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A UNIVERSAL FUNCTION FOR CONTINUOUS FUNCTIONS

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In the descriptive set theory the codings of continuous functions play important role. Namely, the proof of so called Coding lemma (see [2] or [3], p. 426) uses a special kind of such a coding satisfying the Kleene recursion theorem. The classical construction of those codings is based on the theory of recursive functions. In this note we shall try to show that such codings possesse many typical recursive theoretic properties independently of their construction.

Let $\mathcal{N}$ denote the Baire space ${}^{\omega}\omega$. If F is a function defined on a set $A \subseteq X_1 \times \ldots \times X_n$ and $\alpha \in X_1$ we denote by F_α the function with the domain

$$\mathcal{D}(F_\alpha) = \{[x_2, \ldots, x_n] \in X_2 \times \ldots \times X_n ; [\alpha, x_2, \ldots, x_n] \in A\}$$

defined by

$$F_\alpha(x_2, \ldots, x_n) = F(\alpha, x_1, \ldots, x_n) .$$

Let X be a Polish space, U being a function defined on a subset A of $\mathcal{N} \times X$ with values in $\mathcal{N}$. U is said to be a <u>universal function</u> for continuous functions on X iff the following holds true:

(1) the domain $A = \mathcal{D}(U)$ is a G_δ-subset of $\mathcal{N} \times X$ and U is continuous on A ;

(2) for every continuous function f from a G_δ-subset of X into $\mathcal{N}$ there exists <u>a code</u> $\alpha \in \mathcal{N}$ such that $f = U_\alpha$.

Let us remark that the conditions (1) and (2) are stronger than (i) and (ii) of the theorem 7A.1 in [3], p. 382. However, we need this strengthening in our investigations.

A system $\{U^X ; X \text{ Polish space}\}$ is called <u>a universal system</u> for continuous functions iff U^X is a universal function for continuous functions on X for every Polish space X .

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One can easily check that the Kleene's system as described in [3], pp. 381-382 is a universal system. Moreover, this system possesses the <u>s-property</u>:

(3) there exist continuous functions

$$s^{X,Y}: \mathcal{N} \times X \longrightarrow \mathcal{N}$$

such that for each $\alpha \in \mathcal{N}$, $x \in X$ the following holds true

$$(U_\alpha^{X\times Y})_x = U^Y_{s^{X,Y}(\alpha,x)} .$$

The superscripts X,Y will be usually omitted.

Analogously to the proof of the Rice theorem in the recursion theory (see [4] or [1], p. 102), we can prove

<u>Theorem 1</u>. Let $\{U^X;\ X \text{ Polish space}\}$ be a universal system satisfying the s-property. Let f be a continuous function from a G_δ-subset A of X into $\mathcal{N}$. Then the set C of codes of f

$$C = \{\alpha \in \mathcal{N};\ U_\alpha^X = f\}$$

contains a perfect subset.

PROOF. Let D be a G_δ-subset of $\mathcal{N}$ such that $\mathcal{N}-D$ is not a G_δ-set. We set

$$F(x,y) = f(y) \quad \text{for} \quad x \in D,\ y \in A;$$
$$\text{undefined otherwise.}$$

Then F is a continuous function with a G_δ-domain $D \times A \subseteq \mathcal{N} \times X$. Hence there exists a code $\beta \in \mathcal{N}$ of F:

$$F = U_\beta^{\mathcal{N}\times X} .$$

By the definition of the function F we have

$$x \in D \equiv F_x = f .$$

By the s-property we have

$$(U_\beta^{\mathcal{N}\times X})_x = U^X_{s(\beta,x)} .$$

Thus

$$x \in D \equiv U^X_{s(\beta,x)} = f \equiv s(\beta,x) \in C ,$$

i.e.

$$D = s_\beta^{-1}(C) = s_\beta^{-1}(s_\beta(D)) .$$

Since s is continuous, $s_\beta(D)$ is an analytic subset of C. Moreover, since D is not a F_σ-set, $s_\beta(D)$ is uncountable. Therefore $s_\beta(D) \subseteq C$ contains a perfect subset.

q.e.d.

Similarly we can prove

<u>Theorem 2</u>. Let $\{U^X;\ X \text{ Polish space}\}$ be a universal system satisfying the s-property. Then there exists a continuous function c^X from $\mathcal{N} \times \mathcal{N}$ into $\mathcal{N}$ such that

$$U^X_{c^X(\alpha_1,\alpha_2)} = U^X_{\alpha_1} \circ U^{\mathcal{N}}_{\alpha_2}$$

for any $\alpha_1, \alpha_2 \in \mathcal{N}$.

