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SIXTY QUESTIONS ON REGULAR NOT PARACOMPACT SPACES

compiled by W. STEPHEN WATSON

This is a list of sixty open problems on regular not paracompact spaces. A variation of almost every open problem on normal not collectionwise normal spaces of which I am aware has been listed. The omission of an interesting open problem should be taken as indicative of my lack of awareness. I would be grateful to receive those portions of the lengthy list of such omissions that exist and I would be pleased to refer anyone interested in a specific question to who I believe to be its source. Less than half of these questions are due to the compiler. These questions have been asked by Balogh, Daniels, Fleissner, Hanazawa, Kunen, Navy, Nyikos, Przymusiński, Reed, Rudin, Tall, Wicke, Zenor and, I am sure, at least one other person I have unfortunately forgotten to mention.

A: NORMAL NOT COLLECTIONWISE HAUSDORFF SPACES

1. Does GCH imply that normal first countable \aleph_1 -collectionwise Hausdorff spaces are collectionwise Hausdorff ?
2. Does CH imply that there is a normal not collectionwise Hausdorff space of character \aleph_2 ?
3. Does CH imply that normal first countable spaces are weakly \aleph_2 -collectionwise Hausdorff ?
4. If normal spaces of character ω are collectionwise Hausdorff, then does CH hold ?
5. If $\diamond(S)$ holds for each stationary set S , then are normal first countable spaces \aleph_1 -collectionwise Hausdorff ?
6. If \mathcal{K} is a strong limit, then are normal first countable spaces which are $<\mathcal{K}$ -collectionwise Hausdorff, \mathcal{K} -collectionwise Hausdorff ?
7. Is there an axiom which implies that first countable normal spaces are \aleph_1 -collectionwise Hausdorff, which follows from

PMEA, \diamond^* , \diamond_{SS} and which holds in the reverse Easton model ?

B: NONMETRIZABLE NORMAL MOORE SPACES

8. Does $2^{\aleph_0} = 2^{\aleph_2}$ imply the existence of a nonmetrizable normal Moore space ?

9. Does the existence of a nonmetrizable normal Moore space imply the existence of a metacompact nonmetrizable normal Moore space ?

10. Does the existence of a nonmetrizable normal Moore space imply the existence of a paralindelöf nonmetrizable normal Moore space ?

11. Does the existence of a nonmetrizable normal Moore space imply the existence of a nonmetrizable normal Moore space of cardinality \mathfrak{c} ?

12. Does the existence of a nonmetrizable normal Moore space imply the existence of a normal Moore space which is not collectionwise normal with respect to metrizable sets ?

13. Are normal Moore spaces submetrizable ?

14. Are Čech-complete locally connected normal Moore spaces metrizable ?

C: LOCALLY COMPACT NORMAL SPACES

15. Does ZFC imply that normal locally compact metacompact spaces are paracompact ?

16. Does ZFC imply that there is a normal locally compact space which is not collectionwise normal ?

17. Does ZFC imply that there is a perfectly normal locally compact space which is not paracompact ?

18. Does ZFC imply that normal manifolds are collectionwise normal ?

19. Does the existence of a locally compact normal space which is not collectionwise Hausdorff imply the existence of a first countable normal space which is not collectionwise Hausdorff ?

20. Does normal + locally compact + locally connected imply collectionwise normal ?

21. Does ZFC imply that normal manifolds are collectionwise Hausdorff ?

22. Does $V = L$ imply that normal locally compact spaces are collectionwise normal ?

23. Does $V = L$ imply that perfectly normal locally compact spaces are collectionwise normal ?

24. Does ZFC imply that locally compact normal collectionwise Hausdorff spaces are collectionwise normal ?

25. Does ZFC imply that there is a normal extremally disconnected locally compact space which is not paracompact ?

D: COUNTABLY PARACOMPACT NOT COLLECTIONWISE NORMAL SPACES

26. Does ZFC imply that there is a countably paracompact locally compact space which is not collectionwise Hausdorff ?

27. Does $2^{\aleph_0} < 2^{\aleph_1}$ imply that separable first countable countably paracompact spaces are collectionwise Hausdorff ?

28. Does countably paracompact + locally compact + metacompact imply paracompact ?

29. Does \diamond^* imply that countably paracompact first countable spaces are \aleph_1 -collectionwise Hausdorff ?

30. Does PMEA imply that countably paracompact first countable spaces are collectionwise Hausdorff ?

31. Does $2^{\aleph_0} < 2^{\aleph_1}$ imply that special Aronszajn trees are not countably paracompact ?

32. If the continuum function is 1:1 and X is a countably paracompact first countable space, then does $e(X) \leq c(X)$?

33. Does ZFC imply that there is a nonmetrizable countably paracompact Moore space ?

34. Is it consistent that countably paracompact first countable spaces are collectionwise normal with respect to metrizable sets ?

E: NORMAL COLLECTIONWISE HAUSDORFF NOT COLLECTIONWISE NORMAL SPACES

35. Does $V = L$ imply that normal first countable spaces are collectionwise normal with respect to separable sets ?

36. Does $V = L$ imply that normal first countable spaces are collectionwise normal with respect to copies of ω_1 ?

37. Does ZFC imply that there is a metacompact normal space of character ω which is not collectionwise normal ?

38. Is there a normal not collectionwise normal space which

is collectionwise normal with respect to collectionwise normal sets ?

39. Is there a normal collectionwise Hausdorff space which is not collectionwise normal with respect to \aleph_1 -many compact sets ?

40. Is it consistent that there is a normal first countable collectionwise Hausdorff space which is not collectionwise normal with respect to compact sets ?

41. Does ZFC imply that normal first countable collectionwise Hausdorff spaces are collectionwise normal with respect to scattered sets ?

F: PARALINDELÖF SPACES

42. Does paralindelöf + collectionwise normal imply paracompact ?

43. Does paralindelöf imply countably paracompact ?

44. Is there a paralindelöf Dowker space ?

45. Does paralindelöf + screenable imply normal ?

46. Does paralindelöf + screenable + normal imply paracompact ?

47. Does ZFC imply that there is a paralindelöf space of character c which is not paracompact ?

G: DOWKER SPACES

48. Does ZFC imply that there is a hereditarily normal Dowker space ?

49. Does ZFC imply that there is a screenable normal space which is not paracompact ?

50. Does ZFC imply that there is a Dowker space of cardinality less than \aleph_ω ?

51. Does ZFC imply that there is a first countable Dowker space ?

52. Does ZFC imply that normal spaces with a \mathcal{G} -disjoint base are collectionwise normal ?

53. Does the existence of a screenable normal space which is not paracompact imply the existence of a screenable collectionwise normal space which is not paracompact ?

54. Does ZFC imply that there is a scattered Dowker space ?

55. Does ZFC imply that there is a \mathcal{G} -discrete Dowker space ?

H: MISCELLANEOUS

56. Does ZFC imply that the Pixley-Roy space of the countable topology on ω_1 is collectionwise Hausdorff ?

57. Does ZFC imply that there is a first countable $\aleph_1^{\aleph_1}$ -collectionwise Hausdorff space which fails to be collectionwise Hausdorff?

58. Does metalindelöf + collectionwise normal imply paracompact ?

59. Does ZFC imply that collectionwise Hausdorff ω_1 -trees are countably paracompact ?

60. Does ZFC imply that there is a collectionwise Hausdorff Moore space which is not collectionwise normal with respect to compact sets ?

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