = 2,\dots,n$,

$$(30)\qquad ds^2 = \frac{1}{c_1c_2\dots c_m x^1x^2\dots x^m} \prod_{q\neq p}{}' \frac{(x^p-x^q)}{Q(x^p)}(dx^p)^2 + \frac{(a_1-x^1)\dots(a_1-x^m)}{c_1\dots c_m x^1\dots x^m} g^*_{\sigma_1\mu_1}dx^{\sigma_1}dx^{\mu_1} + \dots + \frac{(a_k-x^1)\dots(a_k-x^m)}{c_1\dots c_m x^1\dots x^m} g^*_{\sigma_k\mu_k}dx^{\sigma_k}dx^{\mu_k},$$

where

$Q(z) = 4Kz^{m+1} + B_m z^m + \dots B_1 z + (-1)^{m+1}4C$, $a_1,\dots,a_k$, $C, c_1,\dots,c_m, B_p$ = const. $\neq 0$, $p = 1,2,\dots,m$, $1<m\leq n-4$,

$k \leq m+1$, $Q(a_\lambda) = 0$, $g^*_{\sigma_\lambda \mu_\lambda}$ are metric tensors of Einstein manifolds $(M^{n_\lambda}, g^*_{\sigma_\lambda \mu_\lambda})$ and $R^*_{\sigma_\lambda \mu_\lambda} = K^*_\lambda g^*_{\sigma_\lambda \mu_\lambda}$, $K^*_\lambda = \frac{(n_\lambda - 1)}{4} Q'(a_\lambda)$, $= 1,2,\dots,k$.

(ii) the manifold $(M,\tilde{g})$ is a generalized Einstein manifold.

Proposition 1, in the same way as in the proof of theorem 1 of [1], gives

PROPOSITION 3. If $(M,\tilde{g})$ is a generalized Einstein manifold then the geodesic mapping $\gamma : (M,g) \longrightarrow (M,\bar{g})$ is normal.

If manifolds (M,g) and $(M,\bar{g})$ admit a normal geodesic mapping, then their metrics are of the form ([8], p. 117)

$$ds^2 = f(u^1)(du^1)^2 + \underset{1}{P}{}^\sigma_\alpha \, g^*_{\sigma\beta}(u^\lambda)\, du^\alpha du^\beta , \tag{31}$$

$$d\bar{s}^2 = c \exp(4u^1)\, g_{11}(du^1)^2 + \exp(2u^1)\, P'^\sigma_\alpha\, g_{\sigma\beta}\, du^\alpha du^\beta,$$

$$\underset{1}{P}{}^\sigma_\alpha = \frac{1}{2}(P^\sigma_\alpha(u^\lambda) - \frac{1}{c}\exp(-2u^1)\,\delta^\sigma_\alpha), \quad \Psi = u^1, \quad \alpha, \beta, \sigma, \lambda = 2,\dots,n,$$

where

$g^*_{\alpha\beta}\, du^\alpha du^\beta$ is a metric of an $(n-1)$-dim. manifold,

$P_{\alpha\beta} = P^\sigma_\alpha\, g^*_{\sigma\beta}$ is some $(0,2)$ symmetric tensor on (M^{n-1}, g^*) of the rank $n-1$, covariantly constant with respect to metric g^*, and $P'^\sigma_\alpha(u^\lambda)$ denotes the reciprocal of P^σ_α.

Making use of (6) - (9), (25), (4), Proposition 2 and (31), it is easy to verify that the following cases hold

(i) $P_{\alpha\beta} = A g^*_{\alpha\beta}$, $A = \text{const.} \neq 0$,

(ii) (M^{n-1}, g^*) is decomposable and

$$(g^*_{\alpha\beta}) = \begin{bmatrix} (g^*_{\alpha_1\beta_1}) & 0 \\ 0 & (g^*_{\alpha_2\beta_2}) \end{bmatrix},$$

$$(P_{\alpha\beta}) = \begin{bmatrix} A\cdot(g^*_{\alpha_1\beta_1}) & 0 \\ 0 & B\cdot(g^*_{\alpha_2\beta_2}) \end{bmatrix}.$$

Consequently, we have

THEOREM 4. A manifold (M,g) is a generalized Einstein manifold iff in some coordinate system a metric form of M takes one of the following forms

$$ds^2 = \frac{1}{f}(dx^1)^2 + \frac{cAx^1 - 1}{2cx^1}\, g^*_{\alpha\beta}\, dx^\alpha dx^\beta , \tag{32}$$

$$d\tilde{s}^2 = x^1 ds^2 , \tag{33}$$

where

(34) $$f = 4x^1(cAx^1 - 1)\cdot(-\frac{\omega}{c(n-2)} - \frac{C}{n-1}x^1 + C(x^1)^2),$$