PROOF. We denote

$$F(\alpha_1,\alpha_2,x) = U^{\mathcal{N}}(\alpha_2, U^X(\alpha_1,x)) .$$

Then F is continuous with a G_δ-domain. Hence, there exists a code $\alpha \in \mathcal{N}$ such that

$$F = U_\alpha^{\mathcal{N}\times\mathcal{N}\times X} .$$

It suffices to set

$$c^X = s_\alpha^{\mathcal{N}\times\mathcal{N},X} .$$

q.e.d.

In the theory of recursive functions it is shown that all universal functions are isomorphic in a certain sense. A similar result holds true in our case.

Theorem 3. Let $\{U^X$, X Polish space$\}$, $\{V^X$, X Polish space$\}$ be universal systems, $\{U^X\}$ satisfying the s-property. Then there exists a system $\{f^X$, X Polish space$\}$ of continuous functions such that

$$V^X_\alpha = U^X_{f^X(\alpha)}$$

for each $\alpha \in \mathcal{N}$, X Polish space.

PROOF. Since V^X is a continuous function with a G_δ-domain, there exists a code $\beta \in \mathcal{N}$ such that

$$V^X = U_\beta^{\mathcal{N}\times X} .$$

The function

$$f^X = s_\beta^{\mathcal{N},X}$$

has the required property.

q.e.d.

If $\underset{\sim}{\Gamma}$ is a pointclass in the sense of Y. Moschovakis [3], we can introduce the notion of a universal function and a universal system for $\underset{\sim}{\Gamma}$-measurable functions replacing the words "continuous" and "G_δ" by "$\underset{\sim}{\Gamma}$-measurable" and "$\bigwedge^\omega \underset{\sim}{\Gamma}$". However, the definition of the s-property remains as before.

One can easily formulate and prove similar results for universal systems for $\underset{\sim}{\Gamma}$-measurable functions for a reasonable pointclass $\underset{\sim}{\Gamma}$ (e.g. for a Σ^*-pointclass). However, we were not succesfull in proving the existence of a universal system with the property (1) in the general case. Actually, if the function U^X is defined as in [3], p. 382, then you can immediately see that the graph of U^X is a G_δ-set (in the general case it is a $\bigwedge^\omega \underset{\sim}{\Gamma}$-set). However, we need some kind of topological invariance to show that the domain of U^X is a G_δ-set (a $\Lambda^\omega \underset{\sim}{\Gamma}$-set). In the continuous case it is a trivial consequence of the invariance of G_δ-sets under homeomorphisms.

Using the ideas of the proof of the good parametrization lemma from [3], pp. 183-185, we can easily construct a system $\{U^X$, X Polish space$\}$ of functions satisfying the property (2) for $\underset{\sim}{\Sigma}^0_\xi$-sets and the s-property. A suitable modification of the theorem 1 holds true for such a system, because the property (1) is not used in the proof of this theorem. As an example of possible generalizations, we shall formulate exactly this result.

__Theorem 4__. Let $\xi \in \omega_1$ be an ordinal, $\{U^X$, X Polish space$\}$ being a system of $\underset{\sim}{\Sigma}^0_\xi$-measurable function such that:

(4) U^X is defined on a subset of $\mathcal{N} \times X$ with values in $\mathcal{N}$;

(5) if f is a $\underset{\sim}{\Sigma}^0_\xi$-measurable function from a $\underset{\sim}{\Pi}^0_{\xi+1}$-subset of X into $\mathcal{N}$, then there exists a code $\alpha \in \mathcal{N}$ such that $f = U^X_\alpha$.

Then for any $\underset{\sim}{\Sigma}^0_\xi$-measurable function defined on a $\underset{\sim}{\Pi}^0_{\xi+1}$-set, the set $C = \{\alpha \in \mathcal{N};\ f = U^X_\alpha\}$ contains a perfect subset.

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SUBSETS OF βN WITHOUT AN INFIMUM IN RUDIN-FROLÍK ORDER

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Abstract. We prove the existence of a set of $2^{2^{\aleph_0}}$ ultrafilters incomparable in Rudin-Frolík order of $\beta N - N$ which is bounded from below and no its subset with more than one point has an infimum.

§ 0. Introduction. In [2] we have constructed a Simon point in $\beta N - N$, i.e. a nonminimal ultrafilter in Rudin-Frolík order of $\beta N - N$ (shortly written RF) without an immediate predecessor. By a modification of this construction we shall prove

THEOREM. There exists a set $Q \subseteq \beta N - N$ of mutually incomparable ultrafilters in RF such that

1) $|Q| = 2^{2^{\aleph_0}}$,

2) Q is bounded from below, i.e. there is an ultrafilter p such that $p < q$ for each $q \in Q$,

3) for any subset $A \subseteq Q$, $|A| > 1$ the infimum $\inf A$ does not exist.

We use the technique of independent linked families developed by K.Kunen, his concept of the OK-point and some ideas from his proof of the existence of $2^{2^{\aleph_0}}$ distinct OK-points [4].

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§ 1. Preliminaries. We use the standard notation and terminology to be found e.g. in [1], [4], [5].

Recall that the type of an ultrafilter p is the set $\tau(p) = \{q \in \beta N$; there exists a homeomorphism h of βN onto βN such that $h(p) = q$, i.e. $p \approx q\}$.

If $p \in \beta N$, $X = \{x_n ; n \in \omega\}$ is a countable discrete set of ultrafilters in βN then

$$\Sigma(X, p) = \{A \subseteq N ; \{n ; A \in x_n\} \in p\}.$$

Conversely, if $q \in \overline{X}$ then there exists a unique ultrafilter $\Omega(X, q)$ such that $\Sigma(X, \Omega(X, q)) = q$.

If $q = \Sigma(X,p)$ for some countable discrete set $X \subseteq \beta N$, then $p \leq q$ in Rudin-Frolík order of ultrafilters in βN. Let us remark that for $p \leq q$, $\tau(p) = \tau(p')$, $\tau(q) = \tau(q')$ we have also $p' \leq q'$, i.e. the Rudin-Frolík order of βN is also an order of types of ultrafilters.

If $p \in \overline{X \cap (\overline{Y} - Y)}$, X, Y being countable discrete sets, then $\Omega(X,p) < \Omega(Y,p)$.

An ultrafilter p has an immediate predecessor q in RF iff there exists a countable discrete set X of minimal ultrafilters in RF such that $p = \Sigma(X,q)$.

If $\mathcal{F}$ is a filter then $\mathcal{F}^*$ is the dual ideal. If G is a centered system of sets then (G) denotes a filter generated by this system. F refers to the Fréchet filter.

Definition 1.1. A set of filters $\{\mathcal{F}_{n,m};\, n,m \in \omega\}$ is stratified iff

(1) the set $\{\mathcal{F}_{n,m};\, m \in \omega\}$ is discrete for each $n \in \omega$, i.e. there exists a set $\{A_{n,m} \subseteq N;\, m \in \omega\}$ satisfying $A_{n,m} \in \mathcal{F}_{n,m}$ and $A_{n,m} \cap A_{n,\ell} = \emptyset$ for $m \neq \ell$.

(2) the filter $\mathcal{F}_{n,m}$ is in the closure of the set $\{\mathcal{F}_{n+1,\ell};\, \ell \in \omega\}$ for each $n,m \in \omega$, i.e. for each $A \in \mathcal{F}_{n,m}$ the set $\{\mathcal{F}_{n+1,\ell};\, A \in \mathcal{F}_{n+1,\ell}\}$ is infinite.

Definition 1.2. Let $\{\mathcal{F}_{n,m};\, n,m \in \omega\}$ be a stratified set of filters and C be its subset. We define

$C(0) = C$

$C(\alpha) = \bigcup_{\beta < \alpha} C(\beta)$, if α is limit,

$C(\alpha+1) = C(\alpha) \cup \{\mathcal{F}_{n,m};\, \exists B \in \mathcal{F}_{n,m}$ such that $\{\mathcal{F}_{n+1,\ell};\, B \in \mathcal{F}_{n+1,\ell}\} \subseteq C(\alpha)\}$

and $\widetilde{C} = \bigcup_{\alpha < \omega_1} C(\alpha)$.