$c, C, A, \omega, \tilde{\omega} = \text{const.}$, $\tilde{\omega} = (n-2)AC$,

$g^*_{\alpha\beta}\, dx^\alpha dx^\beta$ is the metric of an $(n-1)$-dimensional Einstein manifold with $R^*_{\alpha\beta} = (n-2)K^* g^*_{\alpha\beta}$ and

$$K^* = \frac{1}{2}\omega A^2 + \frac{n-2}{2(n-1)}AC - \frac{(n-2)}{2}\frac{C}{c},$$

$\alpha, \beta = 2, \ldots, n$,

(35) $$ds^2 = \frac{1}{f}(dx^1)^2 + \frac{cAx^1-1}{2cx^1}\, g^*_{\alpha_1\beta_1}\, dx^{\alpha_1} dx^{\beta_1} + \frac{cBx^1-1}{2cx^1}\, g^*_{\alpha_2\beta_2}\, dx^{\alpha_2} dx^{\beta_2},$$

(36) $$d\tilde{s}^2 = x^1\, ds^2,$$

where

(37) $$f = 4x^1(Dx^1 + E)\cdot(cAx^1 - 1)\cdot(cBx^1 - 1),$$

$g^*_{\alpha_1\beta_1}\, dx^{\alpha_1} dx^{\beta_1}$ is the metric of an n_1-dimensional Einstein manifold with $R^*_{\alpha_1\beta_1} = (n_1-1)K^*_1 g^*_{\alpha_1\beta_1}$,

$$K^*_1 = \frac{(n_1-1)}{2}D(A-B) + \frac{\omega AB}{2(n-2)} + \frac{\omega A^2}{2} - \frac{\omega A}{2(n-2)}(n_1B + n_2A),$$

$g^*_{\alpha_2\beta_2}\, dx^{\alpha_2} dx^{\beta_2}$ is the metric of an n_2-dimensional $(n_1 + n_2 + 1 = n)$ Einstein manifold with $R^*_{\alpha_2\beta_2} = (n_2-1)\cdot K^*_2\, g^*_{\alpha_2\beta_2}$ and

$$K^*_2 = \frac{(n_2-1)}{2}D(B-A) + \frac{\omega AB}{2(n-2)} + \frac{\omega B^2}{2} - \frac{\omega B}{2(n-2)}\cdot(n_2A + n_1B), \quad \omega = (n-2)cE,$$

$\alpha_1, \beta_1 = 2, \ldots, n_1+1$, $\alpha_2, \beta_2 = n_1+2, \ldots, n$.

REFERENCES

1. FORMELLA S. "On geodesic mappings in Einstein manifolds", Coll. Mathem. Soc. J. Bolyai 46 ,(1984), 483-492 ,
2. FORMELLA S. "On generalized Ricci-recurrent manifolds", PN Polit. Szczec. (to appear) ,
3. FORMELLA S., MIKEŠ J. "A classification of Einstein manifolds admitting geodesic mappings" (to appear) ,

4. MIKEŠ J. "On geodesic mappings of Einstein spaces", Mat. Zam. 28, N6 ,(1980), 935-938 ,
5. MIKEŠ J. "On geodesic mappings of Ricci 2-symmetric Riemannian manifolds", Matem. Zam. 28, N2 ,(1980), 313-317 ,
6. SOŁODOVNIKOV A.S. "Projective mappings of Riemannian spaces", Uspehi Mat. Nauk 11,(1956),
7. SCHOUTEN J.A. "Ricci-Calculus", Springer-Verlag 1954 ,
8. SINYUKOV N.S. "Geodesic mappings of Riemannian spaces", Moscow, Nauka ,(1979),
9. SOBČUK V.C. "On geodesic mappings of generalized Ricci symmetric Riemannian manifolds", Černovcy,(1981),
10. VENZI P. "Uber konforme und geodätische Abbildungen", Result. der Mathemat., Vol.5 ,(1982), 184-198 .

INSTITUTE OF MATHEMATICS, TECHNICAL UNIVERSITY OF SZCZECIN,
AL. PIASTOW 48/49 , 70-310 SZCZECIN, POLAND.