Definition 1.3. Let $\{\mathcal{F}_{n,m};\, n,m \in \omega\}$ be a stratified set of filters. We shall say that this set satisfies the property (P) for the partition $\{D_i;\, i \in \omega\}$ of ω iff

(1) D_i or $\omega - D_i$ belongs into $\mathcal{F}_{n,m}$ for each $n,m \in \omega$

(2) if $C = \{\mathcal{F}_{n,m};\, (\exists i \in \omega)(D_i \in \mathcal{F}_{n,m})\}$ and $\mathcal{F}_{k,\ell} \notin \widetilde{C}$ then there exist sets U_α, $\alpha < 2^{\aleph_0}$, $U_\alpha \in \mathcal{F}_{k,\ell}$ such that for each $i \in \omega$ and each $\alpha_1 < \alpha_2 < \ldots < \alpha_i$, $U_{\alpha_1} \cap U_{\alpha_2} \cap \ldots \cap U_{\alpha_i} \cap D_i$ is finite.

The property (P) is derived from the Kunen's definition of OK-point [4].

Definition 1.4. A set of ultrafilters $\{q_{n,m};\, n,m \in \omega\}$ is called well stratified iff

(1) it is stratified

(2) it has the property (P) for each partition of ω.

Definition 1.5. A set of ultrafilters $\{q_{n,m} ; n,m \in \omega\}$ is called well stratified set with uniform predecessor p iff

(1) $\{q_{n,m} ; n,m \in \omega\}$ is a well stratified set

(2) $\Omega(X_{n+1}, q_{n,m}) \approx p$ for every $n,m \in \omega$; $X_n = \{q_{n,m} ; m \in \omega\}$.

Definition 1.6. due to K.Kunen [4]. Let $\mathcal{F}$ be a filter on N, $\mathcal{F} \supseteq F$, $A_\eta \subseteq N$ for each $\eta \in J$.

a) Let $1 \leq n < \omega$. An indexed family $\{A_\eta ; \eta \in J\}$ is precisely n-linked with respect to (w.r.t.) $\mathcal{F}$ iff for all $r \in [J]^n$, $\bigcap_{\eta \in r} A_\eta \notin \mathcal{F}^*$, but for all $r \in [J]^{n+1}$, $\bigcap_{\eta \in r} A_\eta$ is finite.

b) An indexed family $\{A_{\eta n} ; \eta \in J, n \in \omega\}$ is a linked system w.r.t. $\mathcal{F}$ iff for each $n \in \omega$, $\{A_{\eta n} ; \eta \in J\}$ is precisely n-linked w.r.t. $\mathcal{F}$ and for each n and η, $A_{\eta n} \subseteq A_{\eta n+1}$.

c) An indexed family $\{A^\xi_{\eta n} ; \eta \in J, \xi \in I, n \in \omega\}$ is a J by I independent linked family (ILF) w.r.t. $\mathcal{F}$ iff for each $\xi \in I$, $\{A^\xi_{\eta n} ; \eta \in J, n \in \omega\}$ is a linked system w.r.t. $\mathcal{F}$ and $\bigcap_{\xi \in u} (\bigcap_{\eta \in r_\xi} A^\xi_{\eta n_\xi}) \notin \mathcal{F}^*$ whenever $u \in [I]^{<\omega}$ and for each $\xi \in u$, $1 \leq n_\xi < \omega$ and $r_\xi \in [J]^{n_\xi}$.

K.Kunen [4] has proved that there exists a 2^ω by 2^ω independent linked family w.r.t. Fréchet filter.

§ 2. Auxiliary results. In this part we prove some important lemmas.

Lemma 2.1. Let p be a minimal ultrafilter in RF and $W = \{q_{n,m} ; n,m \in \omega\}$ be a well stratified set of ultrafilters with a uniform predecessor p. Let $\mathbb{D}$ be a countable discrete set of ultrafilters. If $q_{k,l} \notin \overline{\mathbb{D} \cap W}$ then $q_{k,l} \notin \overline{\mathbb{D}}$.

Proof. Let us consider a countable discrete set $\mathbb{D} = \{d_i ; i \in \omega\}$ such that $\mathbb{D} \cap W = \emptyset$ (without loss of generality). Let $\{D_i ; i \in \omega\}$ be a partition of ω such that $D_i \in d_i$.

Denote $C = \{q_{n,m} \in W ; (\exists i \in \omega)(D_i \in q_{n,m})\}$.

Evidently, if $q_{k,l} \notin \widetilde{C}$ then $q_{k,l} \notin \overline{\mathbb{D}}$ according to the property (P).

Clearly, $C(0) \cap \overline{\mathbb{D}} = \emptyset$. We proceed by induction. Let $C(\alpha) \cap \overline{\mathbb{D}} = \emptyset$ and suppose $C(\alpha+1) \cap \overline{\mathbb{D}} \neq \emptyset$, i.e. there exists $q_{k,l} \in C(\alpha+1)$ such that $q_{k,l} \in \overline{\mathbb{D} \cap \overline{C(\alpha) \cap X_{k+1}}}$.

Then $q_{k,l} \in \overline{\mathbb{D} \cap C(\alpha) \cap X_{k+1}} \cup \overline{\mathbb{D} \cap (\overline{C(\alpha) \cap X_{k+1}} - C(\alpha) \cap X_{k+1})} \cup \overline{C(\alpha) \cap X_{k+1} \cap (\overline{\mathbb{D}} - \mathbb{D})}$.

Evidently $\overline{\mathbb{D} \cap C(\alpha) \cap X_{k+1}} \cup \overline{C(\alpha) \cap X_{k+1} \cap (\overline{\mathbb{D}} - \mathbb{D})} = \emptyset$.

Assume that

$$q_{k,l} \in \overline{\mathbb{D} \cap (\overline{C(\alpha) \cap X_{k+1}} - C(\alpha) \cap X_{k+1})} .$$

Then we have $\Omega(\mathbb{D}, q_{k,l}) < \Omega(X_{k+1}, q_{k,l})$. However, p is of the same type as $\Omega(X_{k+1}, q_{k,l})$. This is a contradiction with the assumption that p is a minimal ultrafilter.

q.e.d.

Lemma 2.2. Let $W = \{ q_{n,m} ; n, m \in \omega \}$ be a stratified set of ultrafilters and $C \subseteq W$.

If $q_{k,l} \in \tilde{C} - C(1)$ then $q_{k,l} \in \overline{C(1) - C}$.

Proof. We proceed by induction. It is evident that if $q_{k,l} \in C(2) - C(1)$ then $q_{k,l} \in \overline{C(1) - C}$.

Suppose that for each $\beta < \alpha$, $q_{s,t} \in C(\beta) - C(1)$ implies $q_{s,t} \in \overline{C(1) - C}$. Let $q_{k,l} \in C(\alpha) - \bigcup_{\beta<\alpha} C(\beta)$, i.e. $q_{k,l} \in \overline{C(\alpha - 1) \cap X_{k+1}}$. By the induction assumption we obtain $q_{k+1,t} \in \overline{C(1) - C}$ for each $q_{k+1,t} \in C(\alpha-1) - C(1)$. Then also $q_{k,l} \in \overline{C(1) - C}$.

q.e.d.

Lemma 2.3. Let $p, q, y_n \in \beta N - N$, $X_i = \{ x_n^i ; n \in \omega \}$, $i = 1, 2$ be discrete sets of ultrafilters such that $\Omega(X_1, q) = p$ and $\Omega(X_2, x_n^1) = y_n$. Then $\Omega(X_2, q) = \Sigma(Y, p)$ where $Y = \{ y_n ; n \in \omega \}$.

Lemma 2.4. Let $W_i = \{ q_{n,m}^i ; n, m \in \omega \}$, $i \in I$ be well stratified sets of ultrafilters with a uniform predecessor p, p being a minimal ultrafilter in RF. Then

a) for each $j < q \in W_i$, $j \not\approx p$ there exists an immediate predecessor of j ,

b) if $j < q_i \in W_i$, $i \in I$ then there exists $r > j$ such that $r < q_i$ for each $i \in I$.

Proof. a) Assume that $j < q$ and $j \not\approx p$. By Lemma 2.1 there exists a countable discrete set $Y \subseteq W_i$ such that $q = \Sigma(Y, j)$. According to the property (P) the ultrafilter q belongs to $\tilde{Y}$. Since $j \not\approx p$, q does not belong into $Y(1)$.

By Lemma 2.2 we have $q \in \overline{Y(1) - Y} - Y(1)$, this means that $q \in \overline{Y(1) \cap (\overline{Y} - Y)} - Y(1)$. Hence $\Omega(Y(1), q) < j$ and $\Omega(Y(1), q) \notin N$.

Using Lemma 2.3 there is a countable discrete set $J = \{ j_n ; j_n \approx p, n \in \omega \}$ such that $j = \Sigma(J, \Omega(Y(1), q))$. J is the set of minimal ultrafilters therefore $\Omega(Y(1), q)$

is an immediate predecessor of j .

b) Let j be a predecessor of q_i, $i \in I$. Then there exists $Y_i \subseteq W_i$ such that $q_i = \Sigma(Y_i, j)$.

If $q_{k,l} = y_n^i \in Y_i$, $i \in I$, $\{A_n^i ; n \in \omega\}$ is a partition of ω such that $A_n^i \in y_n^i$, we define $Y_i^* = \bigcup_{n \in \omega}\{q_{k+1,t} ; A_n^i \in q_{k+1,t}$ & $q_{k,l} = y_n^i\}$.

Clearly, $q_i \in Y_i \cap (\overline{Y_i^*} - Y_i^*)$, i.e. $j = \Omega(Y_i, q_i) < \Omega(Y_i^*, q_i)$.

Using Lemma 2.3 we see that

$$\Omega(Y_i^*, q_i) \approx \Sigma(J, j), \qquad J = \{j_n ; j_n \approx p\}.$$

Therefore, $s = \Sigma(J, j)$ is a common predecessor of q_i and s is greater than j .

q.e.d.

§ 3. Proof of THEOREM. The assertion of THEOREM follows immediately from the following

Theorem 3.1. For any minimal ultrafilter p there exists a well stratified set of ultrafilters with uniform predecessor p .

We prove this theorem directly by a construction of a well stratified set. Because, the transfinite induction is quite similar to that in the construction of Simon point [2], we give a sketch of the proof, only. More precisely, we describe the first step of the induction. The rest is the same as in the above mentioned construction.

We need the following simple lemma.

Lemma 3.2. Let p be an ultrafilter and $\{A_n ; n \in \omega\}$ be a partition of ω . Let x_n be any ultrafilter extending the filter $\mathcal{F}_n = (F \cup \{A_n\})$, $n \in \omega$ and let q be any ultrafilter extending the filter $\mathcal{G} = (F \cup \{\bigcup_{k \in A} A_k ; A \in p\})$.

If $q \in \overline{X}$, where $X = \{x_n ; n \in \omega\}$, then $\Omega(X, q) = p$, i.e. $\Sigma(X, p) = q$.

The construction. The main difference between this construction and the construction of a Simon point ([2] - Prop. 2.1), is to guarantee that $\Omega(X_{n+1}, q_{n,m}) \approx p$. This can be obtained by Lemma 3.2 and the first step of the induction.

Let $\{A_{\eta k}^{\xi} ; k \in \omega, \eta, \xi \in 2^{\omega}\}$ be ILF w.r.t. F and $\{C_i ; i \in \omega\}$ be a partition of ω into infinite sets. For each ξ , the system $\{A_{\eta 1}^{\xi} ; \eta < 2^{\omega}\}$ is almost disjoint.

Set $B_{1,m} = A_{m1}^{1} - \bigcup_{j<m} A_{j1}^{1}$.

Suppose $B_{n,m}$ is defined for each $m < \omega$.

Set $B_{n+1,m} = B_{n,l} \cap (A_{m1}^{n+1} - \bigcup_{j<m} A_{j1}^{n+1})$ iff $m \in C_l$.

For each $n \in \omega$ the system $\{B_{n,m} ; m \in \omega\}$ is pairwise disjoint.

Let p_i be an ultrafilter of the same type as p and $C_i \in p_i$.

Let $\mathcal{F}^0_{n,m}$ be a filter generated by
$F \cup \{B_{n,m}\} \cup \{\bigcup_{\ell \in A} B_{n+1,\ell} ; A \in p_m\}$ for each $n, m \in \omega$ and $I_0 = 2^\omega - \omega$.

The set $\{A^\xi_{\eta k} ; \xi \in I_0, \eta < 2^\omega, k < \omega\}$ is ILF w.r.t. $\mathcal{F}^0_{n,m}$ for all $n, m \in \omega$.

q.e.d.

One can easily check that by the Kunen's method used in the proof of Theorem 0.1 in [4] we obtain $2^{2^{\aleph_0}}$ distinct stratified sets with the uniform predecessor p.

$\{W_\alpha ; \alpha \in 2^{2^\omega}\}$ are "distinct" well stratified sets whenever for each $W_\alpha, W_\beta, \alpha \neq \beta$ there exists a set A such that $A \in q^\alpha_{n,m}$ for each $n, m \in \omega$ and $\omega - A \in q^\beta_{n,m}$ for each $n, m \in \omega$.

By [2] all ultrafilters from the set W_α are Simon points and by Lemma 2.4 a) no predecessor of $q^\alpha_{n,m}$ can be a Simon point. Therefore, there exists a set Q of $2^{2^{\aleph_0}}$ incomparable Simon points with the common predecessor p and without the greatest common predecessor (by Lemma 2.4 b)). By the same argument no subset of Q has an infimum.

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